Optimal Path Planning for Controllability of Switched Linear Systems Using Multi-level Constrained GA

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Abstract. In this paper, optimal switching signal as well as control input is designed for a general switched linear system using multi-level constrained genetic algorithms (MLCGA). Given any two states in the controllable subspace, the proposed approach automatically finds optimal switching signal and control input which steer the system from initial state to final state in a desired feasible time. From optimization perspective, this problem has several linear constraints such as controllability condition and desired dwell time as well as desired final time. Also, the problem is mixed-variable when the switching indices must be integer. The objective function may be nonlinear, multi-modal and non-analytical. Generally the problem must be solved in two levels. At the bottom level, an optimal control input is found for a candidate switching signal and at the top level an optimizer searches for optimal switching signal. To solve this complex problem, we propose Multi-Level Constrained Genetic Algorithms (MLCGA) which can solve this problem efficiently. As it is demonstrated by a simulation example, using the proposed approach an optimal switching signal with desired dwell time as well as optimal control input in the presence of actuator saturation can be efficiently designed.

1 Introduction

"Hybrid systems" is one of the most recent and hot topics in systems and control theory which attracts researchers of both theoretical and practical domains. A hybrid system includes two distinct types of components, subsystems with continuous dynamics and subsystems with discrete event dynamics that interact with each other. From the perspective of systems and control theory, a hybrid system is considered as continuous systems with switching and a greater emphasis is placed on properties of the continuous state [1]. These systems are called switched systems and stability analysis and control synthesis are the main topics of switched systems theory.

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Generally, a switched dynamical system is composed of a family of subsystems and a rule that governs a switching among them (Figure 1). As a result, besides subsystems a switched system also consists of a switching device usually called supervisor [1]. The supervisor produces the switching rule (also called switching signal or switching law) which orchestrates the switching among the subsystems. The dynamics of the switched system is determined by both the subsystems and the switching signal. Switched linear system is a type of switched system when the subsystems are linear (often linear time invariant). Switched linear system is an important and applied field of switched system that large amount of theorems are available to analyze their stability, controllability and etc. One of the most basic concepts in control theory and applications is controllability. There are various definitions for controllability including local and global. Controllability of a switched system is more complicated than an ordinary system with single dynamics. Controllability plays an important role in control of a system. Although several theorems are available for controllability of a switched linear system, these theorems just find reachable and controllable sets. Path planning for controllability is a valuable concept which has been briefly introduced in [1]. Path planning for controllability means finding a feasible switching signal as well as feasible control input which steer the system from desired initial state to desired final state in desired finite time. With respect to the practical domain, there exist several objectives to reach the desired performance. As a result, optimal path planning is hardly required. As it is discussed in part 2, from optimization perspective, optimal path planning for controllability of a general switched linear system is a constrained problem. There are two types of constraints including constraints on switching signal and constraints on controllability input. Fortunately, both of these types of constraints are linear. Although the decision variables of control input design is often continuous, optimal switching signal design often includes mixed (both continuous and discrete) decision variables. Regarding the desired performance in a practical control system design, objective function may be nonlinear, multi-modal and non-analytical. Furthermore, the design process includes two levels. At the bottom level, an optimal control input is found for a candidate switching signal and at the top level an optimizer searches for optimal switching signals. In this paper, we propose a general framework based on multi-level constrained genetic algorithms (MLCGA) to solve this complex optimization problem.

In continue, several previous works by others are briefly considered. Ge et al. (2001) [2], Hihi et al. (2007) [3] and Zhijian Ji et al. (2008) [4] considered theoretical aspects of controllability of switched linear systems. Zongbo Hu et al. (2004) [5] discussed output controllability of switched power converters as switched linear systems. Xie et al. (2004) [6] considered the controllability of periodically switched linear systems with saturating actuators as well as Liang Lv et al. (2007) [7] which designed switched linear system under actuator saturation. Li et al. (2004) [8] designed an optimal switching law for switched linear systems based on convergence route. Dong Chen et al. (2004) [9] used dynamic programming as well as evolutionary computing to optimally control the switched systems. Zhijian Ji (2006) [10] determined the number of switching as well as designing switching sequences for controllability of switched linear systems.

In comparison with the previous works, in this paper we propose a general framework to design optimal switching signal as well as optimal control input for controllability of switched linear systems. Using the proposed approach all practical constraints and objectives including dwell time, actuator saturation and desired final time as well as any nonlinear and non-analytical objective functions can be efficiently considered. In continue we first introduce the mathematical preliminaries of path planning for controllability of a switched linear system in part 2. Next, in part 3, we generally formulate the problem of path planning for controllability. Then we proposed MLCGA to solve this problem. At part 4, a simulation example is designed using the proposed approach to demonstrate the efficiency of this framework.

2 Path Planning for Controllability of Switched Linear Systems

2.1 General Switched Linear System

A general non-autonomous switched linear system is shown in Figure 1. As figure indicates, a supervisor produces switching signal which switches the subsystems. Generally the subsystems are non-autonomous LTI systems. There are several types of switching signals. Time-driven switching law as well as state/output feedback switching law is the commonest one. The path planning procedure in the next subsection uses time-driven switching law. Regarding real world systems, the subsystems can not be switched very quickly. As a matter of fact for any application a time τ exists which for any two consecutive switching times $t_{k+1} - t_k \succ \tau$ must be satisfied. This critical time is called dwell time. Using the proposed approach in this paper, an optimal switching signal is designed with desired dwell time. In the next subsection, without considering the controllability theorems we directly deal with the path planning problem. Readers can refer to [1] to know more details about the controllability of switched linear systems.

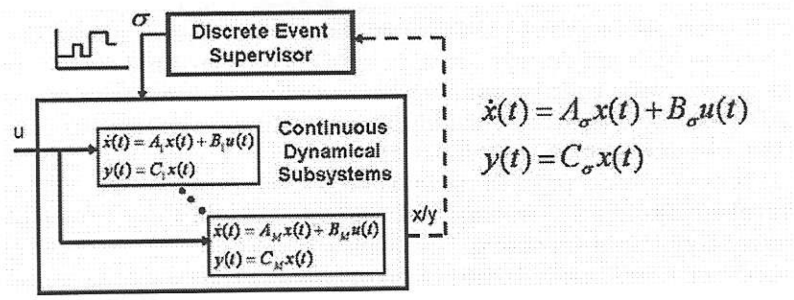

Fig. 1 Switched linear system (left), State space representation (right)

2.2 Path Planning for Controllability

According to [1], given any two states x_0 and x_f in the controllable subspace, path planning means finding feasible switching path σ and control input u to steer the system from x_0 to x_f in a finite time. Using the theorems of controllability of linear switched system, [1] has formulated a procedure for this path planning. Without dealing with the details, we directly introduce the procedure here.

Suppose $t_0 = 0$. According to [1], we can find a natural number L (the number of switching), positive real numbers $h_0,...,h_L$ (switching durations), and an index sequence $i_0,...,i_L$ $\; i_k \in \{1,2,...,M\}$ such that the controllability set requirements are satisfied. It is clear that $t_k = t_{k-1} + h_{k-1} \quad k = 1,...,L+1$. Regarding [1], consider the piecewise continuous control strategy given by $u = B_{i_k}^T e^{A_{i_k}^T (t_{k+1}-t)} a_{k+1}$, $t_k \le t \prec t_{k+1}$, $k = 0,...,L+1$ where $a_k \in R^n$, $k = 1,...,L+1$ is a constant vector variables to be determined. Combining the solution of a switched linear system with this control strategy, the following system of equation is achieved as a linear constraint on a:

$$x_f - e^{A_{i_L} h_{i_L}} ... e^{A_{i_0} h_{i_0}} x_0 = [e^{A_{i_L} h_{i_L}} ... e^{A_{i_1} h_{i_1}} W_{h_0}^{i_0}, ..., W_{h_1}^{i_1}]a \tag{1}$$

where $a = [a_1^T, ..., a_{L+1}^T]^T$ and $W_t^k = \int_0^t e^{A_k(t-\tau)} B_k B_k^T u(\tau) e^{A_k^T (t-\tau)} d\tau$. It is clear that if system (1) has feasible solution(s) then the designed switching signal and control strategy successfully realize the given controllability problem.

3 Proposed Approach

3.1 Constrained Genetic Algorithms

GA is one of the most efficient derivative-free and global optimizers that has been successfully applied in many applications. In an ordinary use of GA, the problem is unconstrained and only the decision variables (genes) are bounded. For this case, several modified versions of GA have been proposed. But if the optimization problem is constrained, the ordinary crossover and mutation operators can not be used and handling these constraints may be difficult. Generally constrained GA is a challenging problem and it is still open for further investigations. As mentioned in [11-13], there are two major approaches for handling constraints in GA: A. Direct constraint handling which contains four methods: 1.eliminating infeasible candidates, 2. repairing infeasible candidates, 3. preserving feasibility by special operators, 4. decoding such as transforming the search space. Direct constraint handling has two advantages. It might perform very well, and might naturally accommodate existing heuristics. Regarding the technique of direct constraint handling is usually problem dependent, designing a method for a given problem may be difficult and using a given method might be computationally expensive. B. Indirect constraint handling: Indirect constraint handling incorporates constraints into a fitness function. Advantages of indirect constraint handling are: generality (problem independent penalty functions), reducing problem to 'simple' optimization, and allowing user preferences by selecting suitable weights. One of disadvantages of indirect constraint handling is loss of information by packing everything in a single number and generally in case of constrained optimization, it is reported to be weak. Generally, if suitable operators can be found for direct handling, undoubtedly direct search in the feasible space is much more efficient and faster than indirect type.

The genetic algorithms toolbox of the recent versions of MATLAB software (2007 and later) efficiently handles linear and nonlinear constraints. We recommend readers to use this toolbox especially for linear constraints since the toolbox handles linear constraints based on direct constraint handling and as a result, the population is kept feasible through generations. As direct constraint handling is faster and more effective than indirect type, we applied this toolbox in the simulation example at part 4.

Table 1 The commonest constraints of two levels

	Constraints on switching signal (Top level)
Dwell time	$h_k \geq dwell, \quad k = 0 : L$
Maximum time to reach final state	$\sum_{k=0}^{L} h_k \leq T_{\max}$
Constraint on the order of index sequence	*For example subsystem 2 can be selected only after subsystem 1, etc.*
	Constraints on control input (Bottom level)
Constraint on vector a	$x_f - e^{A_{i_L} h_{i_L}} \ldots e^{A_{i_0} h_{i_0}} x_0 = [e^{A_{i_L} h_{i_L}} \ldots e^{A_{i_1} h_{i_1}} W_{h_0}^{i_0}, \ldots, W_{h_1}^{i_1}]a$
Constraint on control input amplitude	$u_{\min} \leq u = B_{i_k}^T e^{A_{i_k}^T (t_{k+1} - t)} a_{k+1} \leq u_{\max}$

3.2 Problem Formulation

According section 2, the decision variables as well as constraints can be categorized in two classes including switching signal design as the top level and control input design as the bottom level. Several constraints can be considered for switching signal design including desired dwell time, allowed maximum time to reach the final state, order of switching and etc. The switching indices are discrete variables when the switching durations are continuous. Totally in the top level, $2 \times (L+1)$ decision variables are available including switching indices i_k and switching durations h_k. At the bottom level, as equation (1) shows, a is an $n \times (L+1)$ dimensional vector of decision variables with a linear equality constraint. Note that if u is constrained (such as actuator saturation), this can be simply handled in design process as a linear constraint on a (Table 1). The objective function can be generally formulated as a nonlinear (analytical or non-analytical) function of decision variables. In this paper, although we just consider single objective control design, note that the proposed approach is flexible enough to be applied for multi-objective design. Table 1 shows the details in mathematical form. Note using a simple rounding,

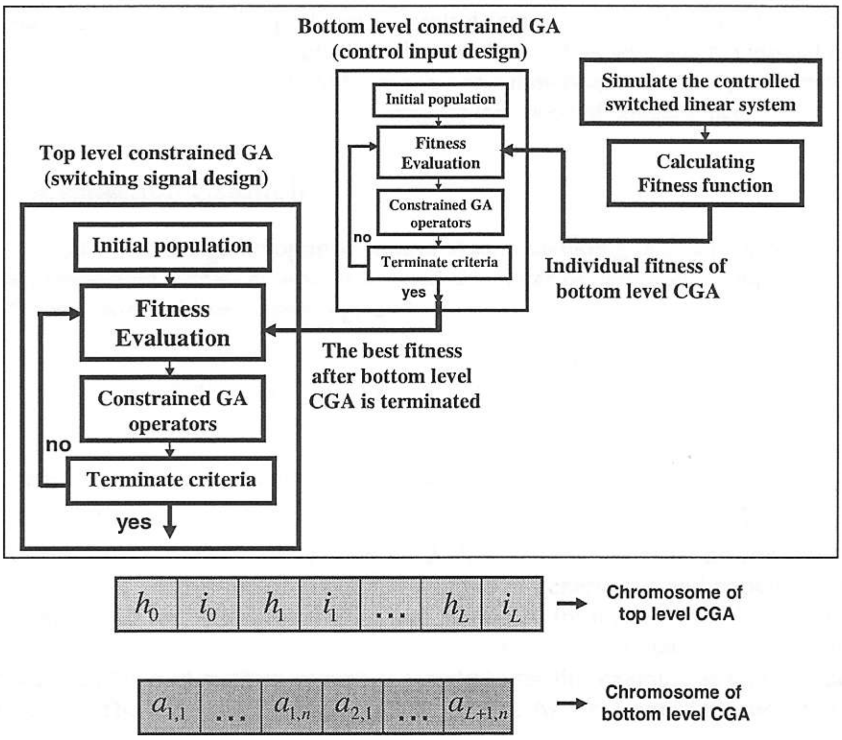

Fig. 2 Proposed MLCGA (top), Chromosomes architecture (bottom)

switching indices can be supposed continuous through optimization and converted to integer just for fitness evaluation.

3.3 Path Planning for Controllability Using MLCGA

Regarding the problem formulation in the previous subsection, this problem must be solved in two levels. From optimization perspective, the problem of each level is constrained and may be nonlinear and multi-modal as well as non-analytical. In these reasons, we propose multi-level constrained genetic algorithms to solve this problem. The top level CGA searches in the feasible space of switching signals. To evaluate the fitness of any chromosome of top level CGA, optimal control input must be found for this candidate switching signal. It means that for any fitness evaluation of top level CGA, a bottom level CGA must be completely run and terminate. Although several termination criterions are available, for the bottom level CGA, we recommend using pre-determined maximum number of generations. The main reason of this recommendation is ensuring a limited time for termination. If you apply another termination criterion, bottom level CGA may be terminated after a long time or even it may never be terminated. Figure 2 shows the MLCGA flowchart as well as chromosomes architecture. Depending on the objective function, if the problem of control input design is analytical, algebraic and single modal, one can replace the bottom level CGA with a conventional nonlinear programming method. Although this recommendation can be performed for specific problems, it can effectively increase the speed of convergence.

4 Simulation Example

In this part, we design an optimal control system including optimal supervisor design (switching signal) as well as optimal controller design (control input) for the 3^{rd} order switched linear system given by

$$A_1 = 0, B_1 = e_1, A_j = e_j e_{j-1}^T, B_j = 0, j = 2,3 \quad x_0 = \begin{bmatrix} 4 \\ 5 \\ 4 \end{bmatrix} \quad x_f = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

where e_j is the unit column vector with the jth entry equal to one. According to Table 1, for this example, $L = 11$, $M = n = 3$, $T_{\max} = 30$, $dwell = 0.5$. The order of switched subsystems is restricted to $\{1, 2, 3, 1, 2, 3 \ldots\}$. There is no constraint on amplitude of control input. Maximum number of generations and population size are 50 and 10 for bottom level CGA and 30 and 20 for top level CGA. The objective is minimizing the value of the integral of 2-norm of state signal. We used Matlab 2007a (GA toolbox as well as simulink) for this example as represented in Figure 3. The following figures show the results. As the figures indicate, the final

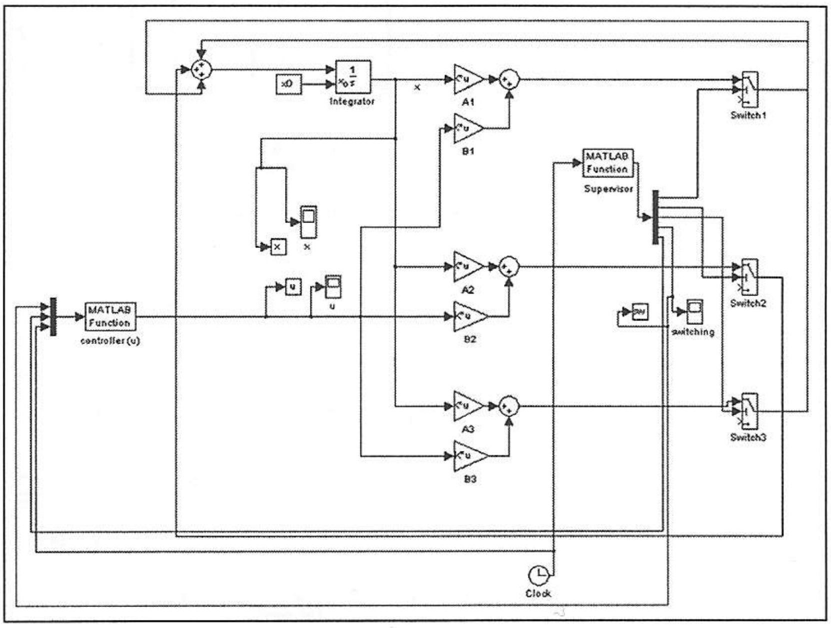

Fig. 3 Simulink file which is used for simulation example (Matlab 2007a)

time is very small and less than 8 seconds. The switching signal has a dwell time larger than 0.5. The 3d plot clearly shows the optimally designed path which achieved using the proposed method.

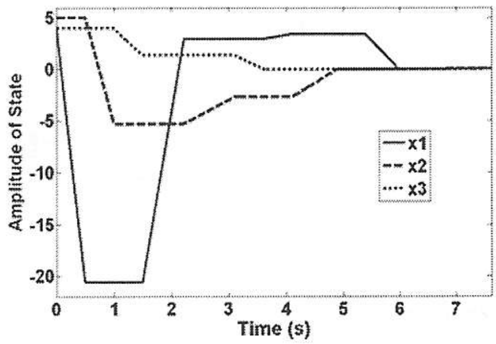

Fig. 4 State signals reach the final state in less than 8 seconds

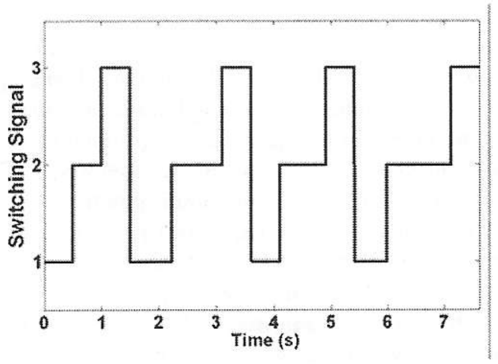

Fig. 5 Switching signal with dwell time larger than 0.5

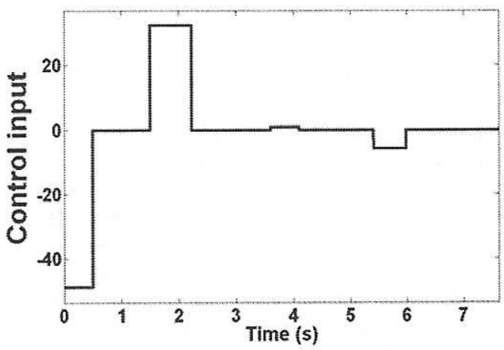

Fig. 6 Control input signal

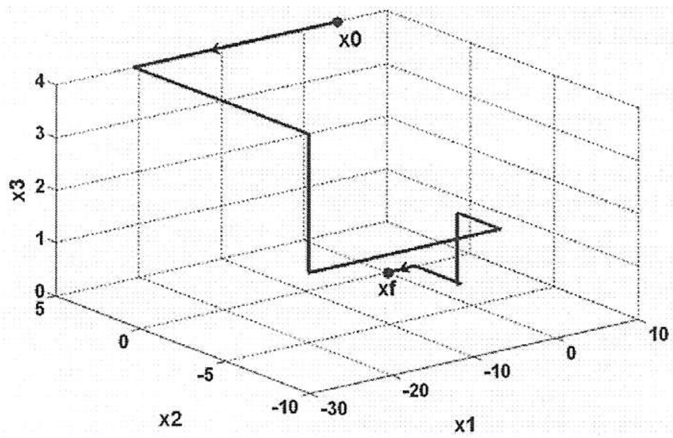

Fig. 7 Optimal path which was designed in the example for controllability of the switched linear system represented in equation (2)

5 Conclusion

In this paper, an optimal path including switching signal and control input is planned for controllability of a general switched linear system based on multi-level constrained genetic algorithms (MLCGA). The problem is solved in two levels. Switching signal design as top level and control input design as bottom level. The problem has several linear constraints such as controllability condition, control input amplitude (actuator saturation) and desired dwell time as well as desired final time. In this paper we formulate this problem as a general nonlinear constrained optimization problem that maybe multi-modal and non-analytical. As simulation example shows, using MLCGA an optimal path can be efficiently found with satisfying all constraints as well as desired performance. Generally the proposed approach can be successfully used for real world applications when the problem of path planning for controllability is complex.

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