

# Simulation of Second Benchmark Model for Sub-Synchronous Resonance Studies Using Reduced-Order Observer Method

Javad Sadeh<sup>1</sup>, Iman Mohammad Hoseiny Naveh<sup>2</sup>

Abstract – In this paper a reduced-order observer controller is proposed to solve the subsynchronous resonance phenomenon in the second benchmark model. In the practical environment (real world), access to all of the state variables of system is limited and measuring all of them is also impossible. So when we have fewer sensors available than the number of states or it may be undesirable, expensive, or impossible to directly measure all of the states, using a reduced-order observer is proposed. A proposed design, which is presented in this paper, has been developed in order to over-come this problem. The proposed strategy is tested on second benchmark model and compared with the optimal full-state feedback method by means of simulation. It is shown that this method creates a suitable estimate with good accuracy for all of states that they are not accessible. The proposed method is applied to the IEEE Second Benchmark system for subsynchronous resonance studies and the results are verified based on comparison with those obtained from digital computer simulation by MATLAB. Copyright © 2009 Praise Worthy Prize S.r.l. - All rights reserved.

**Keywords**: Sub-synchronous resonance, Sub-synchronous oscillations, Reduced-Order Observer, Optimal Full State Feedback

### Nomenclature

|  | $\Delta X_{\text{Gen}}$         | State Vector for Generator System Model     |
|--|---------------------------------|---|
|  | $A_G$                           | State Matrix for Generator System Model     |
|  | $\mathbf{B}_{\mathbf{G}_{i}}$   | ith Input Matrix for Generator System Model |
|  | $\Delta y_{Gen}$                | Output Vector for Generator System Model    |
|  | $\Delta U_{Gen}$                | Input Vector for Generator System Model     |
|  | $\Delta i_{fd}$                 | Variation of Field Winding Current          |
|  | $\Delta i_d, \Delta i_q$        | Variation of Stator Currents in the d-q     |
|  |                                 | Reference Frame                             |
|  | $\Delta i_{kd},\!\Delta i_{kq}$ | Variation of Damping Winding Current in     |
|  |                                 | the d-q RF                                  |
|  | $\Delta\delta_{ m g}$           | Variation of Generator Angle                |
|  | $\Delta \omega_{\mathbf{g}}$    | Variation of Angular Velocity of Generator  |
|  | $\Delta V_{o}$                  | Variation of Infinitive Bus Voltage         |
|  | ΔΕ                              | Variation of Field Voltage                  |
|  | $\Delta X_{Mech}$               | State Vector for Mechanical System Model    |
|  | $A_{M}$                         | State Matrix for Mechanical System Model    |
|  | $B_{\text{M}\text{i}}$          | ith Input Matrix for Mechanical System      |
|  |                                 | Model                                       |
|  | $\Delta y_{Mech}$               | Output Vector for Mechanical System Model   |
|  | $\Delta U_{Mech}$               | Input Vector for Mechanical System Model    |
|  | $\Delta X_{Line}$               | State Vector for Transmission Line System   |
|  | $A_{Line}$                      | State Matrix for Transmission Line System   |
|  | $\mathbf{B}_{Line}$             | Input Matrix for Transmission Line System   |
|  | $\Delta U_{Line}$               | Input Vector for Transmission Line System   |
|  | $\Delta V_{ref}$                | Variation of Refrence Voltage               |
|  | $\Delta X_{Total}$              | State Vector for Overall System Model       |
|  | $A_{Total}$                     | State Matrix for Overall System Model       |
|  | $B_{Total}$                     | Input Matrix for Overall System Model       |
|  |                                 |   |

| $C_{Total}$ $\Delta y_{Total}$ $\Delta X_{Obs}$ $\Delta X_{Base}$ $\Delta z$ | Output Matrix for Overall System Model Output Vector for Overall System Model Unmeasurable State Variables Measurable State Variables State Vector of Reduced Order Observer |
|--|--|
|  | State Vector of Reduced Order Observer<br>The Estimated State Vector   |

# I. Introduction

Fixed capacitors have long been used to increase the steady state power transfer capabilities of transmission lines. A major concern associated with fixed series capacitor is the sub-synchronous resonance (SSR) phenomenon which arise as a result of the interaction between the compensated transmission line and turbinegenerator shaft. This results in excessively high oscillatory torque on machine shaft causing their fatigue and damage. The occurrence of several incidents in different countries during the seventies and the eighties promoted investigations into the cause of turbinegenerator torsional excitation and the effect of the stimulated oscillations on the machine shaft. The best known incidents are the two shaft failures that occurred in the Mohave station in Nevada in 1970 and 1971, which were caused by sub-synchronous resonance [1]-[3]. These failures were caused by sub-synchronous oscillations due to the SSR between the turbine-generator shaft system and the series compensated transmission network. These incidents and others captured the attention of the industry at large and stimulated greater

Input Vector for Overall System Model

 $\Delta U_{Total}$ 

interest in the interaction between power plants and electric systems [4]-[6].

They also fostered the development of advanced analytical methods and sophisticated computer programs to simulate this interaction under transient conditions. In most investigations, idealized assumptions have been used in digital computer simulations for SSR studies. So these simulations distance obtained results from practical environment. This problem can create vivid differences between simulated and real environment. It occurs in simulations in order to exist many unmeasurable states in real world.

In this paper, a reduced order observer is proposed for estimation of unavailable states. The theory of linear observers and observability is a well researched area in control power systems with vast applications in other area as well. The proposed method is applied to the IEEE Second Benchmark system for sub-synchronous resonance studies and the results are verified based on comparison with those obtained from digital computer simulation by MATLAB. Analysis reveals that the proposed technique gives good results. It can be concluded that the application of reduced-order observer controller to mitigate SSR in power system will be provided a practical viewpoint. Also this method can be used in a large power system as a local estimator.

# II. System Model

The system under study is shown in Fig. 1. This is the IEEE Second benchmark model, with a fixed series capacitor connected to it. This system is adopted to explain and demonstrate applications of the proposed method for investigation of the single-machine torsional oscillations. The system includes a T-G unit which is connected through a radial series compensated line to an infinite bus. The rotating mechanical system of the T-G set is composed of two turbine sections, the generator rotor and a rotating exciter. The system parameters are provided in [7]-[9].

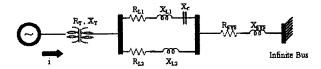


Fig. 1. Schematic diagram of the IEEE Second Benchmark System

#### II.1. Electrical System

Using direct, quadrature (d-q axes) and Park's transformation, the complete mathematical model that describes the dynamics of the synchronous generator system can be found in [10]:

$$\Delta \dot{X}_{Gen} = A_G \Delta X_{Gen} + B_{G1} \Delta U_1 + B_{G2} \Delta U_2 + B_{G3} \Delta U_3 + B_{G4} \Delta U_4$$
 (1)

$$\Delta y_{Gen} = C_G \Delta X_{Gen} \tag{2}$$

It is be noted that  $C_G$  is an identity matrix. The following state variables and input parameters are used in (1):

$$\Delta X_{Gen}^{T} = \begin{bmatrix} \Delta i_{fd} & \Delta i_{d} & \Delta i_{kd} & \Delta i_{q} & \Delta i_{kq} \end{bmatrix}$$
 (3)

$$\Delta U_{Gen}^{T} = \begin{bmatrix} \Delta V_{O} & \Delta \delta_{g} & \Delta \omega_{g} & \Delta E \end{bmatrix}$$
 (4)

Where,  $\Delta V_O$  is variation of infinitive bus voltage. In addition to the synchronous generator, the system also contains the compensated transmission line.

The linearized model of transmission line is given by:

$$\Delta \dot{X}_{Line} = A_{Line} \Delta X_{Line} + B_{Line} \Delta U_{Line} \tag{5}$$

$$\Delta X_{Line}^{T} = \begin{bmatrix} \Delta V_{Cd} & \Delta V_{Cq} \end{bmatrix}$$
 (6)

$$\Delta U_{Line}^{T} = \begin{bmatrix} \Delta i_d & \Delta i_q \end{bmatrix} \tag{7}$$

To obtain the electrical system, we can combine Eq. (1) – Eq. (7) Finally we can illustrate electrical system by below equations:

$$\Delta \dot{X}_{El} = A_{El} \Delta X_{El} + B_{El} \Delta U_{El} \tag{8}$$

$$\Delta X_{El}^{T} = \left[ \Delta X_{Gen}^{T} \quad \Delta X_{Line}^{T} \right] \tag{9}$$

$$\Delta U_{El}^{T} = \left[ \Delta U_{Gen}^{T} \right] \tag{10}$$

### II.2. Mechanical System

The shaft system of the T-G set is represented by four rigid masses. The linearized model of the shaft system, based on a mass-spring-damping model is:

$$\Delta \dot{X}_{Mech} = A_M \Delta X_{Mech} + B_{M1} \Delta U_{M1} + B_{M2} \Delta U_{M2}$$
 (11)

and eq. (12):

$$\Delta U_{Mech}^{T} = \begin{bmatrix} \Delta T_m & \Delta T_e \end{bmatrix} \tag{13}$$

All parameters are available at [7]-[9].

# II.3. Combined Power System Model

The combined power system model is obtained by combining the linearized equations of the electrical

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system and mechanical system. Let us define a state vector as:

$$\Delta X_{Sys}^{T} = [\Delta X_{El}^{T} \Delta X_{Mech}^{T}]$$
 (14)

So we can write:

$$\Delta \dot{X}_{Sys} = A_{Sys} \Delta X_{Sys} + B_{Sys} \Delta U_{Sys}$$
 (15)

$$\Delta U_{Svs}^{T} = \begin{bmatrix} \Delta T_m & \Delta E \end{bmatrix} \tag{16}$$

# III. Controller Design

#### III.1. Linear Optimal Control

Optimal control must be employed in order to damp out the subsynchronous oscillations resulting from the negatively damped mode. For the linear system the control signal U which minimizes the performance index:

$$J = \int \left[ \Delta x_{Sys}^{T}(t) Q \Delta x_{Sys}(t) + \Delta u_{Sys}^{T} R_{\mu} \Delta u_{Sys}(t) \right] dt$$
(17)

is given by the feedback control law in terms of system states:

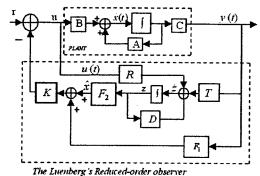


Fig. 2. Shematic diagram of state feedback using the Luenberger's reduced-order observer

$$U(t) = -K\Delta x_{Sys}(t)$$
 (18)

Where P is the solution of Riccati equation:

$$A_{Sys}^{T}.P + P.A_{Sys} - P.B_{Sys}R_{\mu}^{-1}B_{Sys}^{T}.P + Q = 0$$
 (19)

# III.2. Reduced-Order Observer

In this paper, in order to have a complete research, optimal full state feedback control is designed and the results are compared with reduced-order method. Some parameters, such as  $\Delta i_{kd}$  and  $\Delta i_{kq}$ , are not physical variables.  $\Delta V_{Cd}$  and  $\Delta V_{Cq}$  are transmission line parameters that they are not accessible. So let us define:

$$\Delta \hat{X}_{Sys}^{T} = \begin{bmatrix} \Delta i_{kd} & \Delta i_{kq} & \Delta V_{Cd} & \Delta V_{Cq} \end{bmatrix}$$
 (20)

$$\Delta y_{Sys}^{T} = \begin{bmatrix} \Delta T_{EXC-GEN} & \Delta T_{GEN-LP} & \Delta T_{LP-HP} \end{bmatrix}$$
 (21)

Where  $\Delta y_{\mathrm{Sys}}^{\mathrm{T}}$  is used for obtaining variation of torque of the rotating mechanical system of the T-G set [8]. Full order observer estimates all the states in a system, regardless whether they are measurable or unmeasurable. When some of the state variables are measurable using a reduced-order observer is so better.

The Luenberger reduced-order observer is used as a linear observer in this paper. The block diagram of this reduced-order observer is shown in Fig. 2.

For the controllable and observable system that is defined by Eq. (15), there is an observer structure with size of (n-1). The size of state vector is n and output vector is l. The dynamic system of Luenberger reduced-order observer with state vector of z(t), is given by:

$$\Delta z(t) = L \Delta x_{Sys}(t) \tag{22}$$

$$\dot{z}(t) = Dz(t) + Ty_{Sys}(t) + Ru_{Sys}(t)$$
 (23)

To determine L, T and R is basic goal in reduced-order observer. In this method, the estimated state vector  $\Delta \hat{X}_{Sys}(t)$  include two parts. First one will obtain by measuring  $\Delta y_{Sys}(t)$  and the other one will obtain by estimating  $\Delta z(t)$  from (22).

We can take:

$$\begin{bmatrix} \Delta y_{Sys}(t) \\ \Delta z(t) \end{bmatrix} = \begin{bmatrix} C_{Sys} \\ L \end{bmatrix} \Delta \hat{X}_{Sys}(t)$$
 (24)

By assumption full rank  $\begin{bmatrix} C_{Sys}^T & L^T \end{bmatrix}^T$ , we can get:

$$\Delta \hat{X}_{Sys}(t) = \begin{bmatrix} C_{Sys} \\ L \end{bmatrix}^{-1} \begin{bmatrix} \Delta y_{Sys}(t) \\ \Delta z(t) \end{bmatrix}$$
 (25)

By definition:

$$\begin{bmatrix} C_{Sys} \\ L \end{bmatrix}^{-1} = \begin{bmatrix} F_1 & F_2 \end{bmatrix}$$
 (26)

We get:

$$\Delta \hat{X}_{Sys}(t) = F_1.\Delta y_{Sys}(t) + F_2.\Delta z(t)$$
 (27)

Where:

$$F_1 C_{Svs} + F_2 L = I_n \tag{28}$$

Using estimated state variables, the state feedback control law is given by:

$$\Delta U_{Sys}(t) = -K\Delta \hat{X}_{Sys}(t)$$

$$= KF_1 C_{Sys} \cdot \Delta X_{Sys}(t) - KF_2 \Delta z(t)$$
(29)

By assumption  $R=L.B_{Sys}$  in (23), descriptive equations of closed loop control system with reduced-order observer are:

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$$\begin{bmatrix} \Delta \dot{X}_{Sys}(t) \\ \Delta \dot{z}(t) \end{bmatrix} = \begin{bmatrix} A_{Sys} - B_{Sys}F_1C_{Sys} & -B_{Sys}KF_2 \\ TC_{Sys} - LB_{Sys}KF_1C_{Sys} & D - LB_{Sys}KF_2 \end{bmatrix} \begin{bmatrix} \Delta X_{Sys}(t) \\ \Delta z(t) \end{bmatrix}$$
(30)

Dynamic error between linear combination of states of  $L.\Delta X_{Sysl}(t)$  system and observer  $\Delta z(t)$  is defined as:

$$\dot{e}(t) = \Delta \dot{z}(t) - L\Delta \dot{X}_{Sys}(t)$$
 (31)

Combine (36) and (37), we get eq. (32):

$$\begin{bmatrix} \Delta \dot{X}_{Sys}(t) \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A_{Sys} - B_{Sys}.K & -B_{Sys}KF_2 \\ 0 & D \end{bmatrix} \begin{bmatrix} \Delta X_{Sys}(t) \\ e \end{bmatrix}$$

For stability of the observer dynamic system, the eigenvalues of D must lie in the left hand-side of s plane. By choosing D, we can calculate L, T and R [11]-[15].

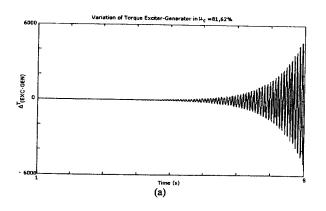
## IV. Simulation Results

Eigenvalue analysis is a fast and well-suited technique for defining behavioral trends in a system that can provide an immediate stability test. The real parts of the eigenvalue represent the damping mode of vibration, a positive value indicating instability, while the imaginary parts denote the damped natural frequency of oscillation. As mentioned earlier, the system considered here is the IEEE second benchmark model. It is assumed that the fixed capacitive reactance ( $X_C$ ) is 81.62% of the reactance of the transmission line ( $X_{LI}$ =0.48 P.u).

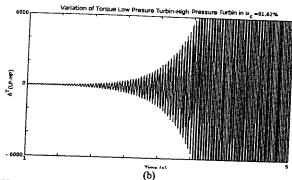
The simulation studies of IEEE-SBM carried out on MATLAB platform is discussed here. The following cases are considered for the analysis.

## IV.1. Without Controller

In the first Case, second benchmark model is simulated without any controller in initial conditions. Fig. 3 shows the variation of torque of the rotating mechanical system of the T-G set.



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Figs. 3. Variation of torque of exciter – generator (a) and low pressure – high pressure turbine (b) in the T-G set for  $\mu_c$ =81.62% without any controller

It can be seen that variations of torque of the mechanical system are severe unstable and then power system tends to approach to the SSR conditions.

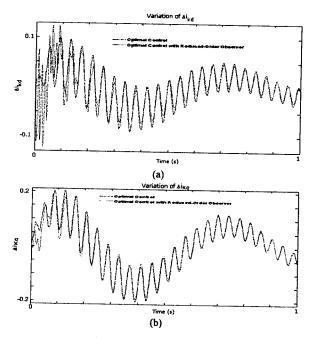
# IV.2. With Controller Design

In the second case, power system is simulated by linear optimal control with using eqs. (17)-(19). Proposed method is carried out on second benchmark model simultaneously. The obtained results have been illustrates in Fig. 4 and Fig. 5.

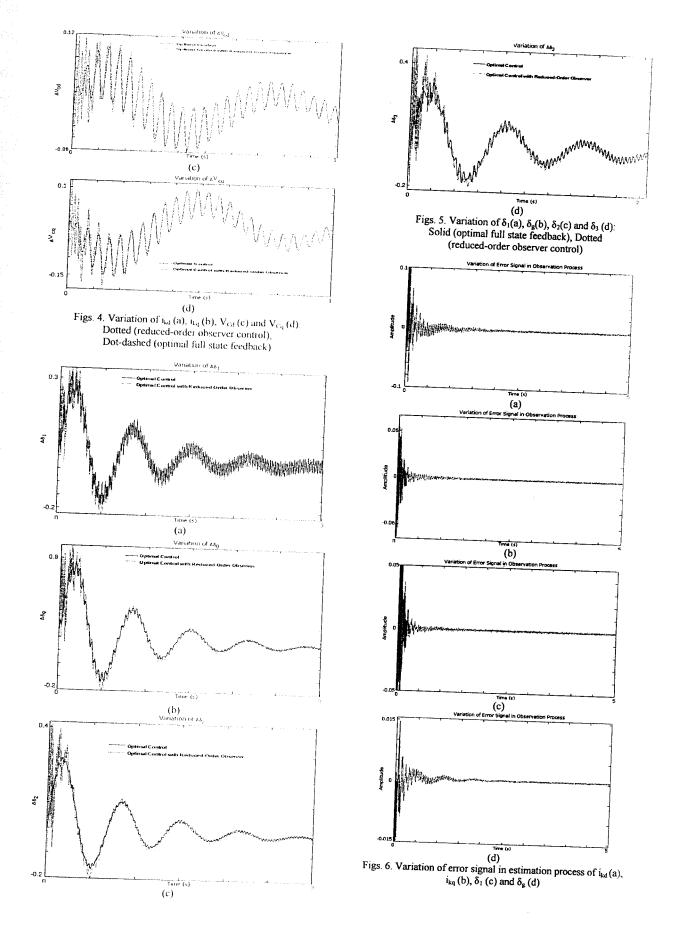
It is observed that the reduced-order method has created a suitable estimation from unmeasurable variables that are introduced in Eq. (20).

Fig. 6 clearly shows amplitude of error signal in the estimation process by using proposed method. It illustrates high capability of this method to estimate unmeasurable variables.

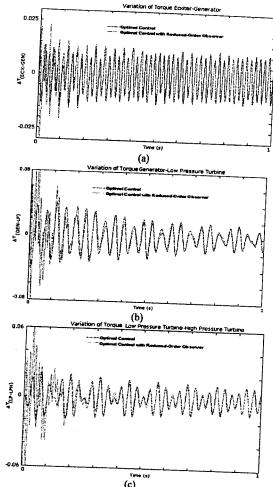
Fig. 7 shows variation of torque of the mechanical system in T-G set. It can be observed that the proposed method has small effect on the output of power system.



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Figs. 7. Variation of torque of exciter – generator (a) and generator – low pressure turbine (b), low pressure – high pressure turbine (c) in the T-G set: Dotted (reduced-order observer control),

Dot-dashed (optimal full state feedback)

# V. Conclusion

In the practical environment (real world), access to all of the state variables of system is limited and measuring all of them is also impossible. So when we have fewer sensors available than the number of states or it may be undesirable, expensive, or impossible to directly measure all of the states, using a reduced-order observer is proposed. Therefore in this paper, a novel approach is introduced by using optimal state feedback, based on the Reduced – order observer structure. Analysis reveals that the proposed technique gives good results. It can be concluded that the application of reduced-order observer controller to mitigate SSR in power system will be provided a practical viewpoint. Also this method can be used in a large power system as a local estimator.

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