

Polynomial Eigen-Beamformer in Time Domain for MIMO-OFDM Systems

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Abstract — Co-space interference (CSI) mitigation is one of the main challenging issues in multiple input multiple output (MIMO) systems. Using beamformer in both transmit and receive sides is an approach to mitigate the CSI. In the MIMO systems employing orthogonal frequency division multiplexing (OFDM) technique, frequency domain or time domain beamforming can be used. In this paper, we propose a new time domain broadband beamforming (TBBF) for MIMO-OFDM systems based on a SVD type of block circular channel matrix. TBBF is attained by polynomial eigen-beamformer when the right and the left polynomial eigenvectors are utilized at the transmitter and the receiver sides, respectively. The new proposed TBBF is able to completely mitigate the CSI due to shaping the multiple beams in orthogonal directions. Performance of the proposed polynomial eigen-based TBBF is evaluated by computer simulations and compared with the previously proposed TBBF scheme based on the SBR2 algorithm. The results show that the proposed time domain beamforming method outperforms the SBR2-based scheme.

Index Terms — Broadband Beamforming, MIMO, OFDM, Polynomial, SVD, Time Domain.

I. INTRODUCTION

Utilizing multiple-input multiple-output (MIMO) antennas in communication systems has been considered as a promising technique to increase the capacity for wireless links. However, multiple signals transmitted over the frequency selective MIMO channel cause co-space interference (CSI) and intersymbol interference (ISI). A lot of attention has been focused on mitigating the CSI and the ISI in order to fulfill the high capacity of the MIMO system. Orthogonal frequency division multiplexing (OFDM) and beamforming are considered as capable techniques to mitigate the CSI and the ISI, respectively. OFDM is a technique that converts the frequency selective MIMO channel to a number of decoupled flat MIMO subchannels. By adding a cyclic prefix to the transmitted signal, the OFDM technique is able to mitigate the ISI effect totally.

Beamforming (BF) at transmit and receive sides is a promising solution for the CSI mitigation in MIMO systems. Singular value decomposition (SVD) technique, which converts a flat MIMO channel into a number of parallel and independent single input single output (SISO) subchannels [1],

is able to mitigate the CSI completely. The SVD-based beamforming shapes the directions of transmit and receive beams in each flat MIMO subchannel to avoid beam interference. This method of beamforming is called frequency-domain broadband beamforming (FBBF) due to shaping beams in each MIMO subchannel independently. However, when the number of subchannels is large, the FBBF method becomes complex due to computing the SVD for all subchannels. The other approach is to employ time-domain broadband beamformer (TBBF) at both transmit and receive sides. This approach is called joint precoding and equalizing as well. Since the channel impulse response duration is less than the number of subchannels in the MIMO-OFDM systems, the TBBF method can be less complex compared to the FBBF method.

Different methods have been proposed to overcome the CSI in literature. A broadband beamforming for single carrier MIMO system has been proposed in [1] based on sequential best rotation (SBR2) algorithm [2] that calculates the eigenvalue decomposition (EVD) of polynomial matrix. In [3], a design of joint precoder and equalizer has been considered based on the SBR2 algorithm for mitigating the CSI. A joint transmit and receive filterbank as a joint precoder and equalizer has been proposed in [4,5] to omit the residual ISI. Broadband MIMO beamforming for frequency selective channels based on using the SBR2 has been discussed in [6] in which, the dominant subchannel with a Viterbi-based receiver is used in order to detect the transmitted data. All time-domain broadband beamforming methods developed based on the SBR2 algorithm are able to partially overcome the CSI effect and improve the BER performance; however, the SBR2 algorithm is not able to completely mitigate the CSI. Thus, as shown in [7], an error floor is happened in the BER performance of the SBR2-based TBBF in frequency selective channels with more time dispersion.

In this paper, we further discuss the effect of the residual CSI in the SBR2 algorithm. Then we propose a new polynomial eigen-beamformer in time-domain for MIMO-OFDM systems based on the SVD of block circular channel matrix. The new polynomial eigen-based broadband beamforming is able to completely overcome the CSI. Computer simulation results show that the proposed polynomial eigen-based TBBF outperforms the SBR2-based TBBF in the sense of BER. Meanwhile, the difference in BER performance between these two TBBF methods becomes larger by increasing the number of transmit or receive antennas.

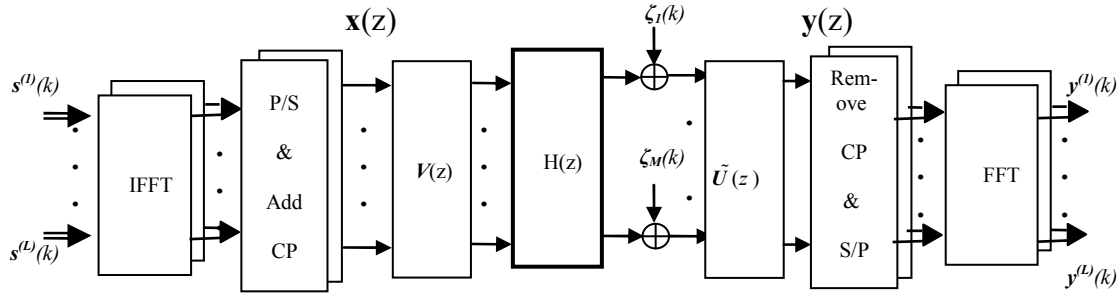


Figure 1. Structure of the MIMO-OFDM system employing time-domain broadband beamforming approach

The structure of this paper is as follows. After introduction, the SBR2-based TBBF method is summarized in section II. Based on the SVD of block circular channel matrix, in section III, the polynomial eigen-beamforming method in time-domain is developed for MIMO-OFDM system. Section IV contains computer simulations and section V concludes the paper.

II. SBR2-BASED TIME-DOMAIN BEAMFORMER

Consider a frequency selective MIMO channel with N transmitting and M receiving antennas. Z -transfer function of $H(k)$, impulse response polynomial matrix of the MIMO channel with the length of L_c , is given as

$$\mathbf{H}(z) = \sum_{k=0}^{L_c-1} H(k)z^{-k} \quad (1)$$

Defining two para-Hermitian polynomial matrix $\mathbf{A}(z) = \mathbf{H}(z)\tilde{\mathbf{H}}(z)$ and $\mathbf{B}(z) = \tilde{\mathbf{H}}(z)\mathbf{H}(z)$ where $\tilde{\mathbf{H}}(z) = \mathbf{H}^H(z^{-1})$ and $(\cdot)^H$ indicates conjugate transpose operation, the SBR2 algorithm [2] can be employed to decompose $\mathbf{A}(z)$ and $\mathbf{B}(z)$ as

$$\mathbf{A}(z) = \mathbf{G}_1(z)\mathbf{D}(z)\tilde{\mathbf{G}}_1(z) \quad (2)$$

$$\mathbf{B}(z) = \mathbf{G}_2(z)\mathbf{D}(z)\tilde{\mathbf{G}}_2(z) \quad (3)$$

where $\mathbf{G}_1(z)$ and $\mathbf{G}_2(z)$ are paraunitary matrices such that $\tilde{\mathbf{G}}_1(z)\mathbf{G}_1(z) = \mathbf{I}$, $\tilde{\mathbf{G}}_2(z)\mathbf{G}_2(z) = \mathbf{I}$ and $\mathbf{D}(z)$ is a diagonal matrix. Meanwhile, $\mathbf{H}(z)$ can be decomposed as

$$\mathbf{H}(z) = \mathbf{U}(z)\mathbf{A}(z)\tilde{\mathbf{V}}(z) \quad (4)$$

Substituting (4) in the defined $\mathbf{A}(z)$ and $\mathbf{B}(z)$ matrices and considering (2) and (3), one can show that $\mathbf{U}(z) = \mathbf{G}_1(z)$, $\mathbf{V}(z) = \mathbf{G}_2(z)$ and $\mathbf{D}(z) = \tilde{\mathbf{A}}(z)\mathbf{A}(z)$. Following [7], the z -transform of the received vector after utilizing $\mathbf{V}(z)$ and $\tilde{\mathbf{U}}(z)$ as transmitter and receiver beamforming matrices becomes

$$\mathbf{X}_r(z) = \tilde{\mathbf{U}}(z)\mathbf{H}(z)\mathbf{V}(z)\mathbf{X}_t(z) + \tilde{\boldsymbol{\xi}}(z) \quad (5)$$

where $\tilde{\boldsymbol{\xi}}(z)$ is the z -transform of the additive white Gaussian noise vector. After passing $\mathbf{X}_r(z)$ through $\tilde{\mathbf{A}}(z)$, the z -transform of the output $\mathbf{X}_d(z)$ vector becomes

$$\mathbf{X}_d(z) = \mathbf{D}(z)\mathbf{X}_r(z) + \boldsymbol{\rho}(z) \quad (6)$$

where $\boldsymbol{\rho}(z) = \tilde{\mathbf{A}}(z)\tilde{\mathbf{U}}(z)\tilde{\boldsymbol{\xi}}(z)$. Since $\mathbf{D}(z)$ is a diagonal polynomial matrix, CSI is eliminated and MIMO-OFDM system is decomposed to $P \leq \min(M, N)$ parallel independent OFDM systems. The SBR2-based time-domain beamforming method is implemented based on (5) and (6).

III. POLYNOMIAL EIGEN-BEAMFORMING IN TIME-DOMAIN

To develop the new time-domain polynomial eigen-beamforming scheme, first we consider a SISO-OFDM system with L subcarriers. Defining $\mathbf{h} = [h_1, h_2, \dots, h_{L_c}]^T$ as the vector of channel impulse response and $\mathbf{x}(k) = [x_1(k), \dots, x_{L-1}(k)]^T$ as the L -point inverse fast Fourier transform (IFFT) of the transmitted signal $\mathbf{s}(k) = [s_1(k), \dots, s_{L-1}(k)]^T$, the k th received OFDM symbol, $\mathbf{r}(k) = [r_1(k), \dots, r_{L-1}(k)]^T$, after removing cyclic prefix (CP), becomes

$$\mathbf{r}(k) = \mathbf{H}\mathbf{x}(k) + \boldsymbol{\zeta}(k) \quad (7)$$

where $\boldsymbol{\zeta}(k)$ is an additive white Gaussian noise vector and \mathbf{H} is the $L \times L$ circular channel matrix that is given as

$$\mathbf{H} = \begin{bmatrix} h_0 & 0 & \cdots & 0 & h_{L_c-1} & \cdots & h_3 & h_2 & h_1 \\ h_1 & h_0 & 0 & \cdots & 0 & h_{L_c-1} & \cdots & h_3 & h_2 \\ h_2 & h_1 & h_0 & 0 & \cdots & 0 & h_{L_c-1} & \cdots & h_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & h_{L_c-1} & \cdots & h_2 & h_1 & h_0 \end{bmatrix} \quad (8)$$

The above process can be extended to a MIMO-OFDM system with N transmitting antennas and M receiving antennas with L subcarriers. Defining \mathbf{x}_i as the transmitted signal from

i th antenna and \mathbf{r}_j as the received signal by the j th antenna, it can be written as

$$\begin{bmatrix} \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_M \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{11} & \cdots & \mathbf{H}_{1N} \\ \vdots & \vdots & \vdots \\ \mathbf{H}_{M1} & \cdots & \mathbf{H}_{MN} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_N \end{bmatrix} + \begin{bmatrix} \zeta_1 \\ \vdots \\ \zeta_M \end{bmatrix} \quad (9)$$

where for simplicity time index k is removed and \mathbf{H}_{ji} for $i=1, \dots, N$ and $j=1, \dots, M$ is the circular channel matrix shaped similar to (8) based on the channel impulse response between i th transmit antenna and j th receive antenna. Equation (9) can be rewritten as

$$\mathbf{r} = \mathcal{H}\mathbf{x} + \zeta \quad (10)$$

where $\mathbf{r} = [\mathbf{r}_1^T, \dots, \mathbf{r}_M^T]^T$, $\mathbf{x} = [\mathbf{x}_1^T, \dots, \mathbf{x}_N^T]^T$, $\zeta = [\zeta_1^T, \dots, \zeta_M^T]^T$ and \mathcal{H} is a block-circular matrix. Defining \mathbf{W}_j as a $JL \times JL$ block diagonal FFT matrix, $\mathbf{x} = \mathbf{W}_N^H \mathbf{s}$ where $\mathbf{s} = [\mathbf{s}_1^T, \dots, \mathbf{s}_N^T]^T$ and the Fourier transform of the received signal vector becomes

$$\begin{aligned} \mathbf{y} &= \mathbf{W}_M \mathbf{r} = \mathbf{W}_M \mathcal{H} \mathbf{W}_N^H \mathbf{s} + \mathbf{W}_M \zeta \\ &= \mathbf{H}_F \mathbf{s} + \mathbf{z} \end{aligned} \quad (11)$$

where $\mathbf{H}_F = \mathbf{W}_M \mathcal{H} \mathbf{W}_N^H$. Due to block-circular property of \mathcal{H} , \mathbf{H}_F would be a block diagonal matrix given as

$$\mathbf{H}_F = \begin{bmatrix} h_{111} & \cdots & h_{1N1} & \cdots & \vdots & \vdots & \vdots \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & h_{11L} & \cdots & \vdots & \vdots & h_{1NL} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{M11} & \cdots & h_{MN1} & \cdots & \vdots & \vdots & \vdots \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & h_{M1L} & \cdots & \vdots & \vdots & h_{MNL} \end{bmatrix} \quad (12)$$

The \mathbf{H}_F matrix can be reshaped to a block-diagonal matrix \mathbf{H}_{FBD} by applying a unitary matrix \mathbf{T}_M and the transposed of \mathbf{T}_N to both right and left sides of it, i.e.

$$\mathbf{H}_{FBD} = \mathbf{T}_M \mathbf{H}_F \mathbf{T}_N^T \quad (13)$$

where for example \mathbf{T}_2 is defined as

$$\mathbf{T}_2 = \begin{bmatrix} \overbrace{1 \ 0 \ \dots \ 0}^{L\text{-element}} & \overbrace{0 \ 0 \ \dots \ 0 \ 0}^{L\text{-element}} & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 \ 0 \ \dots \ 0 & 1 \ 0 \ \dots \ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 \ 0 \ \dots \ 0 & 0 \ 1 \ 0 \ \dots \ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 \ \dots \ 0 & 1 \ 0 \ 0 \ \dots \ 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 \ 0 \ \dots \ 0 & 0 \ 0 \ \dots \ 0 & \vdots & \vdots & \vdots & \vdots & 1 \end{bmatrix} \quad (14)$$

Since $\mathbf{T}_J^T \mathbf{T}_J = \mathbf{T}_J \mathbf{T}_J^T = \mathbf{I}_J$, we have $\mathbf{H}_F = \mathbf{T}_M^T \mathbf{H}_{FBD} \mathbf{T}_N$. Hence, by substituting the SVD of the \mathbf{H}_{FBD} matrix, $\mathbf{H}_{FBD} = \mathbf{U}_{FBD} \mathbf{A}_{FBD} \mathbf{V}_{FBD}^H$ in (13), we have

$$\mathbf{H}_F = \mathbf{T}_M^T \mathbf{U}_{FBD} \mathbf{A}_{FBD} \mathbf{V}_{FBD}^H \mathbf{T}_N \quad (15)$$

Due to unitary property of transformation matrix \mathbf{T} and the SVD definition of \mathbf{H}_F as $\mathbf{H}_F = \mathbf{U}_F \mathbf{A}_F \mathbf{V}_F^H$, it can be inferred that

$$\mathbf{U}_F = \mathbf{T}_M^T \mathbf{U}_{FBD} \mathbf{T}_P \quad (16.a)$$

$$\mathbf{A}_F = \mathbf{T}_P^T \mathbf{A}_{FBD} \mathbf{T}_P \quad (16.b)$$

$$\mathbf{V}_F^H = \mathbf{T}_P^T \mathbf{V}_{FBD}^H \mathbf{T}_N \quad (16.c)$$

Reconsidering the \mathcal{H} matrix and due to definition of the \mathbf{H}_F matrix, it can be shown that

$$\mathcal{H} = \mathbf{W}_M^H \mathbf{U}_F \mathbf{A}_F \mathbf{V}_F^H \mathbf{W}_N \quad (17)$$

Define the block SVD of \mathcal{H} as $\mathcal{H} = \mathbf{U} \mathbf{A} \mathbf{V}^H$ and compare it with (17), we can conclude that

$$\mathbf{U} = \mathbf{W}_M^H \mathbf{U}_F \mathbf{W}_P \quad (18.a)$$

$$\mathbf{A} = \mathbf{W}_P^H \mathbf{A}_F \mathbf{W}_P \quad (18.b)$$

$$\mathbf{V} = \mathbf{W}_N^H \mathbf{V}_F \mathbf{W}_P \quad (18.c)$$

where \mathbf{U} and \mathbf{V} are block circular unitary matrices and \mathbf{A} is a block circular diagonal matrix. By substituting the block SVD of \mathcal{H} from (18) in (10), we have

$$\mathbf{r} = \mathbf{U} \mathbf{A} \mathbf{V}^H \mathbf{x} + \zeta \quad (19)$$

By utilizing \mathbf{V} and \mathbf{U}^H as the transmitter and receiver time-domain beamformer block circular matrices, we have

$$\begin{aligned} \mathbf{y} &= \mathbf{U}^H \mathbf{r} = \mathbf{U}^H \mathbf{U} \mathbf{A} \mathbf{V}^H \mathbf{V} \mathbf{x} + \mathbf{U}^H \zeta \\ &= \mathbf{A} \mathbf{x} + \boldsymbol{\eta} \end{aligned} \quad (20)$$

By converting the block circular diagonal \mathbf{A} matrix and taking z-transform, one can show that

$$\mathbf{y}(z) = \mathbf{A}(z) \mathbf{x}(z) + \boldsymbol{\eta}(z) \quad (21)$$

where $\mathbf{A}(z) = \text{diag}(\lambda_1(z), \dots, \lambda_p(z))$ is a diagonal polynomial matrix. Thus, as shown in Fig. 1, by utilizing $\mathbf{V}(z)$ and $\tilde{\mathbf{U}}(z)$ polynomial matrices, which are attained from the first column of each block of \mathbf{V} and \mathbf{U}^H block circular matrices, respectively, the new time-domain polynomial eigen beamforming method is developed. For MIMO-OFDM system as shown in Fig.2, the polynomial MIMO channel matrix $\mathbf{H}(z)$ is decomposed to the P parallel and decoupled SISO channels with $\lambda_i(z)$ transform function for $i=1, \dots, P$.

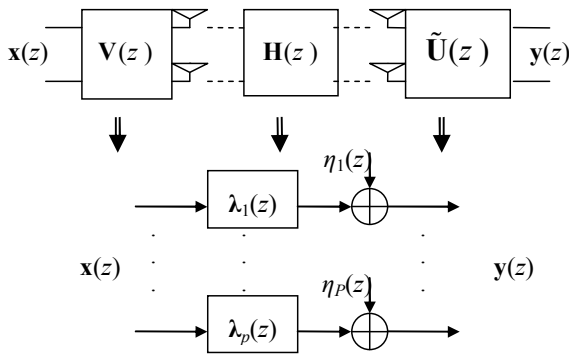


Figure 2. Decomposing the polynomial MIMO channel matrix $\mathbf{H}(z)$ into P parallel polynomial SISO channels $\lambda_1(z), \dots, \lambda_p(z)$.

IV. COMPUTER AND SIMULATION RESULTS

A MIMO-OFDM system with $M = N = 2$ and 4, as depicted in Fig. 1, is considered in simulations. A sequence of independent, identically distributed QPSK or 16QAM signal vector is sent from transmitter antenna array. Each frequency selective channel between Tx-Rx antennas is realized based on an exponentially decaying power delay profile with $L_c = 16$ resolvable paths.

Since SBR2 is a diagonalizing algorithm that computes $\mathbf{D}(z) = \tilde{\mathbf{\Lambda}}(z)\mathbf{\Lambda}(z)$ approximately, $\mathbf{D}(z)$ does not become an exact diagonal polynomial matrix [7]. Hence, the CSI is not completely eliminated. Performance of the SBR2 diagonalizing algorithm is evaluated for a different exponentially decay factors, i.e. $\beta = 0, 0.3, 0.5$ and 0.8 and in Fig. 3. The powers of diagonal and off-diagonal tap weights of $\mathbf{D}(z)$ polynomial matrix are depicted in Fig. 3 when $M = N = 2$. As seen in Fig. 3, for tap index interval of $-L_c < l < L_c$, the power of diagonal tap weights of $\mathbf{D}(z)$ is significantly higher than that of the off-diagonal ones. Thus, with good approximation, $\mathbf{D}(z)$ can be assumed to be a diagonal polynomial matrix for $-L_c < l < L_c$. On the other hand, $\mathbf{\Lambda}(z)$, which is computed based on $\mathbf{\Lambda}(z) = \tilde{\mathbf{U}}(z)\mathbf{H}(z)\mathbf{V}(z)$, does not necessarily become a diagonal polynomial matrix. To highlight this phenomenon, the powers of diagonal and off-diagonal tap weights of $\mathbf{\Lambda}(z)$ have been depicted in Fig. 4 for $\beta = 0.8$. As seen, the powers of diagonal and off-diagonal tap weights are in the same order that indicates $\mathbf{\Lambda}(z)$ attained from the SBR2 is not actually a diagonal polynomial matrix. However, as shown in Section III, $\mathbf{\Lambda}(z)$ attained from the new polynomial eigen-based method becomes an exact diagonal polynomial matrix.

In Fig. 5, the BER performance of the SBR2-based TBBF and the new proposed TBBF schemes are compared for $M = N = 2$ and different β values for QPSK modulation scheme. It can be seen that increment of β has no considerable effect on the performance of the proposed polynomial eigen-beamforming approach. However, in the SBR2-based scheme, BER performance completely saturates after $SNR = 12$ dB for

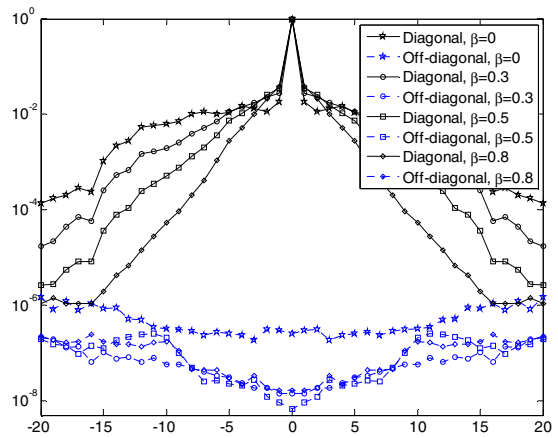


Figure 3. Powers of the diagonal and off-diagonal tap weights of $\mathbf{D}(z) = \tilde{\mathbf{\Lambda}}(z)\mathbf{\Lambda}(z)$ for different β values and $M = N = 2$.

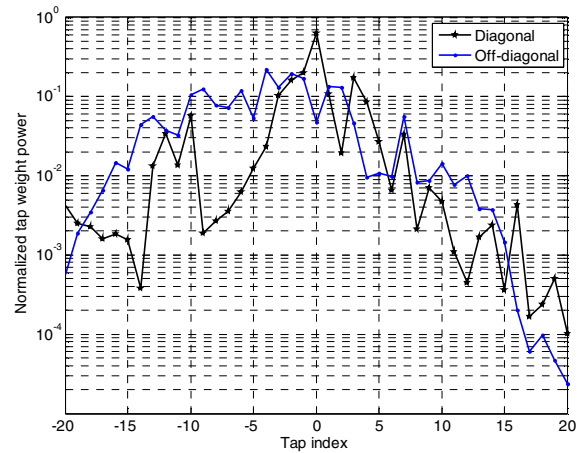


Figure 4. Powers of the diagonal and off-diagonal tap weights of $\mathbf{\Lambda}(z)$ for $\beta = 0.8$ and $M = N = 2$.

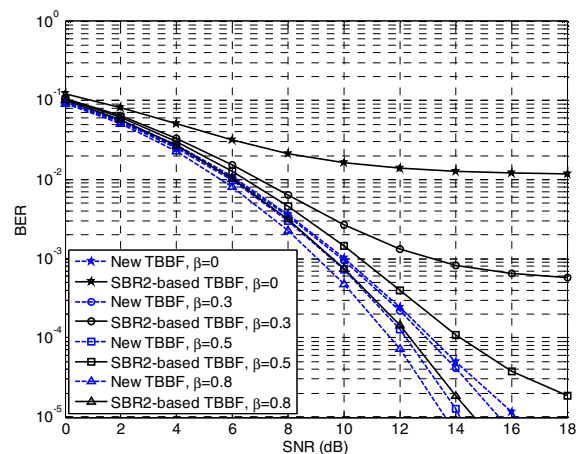


Figure 5. BER versus SNR for the polynomial eigen-beamforming and SBR2-based TBBF for QPSK modulation, different β values and $M = N = 2$.

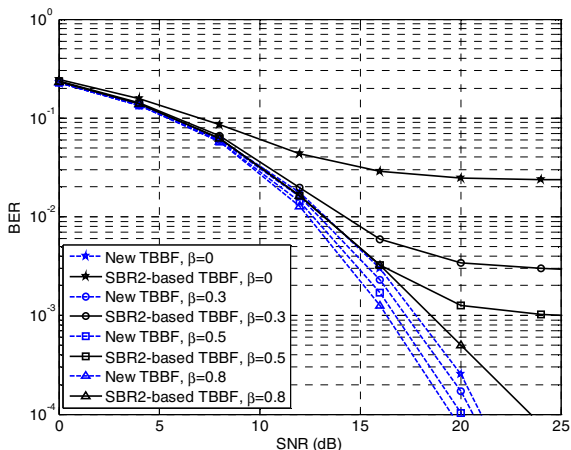


Figure 6. BER versus SNR for the polynomial eigen-beamforming and SBR2-based TBBF for 16QAM modulation, different β values and $M = N = 2$.

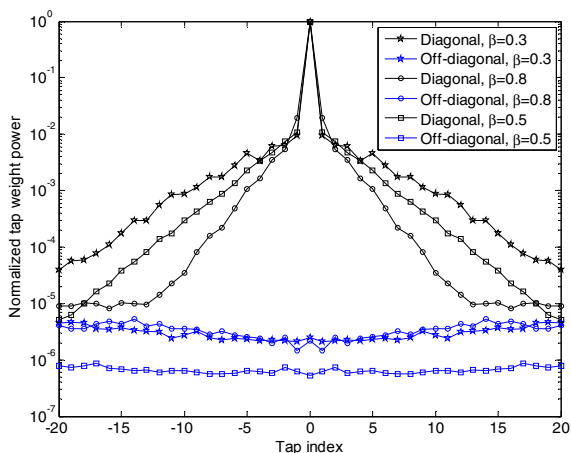


Figure 7. Powers of the diagonal and off-diagonal tap weights of $\mathbf{D}(z) = \tilde{\Lambda}(z)\Lambda(z)$ for different β values and $M = N = 4$.

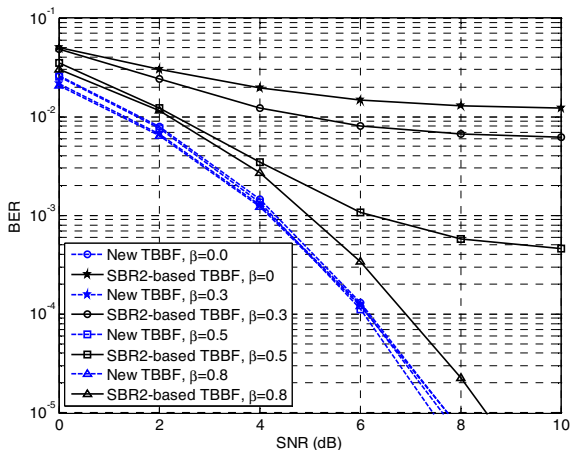


Figure 8. BER versus SNR for the polynomial eigen-beamforming and SBR2-based TBBF for QPSK modulation, different β values and $M = N = 4$.

$\beta = 0$, while it approximately follows the eigen-beamforming scheme performance for $\beta = 0.8$. BER performances of both TBBF approaches are shown in Fig. 6 for 16-QAM modulation scheme. Since the 16-QAM modulation scheme is more sensitive to interference, the floor effect in BER performance occurs for $\beta = 0, 0.3, 0.5$. This effect is also seen in the QPSK modulation scheme. The main reason of the SBR2-based scheme performance degradation is the residual CSI that is produced by the diagonal tap weights out of the considered interval along with the off-diagonal tap weights.

When $M = N = 4$, similar to Fig. 3, the powers of diagonal and off-diagonal tap weights of $\mathbf{D}(z)$ polynomial matrix have been shown in Fig. 7 for different β , exponentially decay factor, values based on one hundred independent channel matrix realizations. As seen in Fig. 7, there is more CSI for $M = N = 4$ in comparison with $M = N = 2$. To evaluate the effect of the CSI in performance, the BER of both TBBF approaches are depicted in Fig. 8 for QPSK modulation scheme. As seen, the new proposed method significantly achieves better performance.

V. CONCLUSIONS

A new time-domain polynomial eigen-beamforming scheme has been proposed in this paper for MIMO-OFDM systems based on the SVD type of the block circular channel matrix. The proposed scheme converts the polynomial channel matrix into a number of parallel and decoupled polynomial SISO channels. Computer simulation results indicates that the new time-domain broadband beamforming (TBBF) method significantly outperforms the previously proposed SBR2-based TBBF method due to its ability in completely eliminating co-space interference (CSI) in time-domain.

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