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Adaptive fuzzy output feedback control for a class of uncertain nonlinear systems with unknown backlash-like hysteresis

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ABSTRACT

An output feedback controller is proposed for a class of uncertain nonlinear systems preceded by unknown backlash-like hysteresis, where the hysteresis is modeled by a differential equation. The unknown nonlinear functions are approximated by fuzzy systems based on universal approximation theorem, where both the premise and the consequent parts of the fuzzy rules are tuned with adaptive schemes. The proposed approach does not need the availability of the states, which is essential in practice, and uses an observer to estimate the states. An adaptive robust structure is used to cope with lumped uncertainties generated by state estimation error, approximation error of fuzzy systems and external disturbances. Due to its adaptive structure the bound of lumped uncertainties does not need to be known and at the same time the chattering is attenuated effectively. The strictly positive real (SPR) Lyapunov synthesis approach is used to guarantee asymptotic stability of the closed-loop system. In order to show the effectiveness of the proposed method simulation results are illustrated.

1. Introduction

The control of nonlinear systems preceded by hysteresis has been a challenging and yet rewarding problem. This is because hysteresis can be seen in a wide range of physical systems and devices [1]. On the other hand since the hysteresis nonlinearity is non-differentiable the system performance is severely limited and usually exhibits undesirable inaccuracies or oscillations and even instability [2].

To address such a challenge, it is important to find a model to describe the hysteresis nonlinearity and utilize this model for controller design. Various models have been proposed for hysteresis nonlinearity, among them, Ishlinskii hysteresis operator [3], Preisach model [4], Krasnosel'skii–Pokrovkii hysteron [3], Duhem hysteresis operator [5], backlash [6] and backlash-like hysteresis [7]. However, from modeling point of view an effective model should be simple enough to facilitate the design, yet complex enough to give the engineer confidence that the model-based designs will work in reality. Inspired by the recent papers and studies [1–12] it can be seen that the backlash-like hysteresis model is simple enough to facilitate the controller design, at the same time is complex enough to mitigate the effects of real hysteresis.

To cope with the drawbacks of hysteresis some adaptive schemes for nonlinear systems have been proposed when the backlash-like hysteresis is used [1,7–12]. The proposed adaptive control schemes assume that the input gain of the plant is constant and also the plant's states are available for measurement. In practical situations the states are fully or partially

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unknown because the states not either available for measurement or the sensors or transducers are very expensive to be used. Thus an observer should be designed to estimate the states. In this situation all the elements of the tracking error cannot be obtained, therefore the conventional adaptive methods cannot be applied. Also in many nonlinear models preceded by hysteresis the input gain is not constant and usually is a function of states and/or time.

There are also functional uncertainties in the models that should be considered. To address the functional uncertainty for the first time Wang in [13] proposed adaptive fuzzy systems where the consequent parts of fuzzy rules were assumed free and were tuned by adaptive laws, derived using Lyapunov method, which also guaranteed stability of the system. The method proposed by Wang [13] was developed and progressed by many researchers in this field for instance [14–18]. In all these methods, parameters of the premise parts of fuzzy rules had to be chosen appropriately, by the designer. However, it is very difficult in situations where the plant is subject to perturbations and unknown disturbances or when the dynamics of the systems are too complex. Recently, in [19–21,7] some novel methods for designing adaptive fuzzy control with both premise and the consequent parts of fuzzy rules have been proposed. However in these proposed methods, all the states of the system need to be available for measurement.

In this paper, a full adaptive fuzzy observer-based controller is proposed for a class of uncertain nonlinear systems preceded by unknown backlash-like hysteresis. The input gain is an unknown nonlinear function. The fuzzy systems based on universal approximation theorem are used to cope with nonlinear functions, where both the premise and the consequent parts of the fuzzy rules are tuned with adaptive schemes. The proposed approach does not need the availability of the states and uses an observer to estimate the states. The selection of the upper bound of lumped uncertainties generated by state estimation error, fuzzy approximation error and external disturbances has a significant effect on the control performance. However, due to uncertainties it is very difficult to know these upper bounds in advance for practical applications. For addressing the lumped uncertainties with unknown bounds a robust structure with adaptive gain is used. Due to its adaptive structure not only the bounds of lumped uncertainties does not need to be known but also since selecting the conservative gain is avoided the chattering is attenuated effectively. The asymptotic stability of the overall system is guaranteed via strictly positive real (SPR) Lyapunov theory.

This paper is organized as follows: Section 2 formulates the class of nonlinear systems under consideration here, describes assumptions, backlash-like hysteresis and fuzzy systems. In Section 3, the proposed observer based adaptive fuzzy controller is presented. The stability analysis of the proposed method is stated in Section 4. To show the effectiveness of the proposed method, in Section 5 it is applied to a model of inverted pendulum preceded by hysteresis as an uncertain nonlinear system amid significant disturbances. Simulation results indicate the effectiveness of the method in the presence of uncertainties, unknown states, disturbances and hysteresis. And finally, Section 6 concludes the main advantages of the proposed method.

2. Problem formulation and fuzzy basis function networks

2.1. Problem formulation

In this paper the control of an uncertain nonlinear system preceded by a backlash-like hysteresis is considered. The nonlinearities, external disturbances, states of the nonlinear system and hysteresis are unknown. It is a challenging task to develop an output feedback controller for uncertain nonlinear systems with unknown backlash-like hysteresis. The details of the nonlinear system and hysteresis will be presented as follows:

Consider a class of SISO n-th order nonlinear systems in the following form:

$$\begin{aligned} x^{(n)} &= f(X,t) + \bar{g}(X,t)\phi(v) + \bar{d}(X,t) \\ y &= x, \end{aligned} \tag{1}$$

where f and $\bar{g}$ are unknown bounded nonlinear functions, $X^T = [x, \dot{x}, \ldots, x^{(n-1)}] = [x_1, x_2, \ldots, x_n] \in U_x \subset \mathbb{R}^n$ is the state vector of the system where U_x is a compact set which is the controllability region of state vector, $y \in \mathbb{R}$ is the output of the system.

It should be noted that the states of the system are unknown and only the output is available for measurement. For controllability condition we should have $\bar{g}(X,t) \neq 0$ and here, without loss of generality, we assume $0 < \bar{g}_l < \bar{g}(X,t) < \infty$ where $\bar{g}_l$ is an unknown positive constant. $\bar{d}(X,t)$ is an unknown but bounded external disturbance, where this bound is also unknown, i.e., we have the following assumption:

Assumption 1. The disturbance $\bar{d}(X,t)$ is bounded by an unknown constant $\overline{D}$, i.e.,

$$|\bar{d}(X,t)| \leqslant \overline{D}.$$

$v \in \mathbb{R}$ is the control input and $\phi(v)$ denotes hysteresis type of nonlinearity described by

$$\frac{d\phi}{dt} = \alpha\left|\frac{dv}{dt}\right|(cv - \phi) + B_1\frac{dv}{dt}, \tag{2}$$

where α, c and B_1 are constants, satisfying $c > B_1$. The dynamic (2) can be used to model a class of backlash-like hysteresis as shown in Fig. 1. In Fig. 1 parameters $\alpha = 1$, $c = 3.1635$ and $B_1 = 0.345$, the input signal $v(t) = 6.5\sin(2.3t)$ and the initial

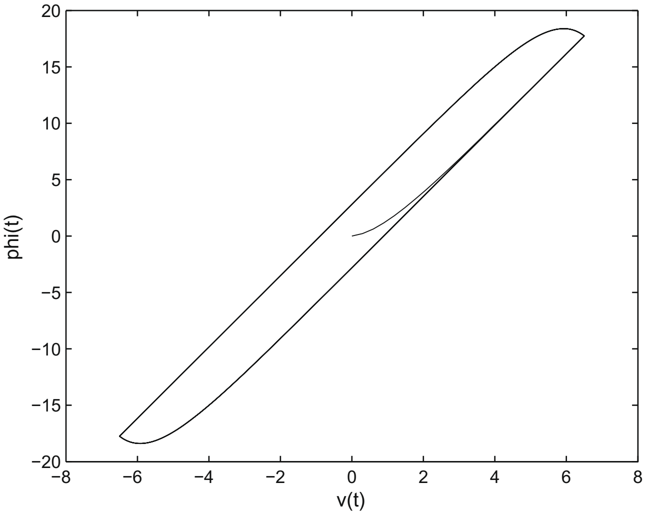

Fig. 1. Hysteresis curves given by (2) with $\alpha = 1,\ c = 3.1635,\ B_1 = 0.345$ and $v(t) = 6.5\sin(2.3t)$.

conditions $v(0) = 0$ and $\phi(0) = 0$ are chosen. In this paper the parameters of the hysteresis in (2), i.e., $\alpha,\ c$ and B_1 are completely unknown.

Control objective: The control objective is to design the output feedback controller v such that the state of the system X follows the desired state $X_d^T = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]$ in the presence of uncertainties and disturbances, that is the tracking error

$$E = X - X_d = [e, \dot{e}, \ldots, e^{(n-1)}]^T, \tag{3}$$

with $e = x - \hat{x}$ should converge to zero.

Assumption 2. The desired trajectory vector X_d is a known, continuous, differentiable and bounded function

$$\|X_d\| \leqslant b_d,$$

where b_d is a positive constant.

2.2. Fuzzy basis function networks and universal approximation theorem

The fuzzy logic systems (FLS) are briefly reviewed below for continuity of discussion [22–24]. FLS perform a mapping from $U_1 \times U_2 \times \cdots \times U_n = U \subset \mathbb{R}^n$ to a compact set $V \subset \mathbb{R}$. Any fuzzy system consists of a fuzzifier, a fuzzy rule base, a fuzzy inference engine and a defuzzifier. The fuzzy rule base consists of a collection of canonical fuzzy IF-THEN rules such as,

$$R^{(l)}: \text{ IF } x_1 \text{ is } F_1^l \text{ and } \cdots \text{ and } x_n \text{ is } F_n^l \text{ THEN } y \text{ is } G^l, \quad l = 1, \ldots, M, \tag{4}$$

where $X = [x_1, x_2, \ldots, x_n]^T \in U$ and $y \in V$ are the input and output of the fuzzy system, respectively; M is the total number of rules; F_i^l and G^l are fuzzy sets in U_i and V, respectively. The fuzzy inference engine performs a mapping from fuzzy sets in U to fuzzy sets in V, based on fuzzy rule base. Furthermore, the fuzzifier maps a crisp point in U to a fuzzy set in U and the defuzzifier maps fuzzy sets in V to a crisp point in V. Using singleton fuzzifier, product inference engine and center average defuzzifier, the output of fuzzy system can be expressed as,

$$y = \frac{\sum_{l=1}^{M} W_l(\prod_{i=1}^{n} \mu_{F_i^l}(x_i))}{\sum_{l=1}^{M}(\prod_{i=1}^{n} \mu_{F_i^l}(x_i))} = \xi^T(x)W, \tag{5}$$

where $W = [W_1, W_2, \ldots, W_M]^T \in \mathbb{R}^M$ is the center of output fuzzy membership functions, F_i^l and G^l are characterized by fuzzy membership functions $\mu_{F_i^l}(x_i)$ and $\mu_{G^l}(y)$, respectively, and $\xi(X) = [\xi_1(X), \xi_2(X), \ldots, \xi_M(X)]^T \in \mathbb{R}^M$ is the vector of fuzzy basis functions defined as below,

$$\xi_l(X) = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{M}(\prod_{i=1}^{n} \mu_{F_i^l}(x_i))}, \quad l = 1, \ldots, M. \tag{6}$$

Based on [22–24,19], since in this paper the linguistic fuzzy IF-THEN rules, are only used for the purpose of approximating the required functions, we define the defuzzifier as a weighted sum of each rule's output. Thus (6) can be written as (Fig. 2),

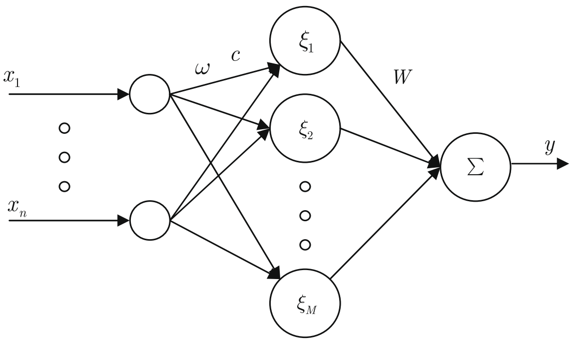

Fig. 2. FBFN structure.

$$\xi_l(X) = \prod_{i=1}^{n} \mu_{F_i^l}(x_i), \quad l = 1, \cdots, M. \tag{7}$$

The shape of each membership function $\mu_{F_i^l}(x_i)$ is chosen as the Gaussian function, i.e., $\mu_{F_i^l}(x_i) = e^{-\omega_l^{i2}(x_i - c_l^i)^2}$ where c_l^i and ω_l^i are the center and the inverse radius of the width of Gaussian membership function. Therefore, the architecture of this fuzzy system called fuzzy basis function network (FBFN) can be represented by a three-layer network with Gaussian functions as its activation functions in the hidden layer and weights W_l connecting hidden layer and output layer (Fig. 2). Thus, the output vector of FBFN can be expressed as

$$y(X, c, \omega, W) = \xi^T(X, c, \omega)W, \tag{8}$$

where $X = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$, $c = [c_1^T, c_2^T, \ldots, c_M^T]^T \in \mathbb{R}^{nM}$, $\omega = [\omega_1^T, \omega_2^T, \ldots, \omega_M^T]^T \in \mathbb{R}^{nM}$, $c_l = [c_l^1, c_l^2, \ldots, c_l^n]^T \in \mathbb{R}^n$, $\omega_l = [\omega_l^1, \omega_l^2, \ldots, \omega_l^n]^T \in \mathbb{R}^n$, $W = [W_1, W_2, \ldots, W_M]^T$, $\xi(X, c, \omega) = [\xi_1, \xi_2, \ldots, \xi_M]^T$, and $\xi_l = e^{-\sum_{i=1}^{n} \omega_l^{i2}(x_i - c_l^i)^2}$.

The following is a proven theorem [22–24]:

Theorem 1. *For any given real continuous function $g(X)$ on the compact set $U \subset \mathbb{R}^n$ and arbitrary $\varepsilon > 0$, there exists a FBFN $f^*(X) = \xi^{*T}(X, c^*, \omega^*)W^*$ in the form of (8) such that*

$$\sup_{X \in U} |f^*(X) - g(X)| < \varepsilon. \tag{9}$$

The above theorem states that the FBFN mentioned above can approximate any real continuous function to any degree of accuracy, which means has universal approximation property as also has been reported earlier in [22–24].

3. The proposed observer-based adaptive fuzzy control

Based on the analysis in [1], (2) can be solved explicitly as

$$\phi(v) = cv(t) + d_1(v), d_1(v) = [\phi_0 - cv_0]e^{-\alpha(v - v_0)\text{sgn}\dot{v}} + e^{-\alpha v \text{sgn}\dot{v}} \int_{v_0}^{v} [B_1 - c]e^{\alpha\eta\,\text{sgn}\,\dot{v}}\,d\eta, \tag{10}$$

where $v(0) = v_0$ and $\phi(v_0) = \phi_0$. Based on above solution it is shown in [1,5] that $d_1(v)$ is bounded. Thus using (10), (1) can be reformulated as

$$\begin{aligned} x^{(n)} &= f(X, t) + g(X, t)v(t) + d(X, t) \\ y &= x, \end{aligned} \tag{11}$$

where $g(X, t) = c\bar{g}(X, t)$ and $d(X, t) = \bar{g}(X, t)d_1(v) + \bar{d}(X, t)$. Since $\bar{g}(X, t)$, $d_1(v)$ and $\bar{d}(X, t)$ are bounded thus $d(X, t)$ is bounded. Therefore there exists positive constant D such that the disturbance-like $d(X, t)$ is bounded by D, i.e., $|d(X, t)| \leqslant D$.

The nonlinear system (11) can be rewritten as follows:

$$\begin{aligned} \dot{X} &= AX + B\{f(t, X) + g(t, X)v + d(t, X)\} \\ y &= CX, \end{aligned} \tag{12}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix}_{n \times n}, \tag{13}$$

and

$$B^T = [0 \ \cdots \ 0 \ \ 1]_{1\times n}, \quad C = [1 \ \cdots \ 0 \ \ 0]_{1\times n}. \tag{14}$$

It is obvious that (A, B) is controllable and (C, A) is observable.

Two FBFNs as (8) are used as follows:

$$f(X, \omega_f, c_f, W_f) = \xi^T(X, \omega_f, c_f)W_f \tag{15}$$

$$g(X, \omega_g, c_g, W_g) = \xi^T(X, \omega_g, c_g)W_g. \tag{16}$$

to approximate the unknown nonlinear functions $f(t, X)$ and $g(t, X)$. Based on Theorem 1 for $f(t, X)$ there exist ideal parameters W_f^*, c_f^* and ω_f^* such that

$$f(t, X) = \xi_f^{*T}(X, \omega_f^*, c_f^*)W_f^* + \Delta_f,$$

where Δ_f is the approximation error of the FBFN (15). Thus

$$f(t, X) - \hat{f}(\widehat{X}, \widehat{W}_f, \hat{\omega}_f, \hat{c}_f) = \xi_f^{*T}(X, \omega_f^*, c_f^*)W_f^* - \hat{\xi}_f^T(\widehat{X}, \hat{\omega}_f, \hat{c}_f)\widehat{W}_f + \Delta_f, \tag{17}$$

where $\hat{c}_f = [\hat{c}_{f1}^T, \hat{c}_{f2}^T, \ldots, \hat{c}_{fM_f}^T]^T \in \mathbb{R}^{nM_f}$, $\hat{c}_{fl} = [\hat{c}_{fl}^1, \hat{c}_{fl}^2, \ldots, \hat{c}_{fl}^n]^T \in \mathbb{R}^n$, $\hat{\omega}_f = [\hat{\omega}_{f1}^T, \hat{\omega}_{f2}^T, \ldots, \hat{\omega}_{fM_f}^T]^T \in \mathbb{R}^{nM_f}$, $\hat{\omega}_{fl} = [\hat{\omega}_{fl}^1, \hat{\omega}_{fl}^2, \ldots, \hat{\omega}_{fl}^n]^T \in \mathbb{R}^n$, $(l = 1, \ldots, M_f)$, $\widehat{W}_f = [\widehat{W}_{f1}, \widehat{W}_{f2}, \ldots, \widehat{W}_{fM_f}]^T$ are the approximation of c_f^*, ω_f^*, W_f^* and $\widehat{X}^T = [\hat{x}, \dot{\hat{x}}, \ldots, \hat{x}^{(n-1)}] = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n] \in U_{\hat{x}}$, ($U_{\hat{x}}$ is a compact set) is the estimate of X, will be defined in (26).

For simplicity considering $\xi_f^*(X, \omega_f^*, c_f^*) = \xi_f^*$, $\hat{\xi}_f(\widehat{X}, \hat{\omega}_f, \hat{c}_f) = \hat{\xi}_f$ and defining $\widetilde{W}_f = W_f^* - \widehat{W}_f$, $\tilde{\xi}_f = \xi_f^* - \hat{\xi}_f$, we have

$$f(t, X) - \hat{f}(\widehat{X}, \widehat{W}_f, \hat{\omega}_f, \hat{c}_f) = (\tilde{\xi}_f + \hat{\xi}_f)^T(\widetilde{W}_f + \widehat{W}_f) - \hat{\xi}_f^T\widehat{W}_f + \Delta_f = \tilde{\xi}_f^T\widehat{W}_f + \hat{\xi}_f^T\widetilde{W}_f + \tilde{\xi}_f^T\widetilde{W}_f + \Delta_f. \tag{18}$$

If the vector of Gaussian membership functions is linearized by using Taylor series expansion then $\tilde{\xi}_f$ can be written as

$$\tilde{\xi}_f = \begin{bmatrix} \tilde{\xi}_{f1} \\ \tilde{\xi}_{f2} \\ \vdots \\ \tilde{\xi}_{fM_f} \end{bmatrix} = \begin{bmatrix} \frac{\partial \xi_{f1}^*}{\partial \omega_f^*} \\ \frac{\partial \xi_{f2}^*}{\partial \omega_f^*} \\ \vdots \\ \frac{\partial \xi_{fM_f}^*}{\partial \omega_f^*} \end{bmatrix}\Bigg|_{\substack{\omega_f^* = \hat{\omega}_f \\ c_f^* = \hat{c}_f \\ X = \widehat{X}}} \tilde{\omega}_f + \begin{bmatrix} \frac{\partial \xi_{f1}^*}{\partial c_f^*} \\ \frac{\partial \xi_{f2}^*}{\partial c_f^*} \\ \vdots \\ \frac{\partial \xi_{fM_f}^*}{\partial c_f^*} \end{bmatrix}\Bigg|_{\substack{\omega_f^* = \hat{\omega}_f \\ c_f^* = \hat{c}_f \\ X = \widehat{X}}} \tilde{c}_f + \begin{bmatrix} \frac{\partial \xi_{f1}^*}{\partial X} \\ \frac{\partial \xi_{f2}^*}{\partial X} \\ \vdots \\ \frac{\partial \xi_{fM_f}^*}{\partial X} \end{bmatrix}\Bigg|_{\substack{\omega_f^* = \hat{\omega}_f \\ c_f^* = \hat{c}_f \\ X = \widehat{X}}} \widetilde{E} + h_f = \Lambda_f\tilde{\omega}_f + \Omega_f\tilde{c}_f + \Gamma_f\widetilde{E} + h_f, \tag{19}$$

where $\tilde{\omega}_f = \omega_f^* - \hat{\omega}_f$, $\tilde{c}_f = c_f^* - \hat{c}_f$, $\widetilde{E} = X - \widehat{X}$ and h_f denotes higher order terms. Moreover, we have;

$$\frac{\partial \xi_{fl}^*}{\partial \omega_f^*} = \Bigg[\underbrace{0 \ \cdots \ 0}_{(l-1)\times n} \ \frac{\partial \xi_{fl}^*}{\partial \omega_{fl}^{*1}} \ \ \frac{\partial \xi_{fl}^*}{\partial \omega_{fl}^{*2}} \ \cdots \ \frac{\partial \xi_{fl}^*}{\partial \omega_{fl}^{*n}} \ \underbrace{0 \ \cdots \ 0}_{(M_f-l)\times n}\Bigg] \tag{20}$$

$$\frac{\partial \xi_{fl}^*}{\partial c_f^*} = \Bigg[\underbrace{0 \ \cdots \ 0}_{(l-1)\times n} \ \frac{\partial \xi_{fl}^*}{\partial c_{fl}^{*1}} \ \ \frac{\partial \xi_{fl}^*}{\partial c_{fl}^{*2}} \ \cdots \ \frac{\partial \xi_{fl}^*}{\partial c_{fl}^{*n}} \ \underbrace{0 \ \cdots \ 0}_{(M_f-l)\times n}\Bigg] \tag{21}$$

$$\frac{\partial \xi_{fl}^*}{\partial X} = \left[\frac{\partial \xi_{fl}^*}{\partial x_1} \ \ \frac{\partial \xi_{fl}^*}{\partial x_2} \ \cdots \ \frac{\partial \xi_{fl}^*}{\partial x_n}\right]. \tag{22}$$

Therefore, using (19) in (18) we have

$$\begin{aligned} f(t, X) - \hat{f}(\widehat{X}, \widehat{W}_f, \hat{\omega}_f, \hat{c}_f) &= (\tilde{\omega}_f^T\Lambda_f^T + \tilde{c}_f^T\Omega_f^T)\widetilde{W}_f + (\tilde{\omega}_f^T\Lambda_f^T + \tilde{c}_f^T\Omega_f^T)\widehat{W}_f + \hat{\xi}_f^T\widetilde{W}_f + (\widetilde{E}^T\Gamma_f^T + h_f^T)(\widehat{W}_f + \widetilde{W}_f) + \Delta_f \\ &= (\omega_f^* - \hat{\omega}_f)^T\Lambda_f^T\widetilde{W}_f + (c_f^* - \hat{c}_f)^T\Omega_f^T\widetilde{W}_f + (\tilde{\omega}_f^T\Lambda_f^T + \tilde{c}_f^T\Omega_f^T)\widehat{W}_f + \hat{\xi}_f^T\widetilde{W}_f + (\widetilde{E}^T\Gamma_f^T + h_f^T)W_f^* + \Delta_f \\ &= (\hat{\xi}_f^T - \hat{\omega}_f^T\Lambda_f^T - \hat{c}_f^T\Omega_f^T)\widetilde{W}_f + (\tilde{\omega}_f^T\Lambda_f^T + \tilde{c}_f^T\Omega_f^T)\widehat{W}_f + \varepsilon_f, \end{aligned} \tag{23}$$

where $\varepsilon_f = (\omega_f^{*^T}\Lambda_f^T + c_f^{*^T}\Omega_f^T)\widetilde{W}_f + (\widetilde{E}^T\Gamma_f^T + h_f^T)W_f^* + \Delta_f$. Since in (23) each term is scalar we have

$$f(t, X) - \hat{f}(\widehat{X}, \widehat{W}_f, \hat{\omega}_f, \hat{c}_f) = (\hat{\xi}_f^T - \hat{\omega}_f^T\Lambda_f^T - \hat{c}_f^T\Omega_f^T)\widetilde{W}_f + \widehat{W}_f^T\Lambda_f\tilde{\omega}_f + \widehat{W}_f^T\Omega_f\tilde{c}_f + \varepsilon_f. \tag{24}$$

The above procedure can be written for function $g(t, X)$ with FBFN (16) and changing the f indices to g. Thus we have:

$$g(t, X) - \hat{g}(\widehat{X}, \widehat{W}_g, \hat{\omega}_g, \hat{c}_g) = (\hat{\xi}_g^T - \hat{\omega}_g^T\Lambda_g^T - \hat{c}_g^T\Omega_g^T)\widetilde{W}_g + \widehat{W}_g^T\Lambda_g\tilde{\omega}_g + \widehat{W}_g^T\Omega_g\tilde{c}_g + \varepsilon_g, \tag{25}$$

where $\widetilde{W}_g = W_g^* - \widehat{W}_g$, $\tilde{\omega}_g = \omega_g^* - \hat{\omega}_g$, $\tilde{c}_g = c_g^* - \hat{c}_g$, $\varepsilon_g = (\omega_g^{*^T}\Lambda_g^T + c_g^{*^T}\Omega_g^T)\widetilde{W}_g + (\widetilde{E}^T\Gamma_g^T + h_g^T)W_g^* + \Delta_g$ and Δ_g is the approximation error of the FBFN (16).

To estimate the states the following observer is proposed:

$$\dot{\widehat{X}}(t) = A\widehat{X}(t) + B\{\hat{f}(\widehat{X}, \widehat{W}_f, \hat{\omega}_f, \hat{c}_f) + \hat{g}(\widehat{X}, \widehat{W}_g, \hat{\omega}_g, \hat{c}_g)v + u_c + u_r\} + K_oC\widetilde{E}, \tag{26}$$

where $\widetilde{E} = X - \widehat{X} = [\tilde{e}, \dot{\tilde{e}}, \ldots, \tilde{e}^{(n-1)}]^T$, $\tilde{e} = x - \hat{x} = x_1 - \hat{x}_1$ and K_o is chosen such that $A - K_oC$ becomes Hurwitz, (since (C, A) is observable such K_o exists). The control term u_r is a robust structure with adaptive gain as follows:

$$u_r(\tilde{e}|\hat{\rho}) = \hat{\rho}\,\mathrm{sgn}\,(\tilde{e}), \tag{27}$$

where $\hat{\rho}$ is the free parameter to be adapted. u_c is a compensation term as follows:

$$u_c = K_o^T P_1 \widehat{E}. \tag{28}$$

Assumption 3. The ideal parameter vectors $W_f^*, \omega_f^*, c_f^*, W_g^*, \omega_g^*, c_g^*$ and ρ^* lie in some compact regions:

$$\begin{aligned}
&U_{W_f} = \{W_f \in \mathbb{R}^{M_f} | \|W_f\| \leqslant m_{W_f}\}, \quad U_{W_g} = \{W_g \in \mathbb{R}^{M_g} | \|W_g\| \leqslant m_{W_g}\},\\
&U_{\omega_f} = \{\omega_f \in \mathbb{R}^{nM_f} | \|\omega_f\| \leqslant m_{\omega_f}\}, \quad U_{\omega_g} = \{\omega_g \in \mathbb{R}^{nM_g} | \|\omega_g\| \leqslant m_{\omega_g}\},\\
&U_{c_f} = \{c_f \in \mathbb{R}^{nM_f} | \|c_f\| \leqslant m_{c_f}\}, \quad U_{c_g} = \{c_g \in \mathbb{R}^{nM_g} | \|c_g\| \leqslant m_{c_g}\},\\
&U_\rho = \{\rho \in \mathbb{R} | |\rho| \leqslant m_\rho\},
\end{aligned}$$

where the radiuses m_{W_f}, m_{ω_f}, m_{c_f}, m_{ω_g}, m_{c_g}, m_{W_g} and m_ρ are constants,

$$\begin{aligned}
&(W_f^*, \omega_f^*, c_f^*) = \arg \min_{\widehat{W}_f \in U_{W_f}, \hat{\omega}_f \in U_{\omega_f}, \hat{c}_f \in U_{c_f}} \left[\sup_{X \in U_X, \widehat{X} \in U_{\hat{X}}} \left| f(t, X) - \hat{f}(\widehat{X}, \widehat{W}_f, \hat{\omega}_f, \hat{c}_f) \right| \right],\\
&(W_g^*, \omega_g^*, c_g^*) = \arg \min_{\widehat{W}_g \in U_{W_g}, \hat{\omega}_g \in U_{\omega_g}, \hat{c}_g \in U_{c_g}} \left[\sup_{X \in U_X, \widehat{X} \in U_{\hat{X}}} \left| g(t, X) - \hat{g}(\widehat{X}, \widehat{W}_g, \hat{\omega}_g, \hat{c}_g) \right| \right],\\
&\rho^* = D + \bar{\delta},
\end{aligned}$$

where $\bar{\delta}$ is an unknown positive constant which will be defined in Lemma 4.

Now, the following control law is proposed:

$$v = \frac{1}{\hat{g}(\widehat{X}, \widehat{W}_g, \hat{\omega}_g, \hat{c}_g)} \left[-\hat{f}(\widehat{X}, \widehat{W}_f, \hat{\omega}_f, \hat{c}_f) - K_c\widehat{E} + x_d^{(n)} - u_r - u_c \right], \tag{29}$$

where K_c is chosen such that $A - BK_c$ becomes Hurwitz, (since (A, B) is controllable such K_c exists). Therefore, there exist positive-definite matrices P_1 and Q_1 such that

$$(A - BK_c)^T P_1 + P_1(A - BK_c) = -Q_1, \tag{30}$$

Thus using (29) in (26),

$$\dot{\widehat{E}} = \dot{\widehat{X}} - \dot{X}_d = \dot{\widehat{X}} - AX_d - Bx_d^{(n)} = (A - BK_c)\widehat{E} + K_oC\widetilde{E}. \tag{31}$$

The observation error dynamics by using (12) and (26) can be given by

$$\begin{aligned}
&\dot{\widetilde{E}} = \dot{X} - \dot{\widehat{X}} = (A - K_oC)\widetilde{E} + B\{(f(t, X) - \hat{f}(\widehat{X}, \widehat{W}_f, \hat{\omega}_f, \hat{c}_f)) + (g(t, X) - \hat{g}(\widehat{X}, \widehat{W}_g, \hat{\omega}_g, \hat{c}_g))v + d - u_r - u_c\}\\
&\tilde{e} = C\widetilde{E}.
\end{aligned} \tag{32}$$

Since only $\tilde{e}$ in (32) is assumed to be measurable, we use the strictly positive real (SPR) Lyapunov design method to analyze stability of the closed-loop system and derive adaptive laws.

The error dynamics (32) can be written as

$$\tilde{e} = H(s)\{(f(t, X) - \hat{f}(\widehat{X}, \widehat{W}_f, \hat{\omega}_f, \hat{c}_f)) + (g(t, X) - \hat{g}(\widehat{X}, \widehat{W}_g, \hat{\omega}_g, \hat{c}_g))v + d - u_r - u_c\}, \tag{33}$$

where $H(s) = C(sI - A + K_oC)^{-1}B$ is the transfer function of (32). If we select $K_o^T = [C_n^1\alpha, C_n^2\alpha^2, \ldots, C_n^n\alpha^n]$ with $C_n^i = n!/((n-i)!i!)$, we can see that

$$H(s) = \frac{1}{(s + \alpha)^n}, \tag{34}$$

where α is a positive constant. Now, the following lemmas are considered:

Lemma 1. [25] *A strictly proper rational transfer function $H(s) = C(sI - A)^{-1}B$ is SPR, if and only if there exist positive-definite symmetric matrices P and Q*

$$\begin{aligned}
&A^T P + PA = -Q,\\
&B^T P = C.
\end{aligned} \tag{35}$$

Lemma 2. [25] *Suppose B and C are full rank. Then there exists a matrix $P = P^T > 0$ that satisfies (35) if and only if*

$$CB = B^T C^T > 0. \tag{36}$$

Considering Lemmas 1 and 2 it can be deduced that $H(s)$ is not an SPR transfer function. In order to use SPR-Lyapunov design, (33) is rewritten as

$$\tilde{e} = H(s)L(s)\Big\{L^{-1}(s)(f(t,X) - \hat{f}(\widehat{X}, \widehat{W}_f, \hat{\omega}_f, \hat{c}_f)) + L^{-1}(s)((g(t,X) - \hat{g}(\widehat{X}, \widehat{W}_g, \hat{\omega}_g, \hat{c}_g))\,v) + L^{-1}(s)d - L^{-1}(s)u_r - L^{-1}(s)u_c\Big\}, \tag{37}$$

where $L(s)$ is chosen so that $L^{-1}(s)$ is a proper stable transfer function and $H(s)L(s)$ is a proper SPR transfer function. Using (24), (25) and (37) we have

$$\begin{aligned}\tilde{e} = H(s)L(s)\Big\{&\left(\hat{\xi}_f^T - \hat{\omega}_f^T \Lambda_f^T - \hat{c}_f^T \Omega_f^T\right)\widetilde{W}_f + \widehat{W}_f^T \Lambda_f \tilde{\omega}_f + \widehat{W}_f^T \Omega_f \tilde{c}_f + \left(\hat{\xi}_g^T - \hat{\omega}_g^T \Lambda_g^T - \hat{c}_g^T \Omega_g^T\right) v\widetilde{W}_g + \widehat{W}_g^T \Lambda_g v\tilde{\omega}_g \\ &+ \widehat{W}_g^T \Omega_g v\tilde{c}_g + \delta + d - u_r - u_c\Big\},\end{aligned} \tag{38}$$

where

$$\begin{aligned}\delta = {} & L^{-1}(s)\Big\{\left(\hat{\xi}_f^T - \hat{\omega}_f^T \Lambda_f^T - \hat{c}_f^T \Omega_f^T\right)\widetilde{W}_f + \widehat{W}_f^T \Lambda_f \tilde{\omega}_f + \widehat{W}_f^T \Omega_f \tilde{c}_f\Big\} + L^{-1}(s)\{\left(\hat{\xi}_g^T - \hat{\omega}_g^T \Lambda_g^T - \hat{c}_g^T \Omega_g^T\right) v\widetilde{W}_g + \widehat{W}_g^T \Lambda_g v\tilde{\omega}_g \\ & + \widehat{W}_g^T \Omega_g v\tilde{c}_g\} - \left(\hat{\xi}_f^T - \hat{\omega}_f^T \Lambda_f^T - \hat{c}_f^T \Omega_f^T\right)\widetilde{W}_f - \widehat{W}_f^T \Lambda_f \tilde{\omega}_f - \widehat{W}_f^T \Omega_f \tilde{c}_f - \left(\hat{\xi}_g^T - \hat{\omega}_g^T \Lambda_g^T - \hat{c}_g^T \Omega_g^T\right) v\widetilde{W}_g - \widehat{W}_g^T \Lambda_g v\tilde{\omega}_g \\ & - \widehat{W}_g^T \Omega_g v\tilde{c}_g - d + u_r + u_c + L^{-1}(s)d - L^{-1}(s)u_r - L^{-1}(s)u_c + L^{-1}(s)\varepsilon_f + L^{-1}(s)(\varepsilon_g v).\end{aligned} \tag{39}$$

Suppose $L(s) = b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \cdots + b_m$, where $m = n - 1$. Then the state space realization of (38) can be written as

$$\begin{aligned}\dot{\widetilde{E}} &= A_c \widetilde{E} + B_c\Big\{\left(\hat{\xi}_f^T - \hat{\omega}_f^T \Lambda_f^T - \hat{c}_f^T \Omega_f^T\right)\widetilde{W}_f + \widehat{W}_f^T \Lambda_f \tilde{\omega}_f + \widehat{W}_f^T \Omega_f \tilde{c}_f\Big\} \\ &\quad + B_c\Big\{\left(\hat{\xi}_g^T - \hat{\omega}_g^T \Lambda_g^T - \hat{c}_g^T \Omega_g^T\right) v\widetilde{W}_g + \widehat{W}_g^T \Lambda_g v\tilde{\omega}_g + \widehat{W}_g^T \Omega_g v\tilde{c}_g + \delta + d - u_r - u_c\Big\} \\ \tilde{e} &= C_c \widetilde{E},\end{aligned} \tag{40}$$

where

$$A_c = A - K_o C, \quad ,B_c^T = [b_0, b_1, \ldots, b_m], \quad C_c = C. \tag{41}$$

Thus based on Lemma 1 and since $H(s)L(s)$ is SPR there exist positive-definite matrices P_2 and Q_2 such that

$$A_c^T P_2 + P_2 A_c = -Q_2, \tag{42}$$
$$B_c^T P_2 = C_c. \tag{43}$$

The following lemma gives the adaptation laws:

Lemma 3. *Suppose the following adaptive laws*

$$\dot{\widehat{W}}_f = \begin{cases} \gamma_1(\hat{\xi}_f^T - \hat{\omega}_f^T \Lambda_f^T - \hat{c}_f^T \Omega_f^T)^T \tilde{e} & \|\widehat{W}_f\| < m_{W_f} \quad \text{or} \quad (\|\widehat{W}_f\| = m_{W_f} \\ & \text{and} \quad (\hat{\xi}_f^T - \hat{\omega}_f^T \Lambda_f^T - \hat{c}_f^T \Omega_f^T)\widehat{W}_f \tilde{e} \leqslant 0) \\ \Pr(\gamma_1(\hat{\xi}_f^T - \hat{\omega}_f^T \Lambda_f^T - \hat{c}_f^T \Omega_f^T)^T \tilde{e}) & \|\widehat{W}_f\| = m_{W_f} \quad \text{and} \quad (\hat{\xi}_f^T - \hat{\omega}_f^T \Lambda_f^T - \hat{c}_f^T \Omega_f^T)\widehat{W}_f \tilde{e} > 0, \end{cases} \tag{44}$$

$$\dot{\hat{\omega}}_f = \begin{cases} \gamma_2(\widehat{W}_f^T \Lambda_f)^T \tilde{e} & \|\hat{\omega}_f\| < m_{\omega_f} \quad \text{or} \quad (\|\hat{\omega}_f\| = m_{\omega_f} \\ & \text{and} \quad \widehat{W}_f^T \Lambda_f \hat{\omega}_f \tilde{e} \leqslant 0) \\ \Pr(\gamma_2(\widehat{W}_f^T \Lambda_f)^T \tilde{e}) & \|\hat{\omega}_f\| = m_{\omega_f} \quad \text{and} \quad \widehat{W}_f^T \Lambda_f \hat{\omega}_f \tilde{e} > 0, \end{cases} \tag{45}$$

$$\dot{\hat{c}}_f = \begin{cases} \gamma_3(\widehat{W}_f^T \Omega_f)^T \tilde{e} & \|\hat{c}_f\| < m_{c_f} \quad \text{or} \quad (\|\hat{c}_f\| = m_{c_f} \quad \text{and} \quad \widehat{W}_f^T \Omega_f \hat{c}_f \tilde{e} \leqslant 0) \\ \Pr(\gamma_3(\widehat{W}_f^T \Omega_f)^T \tilde{e}) & \|\hat{c}_f\| = m_{c_f} \quad \text{and} \quad \widehat{W}_f^T \Omega_f \hat{c}_f \tilde{e} > 0, \end{cases} \tag{46}$$

$$\dot{\hat{\omega}}_g = \begin{cases} \gamma_4(\widehat{W}_g^T \Lambda_g)^T v\tilde{e} & \|\hat{\omega}_g\| < m_{\omega_g} \quad or \quad (\|\hat{\omega}_g\| = m_{\omega_g} \\ & and \quad \widehat{W}_g^T \Lambda_g \hat{\omega}_g v\tilde{e} \leqslant 0) \\ \Pr(\gamma_4(\widehat{W}_g^T \Lambda_g)^T v\tilde{e}) & \|\hat{\omega}_g\| = m_{\omega_g} \quad and \quad \widehat{W}_g^T \Lambda_g \hat{\omega}_g v\tilde{e} > 0, \end{cases} \tag{47}$$

$$\dot{\hat{c}}_g = \begin{cases} \gamma_5(\widehat{W}_g^T \Omega_g)^T v\tilde{e} & \|\hat{c}_g\| < m_{c_g} \quad or \quad (\|\hat{c}_g\| = m_{c_g} \\ & and \quad \widehat{W}_g^T \Omega_g \hat{c}_g v\tilde{e} \leqslant 0) \\ \Pr(\gamma_5(\widehat{W}_g^T \Omega_g)^T v\tilde{e}) & \|\hat{c}_g\| = m_{c_g} \quad and \quad \widehat{W}_g^T \Omega_g \hat{c}_g v\tilde{e} > 0, \end{cases} \tag{48}$$

for avoiding singularity (uncontrollability) if an element $\widehat{W}_{ig}$ of $\widehat{W}_g$ equals ε (arbitrarily small positive number) then

$$\dot{\widehat{W}}_{ig} = \begin{cases} \gamma_6(\hat{\xi}_g^T - \hat{\omega}_g^T \Lambda_g^T - \hat{c}_g^T \Omega_g^T)_i v\tilde{e} & (\hat{\xi}_g^T - \hat{\omega}_g^T \Lambda_g^T - \hat{c}_g^T \Omega_g^T)_i v\tilde{e} > 0 \\ 0 & (\hat{\xi}_g^T - \hat{\omega}_g^T \Lambda_g^T - \hat{c}_g^T \Omega_g^T)_i v\tilde{e} \leqslant 0, \end{cases} \tag{49}$$

where $(\hat{\xi}_g^T - \hat{\omega}_g^T \Lambda_g^T - \hat{c}_g^T \Omega_g^T)_i$ is the i-th component of $\hat{\xi}_g^T - \hat{\omega}_g^T \Lambda_g^T - \hat{c}_g^T \Omega_g^T$. Otherwise,

$$\dot{\widehat{W}}_g = \begin{cases} \gamma_6(\hat{\xi}_g^T - \hat{\omega}_g^T \Lambda_g^T - \hat{c}_g^T \Omega_g^T)^T v\tilde{e} & \|\widehat{W}_g\| < m_{W_g} \quad or \quad (\|\widehat{W}_g\| = m_{W_g} \\ & and \quad (\hat{\xi}_g^T - \hat{\omega}_g^T \Lambda_g^T - \hat{c}_g^T \Omega_g^T)\widehat{W}_g v\tilde{e} \leqslant 0) \\ \Pr(\gamma_6(\hat{\xi}_g^T - \hat{\omega}_g^T \Lambda_g^T - \hat{c}_g^T \Omega_g^T)^T v\tilde{e}) & \|\widehat{W}_g\| = m_{W_g} \quad and \quad (\hat{\xi}_g^T - \hat{\omega}_g^T \Lambda_g^T - \hat{c}_g^T \Omega_g^T)\widehat{W}_g v\tilde{e} > 0, \end{cases} \tag{50}$$

$$\dot{\hat{\rho}} = \begin{cases} \gamma_7|\tilde{e}| & |\hat{\rho}| < m_\rho \quad or \quad (|\hat{\rho}| = m_\rho \quad and \quad \hat{\rho} \leqslant 0) \\ \Pr(\gamma_7|\tilde{e}|) & |\hat{\rho}| = m_\rho \quad and \quad \hat{\rho} > 0, \end{cases} \tag{51}$$

where γ_i, $i = 1, \ldots, 7$ are positive constants and projection operator $(\Pr(.))$ is defined as

$$\dot{\Lambda} = \Pr(\Gamma\Theta\varpi) = \Gamma\Theta\varpi - \Gamma\frac{\Lambda\Lambda^T}{\Lambda^T\Gamma\Lambda}\Gamma\Theta\varpi. \tag{52}$$

Then $\|\widehat{W}_f\| \leqslant m_{W_f}$, $\|\hat{\omega}_f\| \leqslant m_{\omega_f}$, $\|\hat{c}_f\| \leqslant m_{c_f}$, $\|\hat{\omega}_g\| \leqslant m_{\omega_g}$, $\|\hat{c}_g\| \leqslant m_{c_g}$, $0 < \varepsilon \leqslant \|\widehat{W}_g\| \leqslant m_{W_g}$ and $|\hat{\rho}| \leqslant m_\rho$. Also $\|\widetilde{W}_f\| \leqslant 2m_{W_f}$, $\|\tilde{\omega}_f\| \leqslant 2m_{\omega_f}$, $\|\tilde{c}_f\| \leqslant 2m_{c_f}$, $\|\widetilde{W}_g\| \leqslant 2m_{W_g}$, $\|\tilde{\omega}_g\| \leqslant 2m_{\omega_g}$, $\|\tilde{c}_g\| \leqslant 2m_{c_g}$, and $|\tilde{\rho}| \leqslant 2m_\rho$, such that $\tilde{\rho} = \rho^* - \hat{\rho}$.

Proof 1. See Appendix A. □

Lemma 4. *δ in (39) is bounded by a constant, i.e.,*

$$|\delta| \leqslant \bar{\delta}, \tag{53}$$

where $\bar{\delta}$ is an unknown positive constant.

Proof 2. See Appendix B. □

4. Stability analysis

Now, based on above discussions we are ready to analyze the asymptotic stability of the closed-loop system. For stability analysis the following theorem is stated:

Theorem 2. *Consider system (1) that satisfies Assumptions 1–3. Let $\widehat{W}_f$, $\hat{\omega}_f$, $\hat{c}_f$, $\widehat{W}_g$, $\hat{\omega}_g$, $\hat{c}_g$ and $\hat{\rho}$ are adjusted by adaptive laws (44)–(51), respectively. If the control signal and the state estimator are chosen by (29) and (26), respectively, then $\lim_{t\to\infty} E(t) = 0$.*

Proof 3. See Appendix C. □

Remark 1. In order to eliminate/alleviate chattering completely the discontinuous function $\text{sgn}(\tilde{e})$ in (27) can be replaced by the following smooth functions: $\text{sat}(\tilde{e}/r)$, $\tanh(\tilde{e}/r)$, $\tan^{-1}(\tilde{e}/r)$, $\tilde{e}/(|\tilde{e}| + r)$ and $\tilde{e}/(|\tilde{e}| + re^{-qt})$, where r and q are two positive constants. The first four smooth functions eliminate chattering completely in the expense of steady state error, which is proportional to r. However, since the last smooth function has a decaying-width boundary layer re^{-pt}, its ability to alleviate chattering while maintaining asymptotic convergence has been proved [26].

Remark 2. According to the previous analysis and to summarize the above results, the design algorithm and the block diagram of the proposed controller are described as follows:

Design Algorithm

Step 1. Specify the feedback and observer gain K_c and K_o such that the characteristic matrices $(A - BK_c)$ and $(A - K_oC)$ are strictly Hurwitz.
Step 2. Specify positive definite $n \times n$ matrices Q_1 and Q_2 and solve Riccati Eqs. (30) and (42) to obtain positive definite $n \times n$ matrices P_1 and P_2, respectively.
Step 3. Noting $C\widetilde{E} = \tilde{e} = y - x_d$, solve the error Eq. (31) to obtain state estimate $\widehat{X} = \widehat{E} + X_d$.
Step 4. Specify the design parameters $m_{W_f},\ m_{\omega_f},\ m_{c_f},\ m_{W_g},\ m_{\omega_g},\ m_{c_g}$ and m_ρ and positive scalars γ_i, $i = 1, \ldots, 7$, then compute the adaptive laws (44)–(51) to adjust the parameter vectors $\widehat{W}_f,\ \hat{\omega}_f,\ \hat{c}_f,\ \widehat{W}_g,\ \hat{\omega}_g,\ \hat{c}_g$ and $\hat{\rho}$.
Step 5. Calculate the robust structure u_r with adaptive gain in (27) and the compensation controller u_c in (28).
Step 6. Obtain the control law (29) and apply it to the plant. In order to cope with chattering the procedure in Remark 1 can be used.

The block diagram of the proposed controller can be seen in Fig. 3.

5. Simulation example

Actuator hysteresis usually brings many problems in control systems such as tracking errors, limit cycles, undesirable performance and even instability. For example due to ferromagnetic effect of motor drive the hysteresis exists in electrical valve actuators and causes degradation of performance in control of physical and mechanical systems [27], piezoelectric actuators due to its hysteresis cause inaccurate positioning in astronomical adaptive optics [28], the unavoidable hysteresis in piezoceramic actuator brings inaccurate and unsatisfactory performance of helicopter vibration control [29] and many others. In this section we apply our proposed controller to inverted pendulum system with backlash-like hysteresis actuator.

Example 1. Consider the inverted pendulum with backlash-like hysteresis actuator depicted in Fig. 4. Denoting $x_1 = \theta$ and $x_2 = \dot{\theta}$ a 2nd order model of inverted pendulum can be stated as follows [30],

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x_1, x_2) + g(x_1, x_2)\phi(v) + d(t) \\ y = x_1, \end{cases} \tag{54}$$

where

$$f(x_1, x_2) = \frac{9.8 \sin x_1 - \frac{m l x_2^2 \cos x_1 \sin x_1}{m_c + m}}{l\left(\frac{4}{3} - \frac{m \cos^2 x_1}{m_c + m}\right)}, \quad g(x_1, x_2) = \frac{\frac{\cos x_1}{m_c + m}}{l\left(\frac{4}{3} - \frac{m \cos^2 x_1}{m_c + m}\right)} \tag{55}$$

m_c is the mass of the cart, m is the mass of the pole, $2l$ is the pole's length, $d(t)$ is the external disturbance, $\phi(v)$ is the output of the backlash-like hysteresis actuator and $v(t)$ is the applied force (control). In the simulations that follows

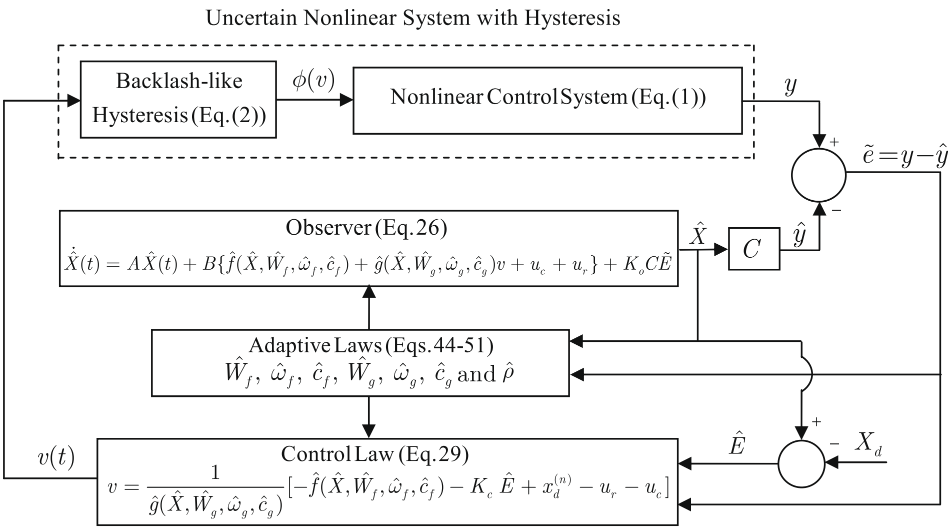

Fig. 3. Block diagram representation of the proposed method.

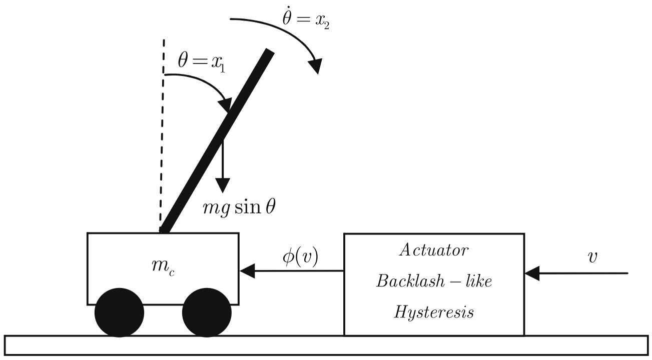

Fig. 4. Inverted pendulum system with backlash-like hysteresis actuator.

$m_c = 1$ kg, $m = 0.1$ kg and $l = 0.5$ m will be chosen. It is assumed that only output state x_1 can be measured. The nonlinear functions $f(x_1, x_2)$ and $g(x_1, x_2)$ are assumed to be unknown. The initial condition of the plant is chosen as $X(0) = \left[-\frac{\pi}{60}, 0\right]^T$.

For state estimation the observer defined in (26) with $K_c = [144, 24]$ and $K_o = [60, 900]^T$ is used. K_c and K_o are chosen such that the matrices $A - BK_c$ and $A - K_oC$ are Hurwitz. Without loss of generality the initial condition of the observer is chosen as $\widehat{X}(0) = \left[-\frac{\pi}{30}, -1\right]^T$ which is different from the initial condition of the plant. The parameters of proposed method are chosen as $\gamma_1 = 100,\ \gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = 1,\ \gamma_6 = 10$ and $\gamma_7 = 4000$. It should be noted that all the adaptive gains are chosen by trial and error to achieve the best transient control performance considering the requirement of stability and possible operating conditions. We note, that for any of positive gains, as it is proved in Theorem 2, asymptotic stability is guaranteed but they are selected to achieve best performance $M_f = M_g = 5$ are selected. The matrices P_1 and P_2 are chosen according to Riccati Eqs. (30) and (42) as follows:

$$P_1 = \begin{bmatrix} 1.4402 & 0.0035 \\ 0.0035 & 0.0101 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.5303 & -0.1015 \\ -0.1015 & 0.0085 \end{bmatrix}$$

The initial conditions of fuzzy parameters are chosen randomly in the interval $[1,\ 2]$. Without loss of generality the parameters of the backlash-like hysteresis model are chosen as $\alpha = 1$ and $c = 3.1635$. To show the effectiveness of the proposed method the backlash-like hysteresis nonlinearities with different width are used. Therefore in simulations $B_1 = 2.345,\ 1.345,\ 0.345$ are used, i.e., for bigger B_1 we have smaller width. Two cases are considered:

5.1. Case 1. (Sinusoidal Desired Trajectory)

For better comparison it is assumed that the unknown disturbance $d(t) = 3 + 2\cos(3t)$ is applied at unknown time $t = 10$ s. The desired trajectory is selected as $x_d = \frac{\pi}{5}\sin(t)$. Figs. 5–7 show the tracking performance, input and output of the backlash-like hysteresis with different hysteresis nonlinearities.

5.2. Case 2. (Periodic Step Desired Trajectory)

The unknown step disturbance $d(t) = 5$ is applied at unknown time $t = 15$ s. A second order transfer function is chosen as the reference model for a periodic step command;

$$\frac{x_d}{u_{command}} = \frac{9}{s^2 + 6s + 9}$$

where, $u_{command}$ is a periodic square wave with amplitude $\frac{\pi}{8}$ and period 12 s. The results are depicted in Figs. 8–10.

Clearly, in both cases the satisfactory tracking in the presence of unknown backlash-like hysteresis, unknown external disturbances and even unknown states are achieved. From the simulation results it can be seen that the observer based adaptive systematic fuzzy controller as proposed in Section 3 has a satisfactory performance to control the system preceded by hysteresis, amid uncertainties and external disturbances. It should be noted that only the input of the backlash-like hysteresis $v(t)$, which is the control signal, can be designed by the designer and its output, $\phi(v)$ is not available, because the hysteresis parameters are unknown.

Remark 3. To cope with the lumped uncertainties $(\delta(t) + d(t))$ usually a discontinuous robust control $u_c = \rho\,\mathrm{sgn}(\tilde{e})$ is used, where ρ is a positive constant that should be larger than the bound of lumped uncertainties. But the bound of lumped uncertainties is difficult to be measured in practical applications. If ρ is chosen too large the control effort results in

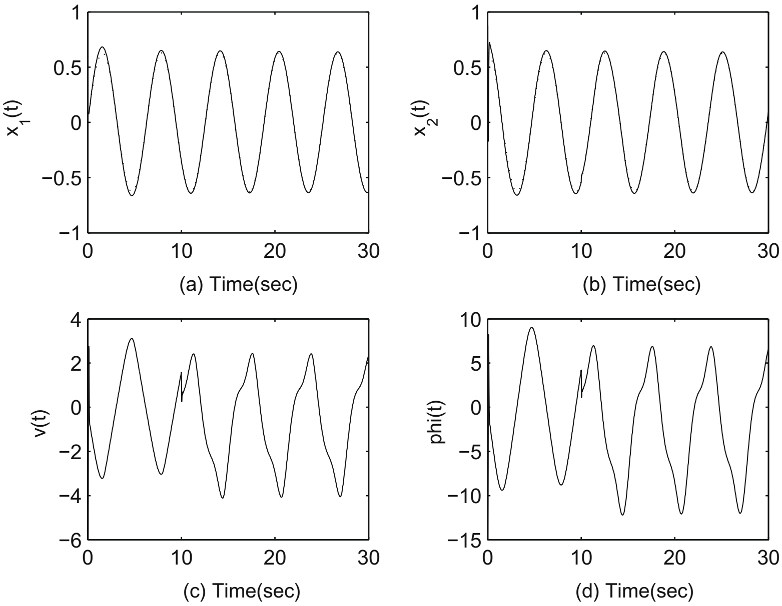

Fig. 5. Simulation results: sinusoidal desired trajectories, $B_1 = 2.345$ and $d(t) = 3 + 2\cos(3t)$: (a) desired state x_{1d} (--), estimated state x_1 (-), (b) desired state x_{2d} (--), estimated state x_2 (-), (c) control signal $v(t)$ acting as the input of the hysteresis and (d) control signal $\phi(v)$ acting as the output of the hysteresis.

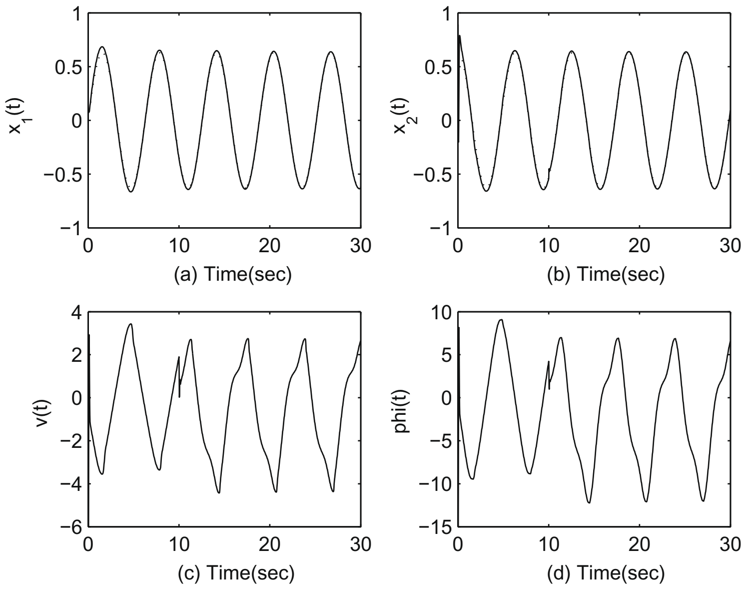

Fig. 6. Simulation results: sinusoidal desired trajectories, $B_1 = 1.345$ and $d(t) = 3 + 2\cos(3t)$: (a) desired state x_{1d} (--), estimated state x_1 (-), (b) desired state x_{2d} (--), estimated state x_2 (-), (c) control signal $v(t)$ acting as the input of the hysteresis and (d) Control signal $\phi(v)$ acting as the output of the hysteresis.

chattering. If ρ is chosen too small the performance degrades or even the closed-loop system may become unstable. Mostly in literature ρ is chosen large enough to avoid instability. To relax the requirement of knowing the bound of lumped uncertainties and to avoid using a conservative one in this paper we proposed of using adaptive gain, i.e., using $\hat{\rho}$ instead of ρ mentioned in robust structure in (27). The adaptive law for $\hat{\rho}$ is derived from Lyapunov approach, therefore, the stability is also guaranteed.

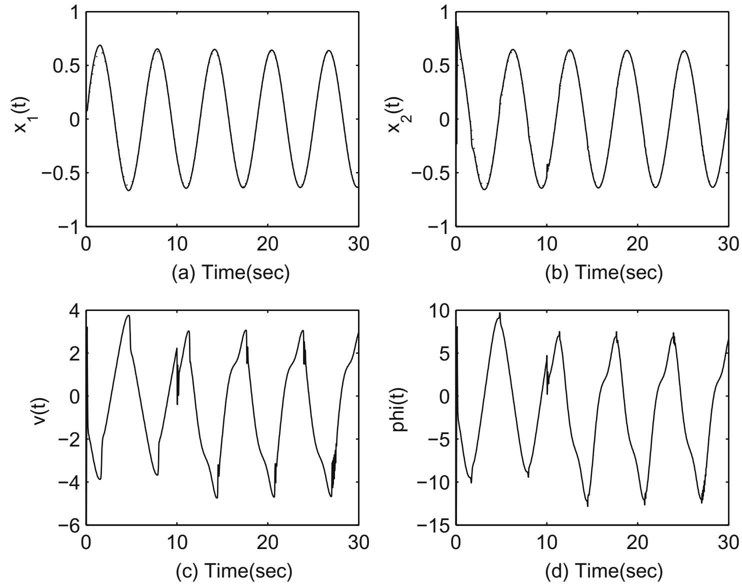

Fig. 7. Simulation results: sinusoidal desired trajectories, $B_1 = 0.345$ and $d(t) = 3 + 2\cos(3t)$: (a) desired state x_{1d} (--), estimated state x_1 (-), (b) desired state x_{2d} (--), estimated state x_2 (-), (c) control signal $v(t)$ acting as the input of the hysteresis and (d) control signal $\phi(v)$ acting as the output of the hysteresis.

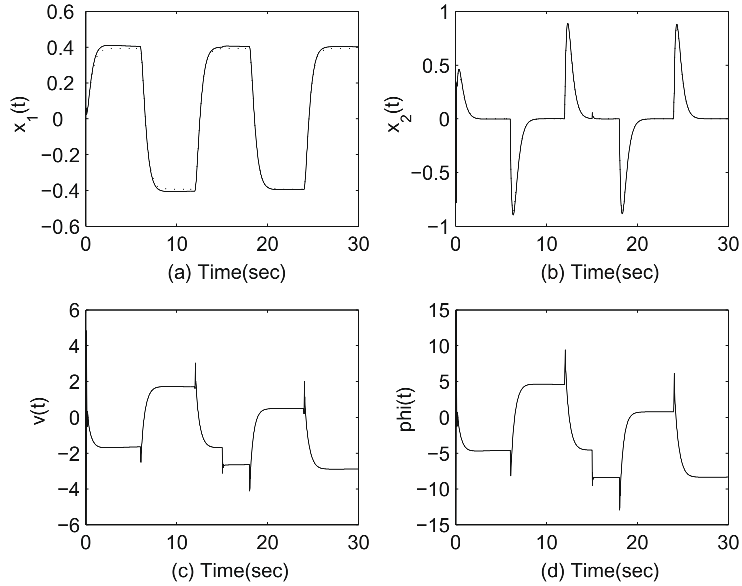

Fig. 8. Simulation results: periodic step desired trajectories, $B_1 = 2.345$ and $d(t) = 5$: (a) desired state x_{1d} (--), estimated state x_1 (-), (b) desired state x_{2d} (--), estimated state x_2 (-), (c) control signal $v(t)$ acting as the input of the hysteresis and (d) control signal $\phi(v)$ acting as the output of the hysteresis.

6. Conclusions

The adaptive control technique has been combined with fuzzy logic systems, as universal approximators, to achieve a robust output feedback control for nonlinear uncertain systems preceded by unknown backlash-like hysteresis and with large uncertainty in plant structure, unknown variations in plant parameters and unknown but bounded sudden external disturbances. Usually only parameters of the consequent parts of fuzzy rules are tuned via adaptive techniques, however, in the proposed controller both consequent and premise parts of fuzzy rules have been adjusted via adaptive laws, making the designing of controller more systematic. In many practical applications the states of the systems are not available. The proposed approach does not need the availability of the states and uses an observer to estimate the states. To cope with lumped

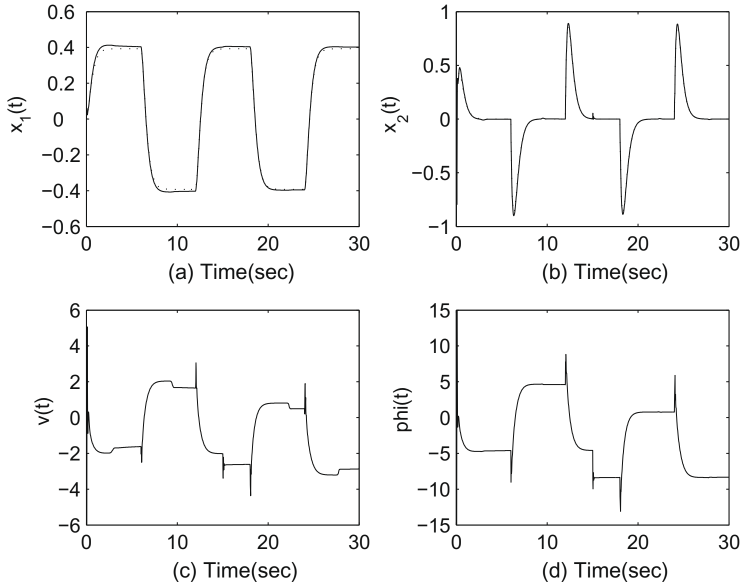

Fig. 9. Simulation results: periodic step desired trajectories, $B_1 = 1.345$ and $d(t) = 5$: (a) desired state x_{1d} (--), estimated state x_1 (-), (b) desired state x_{2d} (--), estimated state x_2 (-), (c) control signal $v(t)$ acting as the input of the hysteresis and (d) control signal $\phi(v)$ acting as the output of the hysteresis.

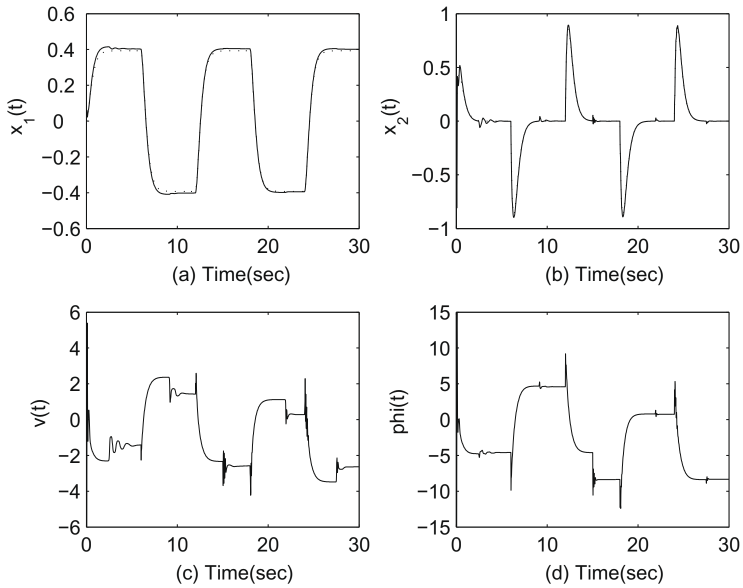

Fig. 10. Simulation results: periodic step desired trajectories, $B_1 = 0.345$ and $d(t) = 5$, (a) desired state x_{1d} (--), estimated state x_1 (-), (b) desired state x_{2d} (--), estimated state x_2 (-), (c) control signal $v(t)$ acting as the input of the hysteresis and (d) control signal $\phi(v)$ acting as the output of the hysteresis.

uncertainties generated by state estimation error, fuzzy approximation error and external disturbances a robust structure with adaptive gain is used which the adaptive gain mechanism not only relaxes the assumption of knowing the bound of lumped uncertainties, but also causes to chattering attenuation. All the adaptive laws are derived via Lyapunov synthesis method and therefore asymptotic stability of overall system is ensured. In future work, we aim to extend this methodology to more general forms of nonlinear systems.

Appendix A. Proof of Lemma 3

Using (44)–(52) the proof of $\|\widehat{W}_f\| \leqslant m_{W_f},\ \|\hat{\omega}_f\| \leqslant m_{\omega_f},\ \|\hat{c}_f\| \leqslant m_{c_f},\ \|\hat{\omega}_g\| \leqslant m_{\omega_g},\ \|\hat{c}_g\| \leqslant m_{c_g},\ \|\widehat{W}_g\| \leqslant m_{W_g}$ and $|\hat{\rho}| \leqslant m_\rho$ can be easily obtained by the results of [13]. By using triangular inequality we have $\|\widetilde{W}_f\| \leqslant$

$2m_{W_f},\ \|\tilde{\omega}_f\| \leqslant 2m_{\omega_f},\ \|\tilde{c}_f\| \leqslant 2m_{c_f},\ \|\tilde{\omega}_g\| \leqslant 2m_{\omega_g},\ \|\tilde{c}_g\| \leqslant 2m_{c_g},\ \|\widetilde{W}_g\| \leqslant 2m_{W_g}$ and $|\tilde{\rho}| \leqslant 2m_\rho$ since from Assumption 3 we have $\|W_f^*\| \leqslant m_{W_f},\ \|\omega_f^*\| \leqslant m_{\omega_f},\ \|c_f^*\| \leqslant m_{c_f},\ \|\omega_g^*\| \leqslant m_{\omega_g},\ \|c_g^*\| \leqslant m_{c_g},\ \|W_g^*\| \leqslant m_{W_g}$ and $|\rho^*| \leqslant m_\rho$. Now, consider the Lyapunov function candidate $V_g = \frac{1}{2}\widehat{W}_g^T\widehat{W}_g$, its time derivative is $\dot{V}_g = \widehat{W}_g^T\dot{\widehat{W}}_g$. Assuming that condition (49) holds, then $\dot{V}_g \geqslant 0$ which implies $0 < \varepsilon \leqslant \|\widehat{W}_g\|$. This completes the proof.

Appendix B. Proof of Lemma 4

(a) From (19) and Lemma 3 we have

$$\begin{aligned}\|h_f^T + \widetilde{E}^T\Gamma_f^T\| &= \|\tilde{\xi}_f^T - \tilde{\omega}_f^T\Lambda_f^T - \tilde{c}_f^T\Omega_f^T\| \leqslant \|\tilde{\xi}_f^T\| + \|\tilde{\omega}_f^T\|\|\Lambda_f^T\| + \|\tilde{c}_f^T\|\|\Omega_f^T\| \leqslant \ell_{f1} + \ell_{f2}\|\tilde{\omega}_f\| + \ell_{f3}\|\tilde{c}_f\| \\ &\leqslant \ell_{f1} + 2\ell_{f2}m_{\omega_f} + 2\ell_{f3}m_{c_f} = c_1,\end{aligned} \tag{B.1}$$

where $\ell_{f1},\ \ell_{f2}$ and ℓ_{f3} are positive constants due to the fact that FBFN and its derivatives are always bounded by constants (because of the boundedness of states and Gaussian membership functions) [19]. Using (B.1), (23), Lemma 3 and Assumption 3 we have:

$$\begin{aligned}|\varepsilon_f| &= |(\omega_f^{*^T}\Lambda_f^T + c_f^{*^T}\Omega_f^T)\widetilde{W}_f + (\widetilde{E}^T\Gamma_f^T + h_f^T)W_f^* + \Delta_f| \leqslant \|\omega_f^*\|\|\Lambda_f\|\|\widetilde{W}_f\| + \|c_f^*\|\|\Omega_f\|\|\widetilde{W}_f\| + \|W_f^*\|c_1 + |\Delta_f| \\ &\leqslant 2\ell_{f2}m_{\omega_f}m_{W_f} + 2\ell_{f3}m_{c_f}m_{W_f} + c_1m_{W_f} + b_{\Delta_f} = c_2,\end{aligned} \tag{B.2}$$

where b_{Δ_f} is a positive constant based on Theorem 1 and boundedness of $f(t, X)$.

(b) Based on (B.1) and Lemma 3

$$\|\hat{\xi}_f^T - \hat{\omega}_f^T\Lambda_f^T - \hat{c}_f^T\Omega_f^T\| \leqslant \|\hat{\xi}_f^T\| + \|\hat{\omega}_f^T\|\|\Lambda_f^T\| + \|\hat{c}_f^T\|\|\Omega_f^T\| \leqslant \ell_{f1} + \ell_{f2}m_{\omega_f} + \ell_{f3}m_{c_f} = c_3, \tag{B.3}$$

$$\|\widehat{W}_f^T\Lambda_f\| \leqslant \|\widehat{W}_f^T\|\|\Lambda_f\| \leqslant \ell_{f2}m_{W_f} = c_4, \tag{B.4}$$

$$\|\widehat{W}_f^T\Omega_f\| \leqslant \|\widehat{W}_f^T\|\|\Omega_f\| \leqslant \ell_{f3}m_{W_f} = c_5. \tag{B.5}$$

Thus based on Lemma 3 and B.3, B.4 and B.5

$$\left|\left(\hat{\xi}_f^T - \hat{\omega}_f^T\Lambda_f^T - \hat{c}_f^T\Omega_f^T\right)\widetilde{W}_f\right| \leqslant c_{6_1}, \left|\widehat{W}_f^T\Lambda_f\tilde{\omega}_f\right| \leqslant c_{6_2}, \left|\widehat{W}_f^T\Omega_f\tilde{c}_f\right| \leqslant c_{6_3}, \tag{B.6}$$

$$\left|\left(\hat{\xi}_f^T - \hat{\omega}_f^T\Lambda_f^T - \hat{c}_f^T\Omega_f^T\right)\widetilde{W}_f + \widehat{W}_f^T\Lambda_f\tilde{\omega}_f + \widehat{W}_f^T\Omega_f\tilde{c}_f\right| \leqslant c_{6_1} + c_{6_2} + c_{6_3} = c_6. \tag{B.7}$$

(c) Based on (27), Lemma 3 and boundedness of d

$$|-d + u_r| \leqslant |d| + |u_r| = D + |\hat{\rho}| \leqslant D + m_\rho = c_7. \tag{B.8}$$

(d) Since $\widehat{X} \in U_{\hat{x}}$ and based on Assumption 2, there exists a compact set $U_{\hat{e}}$ so that $\widehat{E} \in U_{\hat{e}}$, thus

$$|u_c| = |K_o^TP_1\widehat{E}| \leqslant c_8. \tag{B.9}$$

(e) Using Assumption 2, Lemma 3, boundedness of FBFNs and (B.9) it can be deduced that all the terms in the proposed control signal (29) are bounded. Thus there exists a positive constant $\bar{v}$ such that $|v| \leqslant \bar{v}$. Similarly using (25) and above fact we have

$$|\varepsilon_g v| \leqslant c_{g3}\bar{v}. \tag{B.10}$$

Similarly using (25) for FBFN (16) we have

$$\left|\left(\hat{\xi}_g^T - \hat{\omega}_g^T\Lambda_g^T - \hat{c}_g^T\Omega_g^T\right)\widetilde{W}_g\right| \leqslant c_{9_1}, \left|\widehat{W}_g^T\Lambda_g\tilde{\omega}_g\right| \leqslant c_{9_2}, \left|\widehat{W}_g^T\Omega_g\tilde{c}_g\right| \leqslant c_{9_3}, \tag{B.11}$$

$$\left|\left(\hat{\xi}_g^T - \hat{\omega}_g^T\Lambda_g^T - \hat{c}_g^T\Omega_g^T\right)\widetilde{W}_g v + \widehat{W}_g^T\Lambda_g\tilde{\omega}_g v + \widehat{W}_g^T\Omega_g\tilde{c}_g v\right| \leqslant (c_{9_1} + c_{9_2} + c_{9_3})\bar{v} = c_9. \tag{B.12}$$

(f) Since $L^{-1}(s)$ is a stable filter and if $\chi(t)$ is a bounded signal, then clearly there exists a positive constant β such that $|L^{-1}(s)\chi(t)| \leqslant \beta$. Thus based on (B.1)–(B.11) and (B.12) we have

$$\left|L^{-1}(s)\left\{\left(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T\right)\widetilde{W}_f+\widehat{W}_f^T\Lambda_f\tilde{\omega}_f+\widehat{W}_f^T\Omega_f\tilde{c}_f\right\}\right|\leqslant c_{10}, \tag{B.13}$$

$$\left|L^{-1}(s)\left\{\left(\hat{\xi}_g^T-\hat{\omega}_g^T\Lambda_g^T-\hat{c}_g^T\Omega_g^T\right)\widetilde{W}_g v+\widehat{W}_g^T\Lambda_g\tilde{\omega}_g v+\widehat{W}_g^T\Omega_g\tilde{c}_g v\right\}\right|\leqslant c_{11}, \tag{B.14}$$

$$|L^{-1}(s)(d-u_r)|\leqslant c_{12}, \tag{B.15}$$

$$|L^{-1}(s)\varepsilon_f|\leqslant c_{13}, \tag{B.16}$$

$$|L^{-1}(s)(\varepsilon_g v)|\leqslant c_{14}, \tag{B.17}$$

$$|L^{-1}(s)u_c|\leqslant c_{15}. \tag{B.18}$$

Therefore, using the above inequalities we have

$$|\delta|\leqslant c_6+c_7+c_8+c_9+c_{10}+c_{11}+c_{12}+c_{13}+c_{14}+c_{15}=\bar{\delta}. \tag{B.19}$$

Appendix C. Proof of Theorem 2

Consider the following Lyapunov function candidate:

$$V=\frac{1}{2}\widehat{E}^TP_1\widehat{E}+\frac{1}{2}\widetilde{E}^TP_2\widetilde{E}+\frac{\widetilde{W}_f^T\widetilde{W}_f}{2\gamma_1}+\frac{\tilde{\omega}_f^T\tilde{\omega}_f}{2\gamma_2}+\frac{\tilde{c}_f^T\tilde{c}_f}{2\gamma_3}+\frac{\tilde{\omega}_g^T\tilde{\omega}_g}{2\gamma_4}+\frac{\tilde{c}_g^T\tilde{c}_g}{2\gamma_5}+\frac{\widetilde{W}_g^T\widetilde{W}_g}{2\gamma_6}+\frac{\tilde{\rho}^2}{2\gamma_7}. \tag{C.1}$$

The time derivative of (C.1) along (31) and (40), using (30) and (42)

$$\begin{aligned}\dot{V}=&-\frac{1}{2}\widehat{E}^TQ_1\widehat{E}-\frac{1}{2}\widetilde{E}^TQ_2\widetilde{E}+\left\{\widehat{E}^TP_1K_oC\widetilde{E}-\widetilde{E}^TP_2B_cu_c\right\}+\widetilde{W}_f^T\left\{\widetilde{E}^TP_2B_c(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T)^T-\frac{\dot{\widehat{W}}_f}{\gamma_1}\right\}\\&+\tilde{\omega}_f^T\left\{\widetilde{E}^TP_2B_c(\widehat{W}_f^T\Lambda_f)^T-\frac{\dot{\hat{\omega}}_f}{\gamma_2}\right\}+\tilde{c}_f^T\left\{\widetilde{E}^TP_2B_c(\widehat{W}_f^T\Omega_f)^T-\frac{\dot{\hat{c}}_f}{\gamma_3}\right\}+\tilde{\omega}_g^T\left\{\widetilde{E}^TP_2B_c(\widehat{W}_g^T\Lambda_g)^Tv-\frac{\dot{\hat{\omega}}_g}{\gamma_4}\right\}\\&+\tilde{c}_g^T\left\{\widetilde{E}^TP_2B_c(\widehat{W}_g^T\Omega_g)^Tv-\frac{\dot{\hat{c}}_g}{\gamma_5}\right\}+\widetilde{W}_g^T\left\{\widetilde{E}^TP_2B_c(\hat{\xi}_g^T-\hat{\omega}_g^T\Lambda_g^T-\hat{c}_g^T\Omega_g^T)^Tv-\frac{\dot{\widehat{W}}_g}{\gamma_6}\right\}+\left\{\widetilde{E}^TP_2B_c(\delta+d-u_r)-\frac{\tilde{\rho}\dot{\hat{\rho}}}{\gamma_7}\right\}\end{aligned} \tag{C.2}$$

From Lemma 4 and the boundedness of d we have $|\delta+d|\leqslant\bar{\delta}+D$. Nominating $\rho^*\triangleq\bar{\delta}+D$, considering (43), (27) and (28), (C.2) can be written as

$$\begin{aligned}\dot{V}\leqslant&-\frac{1}{2}\widehat{E}^TQ_1\widehat{E}-\frac{1}{2}\widetilde{E}^TQ_2\widetilde{E}+\widetilde{W}_f^T\left\{\tilde{e}\left(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T\right)^T-\frac{\dot{\widehat{W}}_f}{\gamma_1}\right\}+\tilde{\omega}_f^T\left\{\tilde{e}\left(\widehat{W}_f^T\Lambda_f\right)^T-\frac{\dot{\hat{\omega}}_f}{\gamma_2}\right\}\\&+\tilde{c}_f^T\left\{\tilde{e}\left(\widehat{W}_f^T\Omega_f\right)^T-\frac{\dot{\hat{c}}_f}{\gamma_3}\right\}+\tilde{\omega}_g^T\left\{\tilde{e}\left(\widehat{W}_g^T\Lambda_g\right)^Tv-\frac{\dot{\hat{\omega}}_g}{\gamma_4}\right\}+\tilde{c}_g^T\left\{\tilde{e}\left(\widehat{W}_g^T\Omega_g\right)^Tv-\frac{\dot{\hat{c}}_g}{\gamma_5}\right\}\\&+\widetilde{W}_g^T\left\{\tilde{e}\left(\hat{\xi}_g^T-\hat{\omega}_g^T\Lambda_g^T-\hat{c}_g^T\Omega_g^T\right)^Tv-\frac{\dot{\widehat{W}}_g}{\gamma_6}\right\}+\left\{\tilde{\rho}\left(|\tilde{e}|-\frac{\dot{\hat{\rho}}}{\gamma_7}\right)\right\}.\end{aligned} \tag{C.3}$$

If $\|\widehat{W}_f\|<m_{W_f}$ or $(\|\widehat{W}_f\|=m_{W_f}$ and $\left(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T\right)\widehat{W}_f\tilde{e}\leqslant 0))$, then from the first line of adaptation law (44) we have $\dot{\widehat{W}}_f=\gamma_1\left(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T\right)^T\tilde{e}$, thus $\widetilde{W}_f^T\{\tilde{e}\left(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T\right)^T-\frac{\dot{\widehat{W}}_f}{\gamma_1}\}=0$. Now, if $\|\widehat{W}_f\|=m_{W_f}$ and $\left(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T\right)\widehat{W}_f\tilde{e}>0$ then from the second line of adaptation law (44) and (52) we have $\dot{\widehat{W}}_f=\Pr\left(\gamma_1(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T)^T\tilde{e}\right)=\gamma_1\left(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T\right)^T\tilde{e}-\gamma_1\frac{\widehat{W}_f\widehat{W}_f^T}{\|\widehat{W}_f\|^2}\left(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T\right)^T\tilde{e}$, thus

$$\widetilde{W}_f^T\left\{\tilde{e}\left(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T\right)^T-\frac{\dot{\widehat{W}}_f}{\gamma_1}\right\}=\widetilde{W}_f^T\frac{\widehat{W}_f\widehat{W}_f^T}{\|\widehat{W}_f\|^2}\left(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T\right)^T\tilde{e}=\frac{\widetilde{W}_f^T\widehat{W}_f}{\|\widehat{W}_f\|^2}\widehat{W}_f^T\left(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T\right)^T\tilde{e}. \tag{C.4}$$

Since $\left(\hat{\xi}_f^T-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T\right)\widehat{W}_f\tilde{e}>0$ and $\widetilde{W}_f^T\widehat{W}_f\leqslant 0$ (because $\|W_f^*\|\leqslant m_{W_f}$ and $\|\widehat{W}_f\|=m_{W_f}$) we have

$$\widetilde{W}_f^T\left\{\tilde{e}(\hat{\xi}_f-\hat{\omega}_f^T\Lambda_f^T-\hat{c}_f^T\Omega_f^T)^T-\frac{\dot{\widehat{W}}_f}{\gamma_1}\right\}\leqslant 0. \tag{C.5}$$

Similarly, the same procedure can be done for other adaptive laws, after doing this we have:

$$\dot{V} \leqslant -\frac{1}{2}\widehat{E}^T Q_1 \widehat{E} - \frac{1}{2}\widetilde{E}^T Q_2 \widetilde{E}. \quad (C.6)$$

Denoting $Q = diag[Q_1, Q_2]$ and $E_a^T = [\widehat{E}^T, \widetilde{E}^T]$, (C.6) can be rewritten as

$$\dot{V} \leqslant -\frac{1}{2}E_a^T Q E_a \leqslant 0. \quad (C.7)$$

Since Q_1 and Q_2 are positive definite matrices then matrix $Q = \mathrm{diag}[Q_1, Q_2]$ is also positive definite, therefore $\dot{V}$ is negative semi-definite, i.e., $V(\widetilde{W}_f(t),\ \tilde{\omega}_f(t),\ \tilde{c}_f(t),\ \widetilde{W}_g(t),\ \tilde{\omega}_g(t),\ \tilde{c}_g(t),\ \tilde{\rho}(t)) \leqslant V(\widetilde{W}_f(0),\ \tilde{\omega}_f(0),\ \tilde{c}_f(0),\ \widetilde{W}_g(0),\ \tilde{\omega}_g(0),\ \tilde{c}_g(0),\ \tilde{\rho}(0))$, which shows V is non-increasing and bounded. Defining $\Theta(t) = \frac{1}{2}E_a^T Q E_a \leqslant -\dot{V}$ and integrating it with respect to time yields:

$$\int_0^t \Theta(\tau)\,d\tau \leqslant V(\widetilde{W}_f(0), \tilde{\omega}_f(0), \tilde{c}_f(0), \widetilde{W}_g(0), \tilde{\omega}_g(0), \tilde{c}_g(0), \tilde{\rho}(0)) - V(\widetilde{W}_f(t), \tilde{\omega}_f(t), \tilde{c}_f(t), \widetilde{W}_g(t), \tilde{\omega}_g(t), \tilde{c}_g(t), \tilde{\rho}(t)). \quad (C.8)$$

Because $V(\widetilde{W}_f(0), \tilde{\omega}_f(0), \tilde{c}_f(0), \widetilde{W}_g(0), \tilde{\omega}_g(0), \tilde{c}_g(0), \tilde{\rho}(0))$ is bounded and $V(\widetilde{W}_f(t), \tilde{\omega}_f(t), \tilde{c}_f(t), \widetilde{W}_g(t), \tilde{\omega}_g(t), \tilde{c}_g(t), \tilde{\rho}(t))$ is non-increasing and bounded, the following result is obtained:

$$\lim_{t\to\infty}\int_0^t \Theta(\tau)\,d\tau < \infty. \quad (C.9)$$

Also, $\dot{\Theta}(t)$ is bounded, so by using the Barbalat's lemma [30], (if the differentiable function $h(t)$ has a finite limit as $t \to \infty$, and is such that $\ddot{h}$ exists and is bounded, then $\dot{h}(t) \to 0$ as $t \to \infty$), it can be shown that $\lim_{t\to\infty}\Theta(t) = 0$. Therefore, we have $\lim_{t\to\infty}E_a(t) = 0$. Thus $\lim_{t\to\infty}\widehat{E} = 0$ and $\lim_{t\to\infty}\widetilde{E} = 0$. On the other hand, $E = \widehat{E} + \widetilde{E}$, so $\lim_{t\to\infty}E = 0$.

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