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TRANSIENT ANALYSIS OF FUNCTIONALLY GRADED THICK HOLLOW CIRCULAR CYLINDERS UNDER MECHANICAL LOADINGS

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1. Summary

In this article, functionally graded hollow cylinders with finite length under axisymmetric dynamic loads are analyzed. The dynamic loads applied on the cylinder are axisymmetric in the hoop direction. The governing equations of motion are solved by the two-dimensional finite element and Newmark methods. An especial element is introduced to model the distribution of material properties through the thickness of cylinder exactly according to the power law distribution. The transient responses of functionally graded cylinders which are excited in radial direction by suddenly internal pressure and line load are calculated. The results in the present study are obtained for thick cylinder and cylindrical shell and compared with the results for isotropic cylinders.

2. Introduction

A functionally graded material (FGM) is usually a combination of two material phases that has a gradual transition from one material at one surface to another material at the opposite surface. This transition allows the creation of multiple properties without any mechanically weak junction or interface.

Some research has been done related to response of functionally graded cylinders under dynamic thermal and mechanical loads. Gong et al. [1] used Reddy’s third-order shear deformation theory to present an analytical solution to predict the transient response of simply supported FGM cylindrical shells subjected to low-velocity impact by a solid striker. Asgari et al. [2] considered a thick hollow cylinder with finite length made of two-dimensional functionally graded material subjected to impact internal pressure and investigated the time histories of displacements, stresses, and two-dimensional wave propagation. Pradhan et al. [3] investigated the vibration behavior of functionally graded cylindrical shells based on Love’s theory and the Rayleigh–Ritz method. Han et al. [4] presented a numerical method for analyzing transient waves in FGM cylinders. In their method, the FGM shell was divided into layer elements with three nodal lines along the wall thickness. Yang and Shen [5] used Reddy’s higher-order shear deformation shell theory to investigate free vibration and dynamic instability of functionally graded cylindrical panels subjected to thermo-mechanical loads consisting of a steady temperature change as well as static and periodically pulsating forces in axial direction. Shakeri et al. [6] studied vibration and radial wave propagation in FGM thick hollow cylinders with assumption that the FGM cylinder was made from many isotropic subcylinders. Material properties in each layer were constant and functionally graded properties were resulted by suitable arrangement of layers in the multilayer cylinder.

In the aforementioned works, multi-layered method has been used widely, in which a FGM cylinder is divided into several layers and each layer is divided into a number of 2-node elements along the radial direction. Also, the mechanical properties have been considered to be constant inside each of elements. In the present study, by introducing especial elements in which mechanical properties can be considered variable, the power law distribution of the material properties through the thickness of cylinder is modeled exactly. Two-dimensional finite element method in conjunction with the Newmark method is used to solve the system of time-dependent coupled equations that govern the dynamic responses. The results in the present study are obtained for thick cylinder and cylindrical shell and compared with the results for isotropic cylinders.

3. Theoretical Formulations

Consider a FGM circular hollow cylinder, which is made of a mixture of ceramic and metal with an inner radius r_{in} and an outer radius r_{out}. The material properties are graded in the thickness direction of the cylinder according to a power law distribution. A cylindrical coordinate system is introduced with the origin located at the center of one end of the cylinder and coordinates r, θ, and z are in the thickness, circumferential, and axial directions, respectively. Since the geometry of the cylinder and the loads are assumed to be independent of the circumferential direction, the problem is axisymmetric. The governing equations of motion for this case are:

$$\frac{1}{r}\frac{\partial(r\sigma_r)}{\partial r}-\frac{\sigma_\theta}{r}+\frac{\partial\sigma_{rz}}{\partial z}=\rho\frac{\partial^2 u_r}{\partial t^2}, \quad \frac{1}{r}\frac{\partial(r\sigma_{rz})}{\partial r}+\frac{\partial\sigma_z}{\partial z}=\rho\frac{\partial^2 u_z}{\partial t^2} \tag{1}$$

Upon substitution of stress-strain relations into Eqs. (1) and by applying the Rayleigh-Ritz technique weak formulations are obtained as:

$$\begin{aligned}&\int\left\{\frac{\partial w_1}{\partial r}\left[C_{11}r\frac{\partial u_r}{\partial r}+C_{12}\left(u_r+r\frac{\partial u_z}{\partial z}\right)\right]+\frac{\partial w_1}{\partial z}\left[C_{66}r\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right)\right]+w_1\left[C_{11}\frac{u_r}{r}+C_{12}\left(\frac{\partial u_z}{\partial z}+\frac{\partial u_r}{\partial r}\right)\right]\right.\\&\qquad\left.+\rho r w_1\ddot{u}_r\right\}drdz-\oint w_1 t_r\,ds=0\\&\int\left\{\frac{\partial w_2}{\partial z}\left[C_{11}r\frac{\partial u_z}{\partial z}+C_{12}\left(u_r+r\frac{\partial u_r}{\partial r}\right)\right]+\frac{\partial w_2}{\partial r}\left[C_{66}r\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right)\right]+\rho r w_2\ddot{u}_z\right\}drdz-\oint w_2 t_z\,ds=0\end{aligned} \tag{2}$$

where w_1 and w_2 are weight functions and t_r and t_z are the force tractions on the boundary of the cylinder, which are defined as:

$$t_r=\sigma_r n_r+\sigma_{rz}n_z, \quad t_z=\sigma_z n_z+\sigma_{rz}n_r \tag{3}$$

In Eqs. (2) the dot represents the derivative with respect to time. Next, consider a section of the cylinder which is divided into a finite number of elements. The rectangular elements have two degrees of freedom in each node and they are placed in the radial and longitudinal directions of the cylinder. Also, the linear interpolation functions are selected as:

$$\psi_1=\left(1-\frac{r}{a}\right)\left(1-\frac{z}{b}\right), \quad \psi_2=\frac{r}{a}\left(1-\frac{z}{b}\right), \quad \psi_3=\frac{z}{b}\left(1-\frac{r}{a}\right), \quad \psi_4=\frac{z}{b}\frac{r}{a} \tag{4}$$

Displacement components are approximated with summation of interpolation functions as:

$$u_r=\sum_{j=1}^{n_c}u_j\psi_j, \quad u_z=\sum_{j=1}^{n_c}v_j\psi_j \tag{5}$$

Next, the weight functions are replaced by the interpolation functions to obtain the following governing equation of motion for each element:

$$\begin{bmatrix} [M^{11}] & [0] \\ [0] & [M^{22}] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}\} \\ \{\ddot{v}\} \end{Bmatrix} + \begin{bmatrix} [K^{11}] & [K^{12}] \\ [K^{21}] & [K^{22}] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{v\} \end{Bmatrix} = \begin{Bmatrix} \{F^{1}\} \\ \{F^{2}\} \end{Bmatrix} \tag{6}$$

The mass and stiffness matrices are calculated for each individual element and then through an assemblage process the global assembled matrices for the whole FGM cylinder are calculated. In this study, all of the integrations in mass and stiffness components are calculated analytically over each element domain to avoid any numerical errors. Here, the Newmark direct integration method with suitable time step that is used widely in structural dynamics is employed and the equations of motion are solved.

4. Numerical Results and Discussion

The accuracy and effectiveness of the present study are demonstrated by comparing the results of the present method with the results of hybrid numerical method (HNM) presented by Han et al. [4]. In the HNM, the FGM cylinder is divided into N cylindrical elements with three-nodal line in the wall thickness and the element material properties are assumed to vary linearly in the thickness direction. The thick FGM cylinder ($r_{in} = h$) with Silicon nitride on its inner surface and Stainless steel on its outer surface with the power law exponent $n = 4$ is considered (see [4]). The cylinder is subjected to radial line load of $q = q_0\delta(z)f(t)$ uniformly distributed along the circumferential direction, where δ is the Dirac delta function, and $f(t)$ is a function of time. In the calculations, the dimensionless parameters and loading are assumed as considered by Han et al. [4]. The loading is one cycle of a sine function. Time history of the radial displacement at $z = 10h$ on the outer surface of the cylinder excited at z=0, is shown in Fig. 1. It is seen that there is a good agreement between the present solution and that of Han et al. [4].

To illustrate dynamic response of the functionally graded cylinder, loads are applied to the cylinder in radial direction as suddenly applied internal pressure or concentrated as radial line load. The functionally graded cylinder is assumed to be made of a combination of metal (Ti–6Al–4V) and ceramic (ZrO_2) (see [7] for material properties). The effects of FGM configuration are studied by considering the responses of two FGM cylinders: Type 1 and Type 2. The former has metal on its outer surface and ceramic on its inner surface, while the latter is reverse. For both types of cylinders, power law exponent $n = 1$ is considered.

Next, two ratios of the inner radius to thickness, are employed in calculations; the cylinder with $r_{in}/h = 1$ is viewed as a thick cylinder, and with $r_{in}/h = 20$ is viewed as a cylindrical shell. Also, the ratios of the length to inner radius are considered as $L/r_{in} = 20$ for distributed load and $L/r_{in} = 10$ for radial line load. For radial loading, it is assumed that the cylinders have clamped boundary conditions at the ends.

As an example, consider load $P_0 = 100$ kPa is applied suddenly (i.e., step excitation) to the cylinder for case of the internal pressure loading. Also in radial line loading case the load $P_0 = -10^4$ N/m is applied at the middle length of the cylinder and uniformly distributed along the circumferential direction. The time histories of the radial displacement at the middle length on the outer surface of the cylinders are shown in Fig. 2 for internal pressure loading and in Fig. 3 for radial line loading. Also, the dynamic load factors are displayed in Table 1. Dynamic load factor is calculated by dividing the maximum value of the radial displacement in dynamic loading condition to the maximum value of the radial displacement in static loading condition.

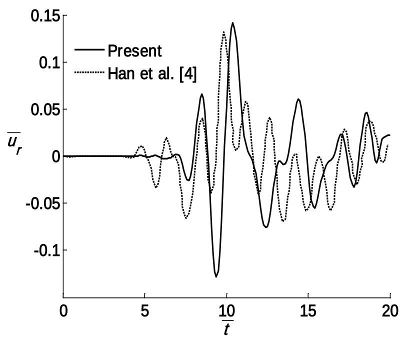

Figure 1 Time history of radial displacement at z=10h on the outer surface of the thick cylinder.

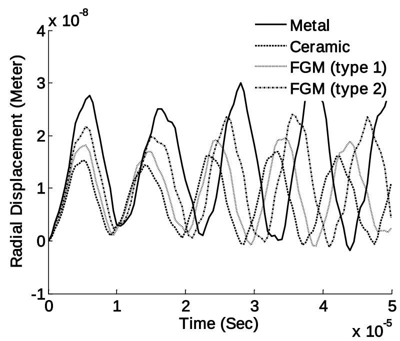

Figure 2 Time history of radial displacement in thick cylinder subjected to suddenly internal pressure loading.

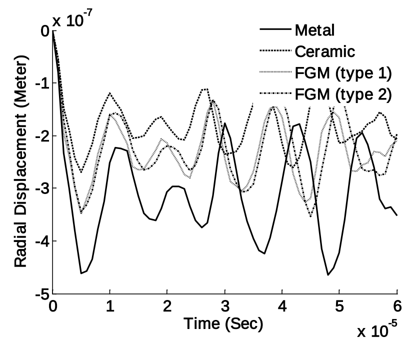

Figure 3 Time history of radial displacement in thick cylinder subjected to suddenly radial line loading.

Table 1 Values of dynamic load factor in the case of suddenly loading.

Loading	Cylinder	Thick cylinder	Cylindrical shell
Internal pressure loading	Metal	1.9145	1.9865
	Ceramic	1.8640	1.9837
	FGM (type 1)	1.8835	1.9833
	FGM (type 2)	1.9037	1.9839

5. Conclusions

Analysis of functionally graded hollow cylinders with finite length under axisymmetric dynamic loads is presented. Two-dimensional finite element and the Newmark methods are used for solving governing equations of motion. For increasing accuracy of the solution, an especial element is introduced in which material properties can be considered variable inside the element exactly according to the distribution of material properties in FGM cylinders. Step excitation axisymmetric loads are applied and the results are compared. Numerical results reveal that dynamic responses of FGM cylinders are between the responses of metal and ceramic cylinders. It is also found that the dynamic load factors of FGM cylinders are close to those of pure cylinders.

6. References

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