Joint Subspace Beamforming and Space-Time Coding in Wireless MIMO Systems

S. Morteza Razavi
Department of Electrical Engineering
Ferdowsi University of Mashhad
Mashhad, Iran

H. Zamiri-Jafarian
Department of Electrical Engineering
Ferdowsi University of Mashhad
Mashhad, Iran

Abstract— **In this paper, we propose a new joint beamforming and space-time coding (STC) scheme for wireless multiple-input multiple-output (MIMO) systems based on block singular value decomposition (BSVD). By utilizing the BSVD, transmit and receive beamformers divide the MIMO channel into some decoupled MIMO subchannels, which have different sizes. Due to flexibility in dividing the MIMO channel and applying the STC to each MIMO subchannel independently, the system can be designed based on trade off between diversity, multiplexing and complexity. Combining the dominant subspace beamformer and Alamouti STC, which is developed based on one dimensional BSVD (1D-BSVD), can be considered as one approach. To evaluate the performance of the new approach, the bit error rate (BER) of the proposed joint beamforming and ST coding is compared with the BERs of the orthogonal space-time block codes (OSTBCs) and the quasi-OSTBCs (QOSTBCs) via computer simulations. The results show that the superior performance of the proposed approach.**

Keywords- **MIMO system; beamforming; space-time block coding; orthogonal space-time block coding; singular value decomposition**

I. INTRODUCTION

Increasing data transmission rate in wireless communications is efficiently achievable by employing space dimension via antenna array in both transmitter and receiver. In this system that is called multiple input multiple output (MIMO), space-time (ST) coding is used in order to utilize both space and time diversities. Due to mitigating fading phenomena, the ST coding improves the performance of the MIMO system [1].

Complexity of decoding is the main challenging issue of the ST codes. With respect to this issue, orthogonal space-time block codes (OSTBCs) attract more attention due to their simplicity in maximum likelihood (ML) decoding. However, complex square OSTBCs can't attain both full diversity and full transmission rate simultaneously except the Alamouti code [2]. The maximum transmission rate of the complex OSTBCs using three and four transmit antennas is 0.75, and constructing complex OTSBCs with rates higher than 0.5 for more than four transmit antennas is not straightforward [3].

Compromising in complexity, quasi OSTBCs (QOSTBCs) have been proposed in that two symbols should be decoded simultaneously. Thus the decoding of the QOSTBCs is more complex in comparison with that of OSTBCs. The QOSTBCs can be able to achieve the transmission rate one for more than two transmit antennas, but they cannot attain full diversity. Therefore, in high signal to noise ratios (SNRs), they have inferior performance compared with the OSTBCs [4].

In [5], a new scheme based on rotated constellations has been introduced. This scheme has led to generate a class of QOSTBCs with full diversity that achieves better performance than the unrotated one. However, by increasing the number of transmit antennas, the proposed scheme in [5] has no more improvement in the performance compared with the unrotated QOSTBCs .

Joint transmit and receive beamformer is able to improve the performance of the MIMO system when channel information is available at both transmitter and receiver. The channel matrix can be decomposed by utilizing singular value decomposition (SVD). The right and the left singular vectors are employed as the beamforming vectors at the receive and transmit sides, respectively. Due to using eigen-vectors in the beamformers, this method is called eigen-beamforming. In this situation, the ST coding is not as useful as the beamforming. However, the eigen-beamforming is very susceptible to channel variations, hence joint beamforming and ST coding is an approach to mitigate this circumstance and achieve diversity gain.

In this paper, we propose a joint Alamouti space-time coding and one dimensional block SVD (1-D BSVD) to improve the performance of MIMO systems. Simulation results show that the proposed scheme has more transmission rate and less BER in comparison with the OSTBCs at all SNR ranges. Moreover, the new scheme has more or at least equal transmission rate and less BER compared with the QOSTBCs.

The paper is organized as follow. After introduction, the system model is described in Section II. In Section III, beamforming based on the BSVD is proposed. Combining beamforming and Alamouti space-time coding based on 1-D BSVD is described in section IV. The comparisons between the proposed scheme, the OSTBCs and the QOSTBCs are given via computer simulations in Section V and finally Section VI contains conclusions.

II. SYSTEM MODEL

Consider a system with N transmit and M receive antennas. The channel matrix of the system can be illustrated by $H \in \mathbf{C}^{M \times N}$ in which $\mathbf{C}$ stands for complex numbers set. The channel input and output are shown by $X \in \mathbf{C}^{N \times T}$ and $Y \in \mathbf{C}^{M \times T}$,respectively. T is the time duration of the ST block code (BSTC). The relation between X and Y is given by

$$Y = \sqrt{\frac{\rho}{N}} HX + Z \tag{1}$$

where ρ is the transmitted power and $Z \in \mathbf{C}^{M \times T}$ is an AWGN matrix that all of its elements are modeled by a *i.i.d* Gaussian random variables with zero mean and unit variance. Suppose A is a matrix. $A(i:j,:)$ and $A(:,i:j)$ denotes ith through jth rows and ith through jth columns of A ,respectively. A^* is the conjugate of A , and A^T is the transpose of A . A^{H} is the complex conjugate of A , and $\|A\|_F^2$ is the Frobenius norm of A . $[A]_{i,j}$ denotes the element of ith row and jth column. I_m is an identity matrix with $m \times m$ dimensions. $Row(A)$ denotes the number of A 's rows, and $Col(A)$ denotes the number of A 's columns.

III. BEAMFORMING BASED ON BSVD

The SVD of the channel matrix can be shown by

$$H = U \Sigma V^{\mathrm{H}} \tag{2}$$

where U and V are unitary matrices, i.e. $U^{\mathrm{H}} U = I_M$ and $V^{\mathrm{H}} V = I_N$. Σ is a diagonal matrix whose diagonal elements are singular values of channel matrix arranged in descending order as $\sigma_1 \geq \cdots \geq \sigma_r$. σ_i indicates the ith singular value of H , and it is a positive real number. r is the rank of H .

Here, we propose to decompose H more generally than regular SVD. In this method, U and V are chosen so that Σ is to be a block diagonal matrix. In other words, H can be decomposed as

$$H = U_b \Sigma_b V_b^H \tag{3}$$

where $U_b^{\mathrm{H}} U_b = I_M$ and $V_b^{\mathrm{H}} V_b = I_N$ and Σ_b is a block diagonal matrix as illustrated bellow

$$\Sigma_b = \begin{pmatrix} \Sigma_1 & 0 & 0 & \cdots & 0 \\ 0 & \Sigma_2 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & 0 & \cdots & \Sigma_k \end{pmatrix} \tag{4}$$

Subblocks $\Sigma_i, i = 1,\dots,k$ are rectangular matrices and independent of each other and $\|\Sigma_1\|_F^2 \geq \|\Sigma_2\|_F^2 \geq \cdots \geq \|\Sigma_k\|_F^2$.

We name this decomposition method as block SVD (BSVD).

By decomposing channel matrix based on (3) and using beamforming matrices V_b and U_b^H at transmitter and receiver, respectively, the $M \times N$ space between transmitter and receiver can be divided into some subspaces with smaller dimensions $M_i \times N_i, i = 1,\dots,k$.

If we consider that the channel matrix is square, i.e. $M = N$, then subblocks $\Sigma_i, i = 1,\dots,k$ will be square as well. The transmitted matrix is given as

$$X = V_b S \tag{5}$$

where $S \in \mathbf{C}^{N \times T}$ is a STBC with T symbol period that transmitted from N antennas, and it can be used as STBCs or ST trellis codes (STTCs) or ST turbo trellis codes (STTuTCs). S can have several forms such as Σ_i in (4) or

$$S = \left(S_b^1, \dots, S_b^k\right)^T \tag{6}$$

where $Col(\Sigma_i) = Row(S_b^i), i = 1,\dots,k$.

At the receiver, after multiplying Y with beamforming matrix U_b^H we have

$$R = U_b^{\mathrm{H}} Y = \sqrt{\frac{\rho}{t}} U_b^{\mathrm{H}} U_b \Sigma_b V_b^{\mathrm{H}} X$$
$$= \sqrt{\frac{\rho}{t}} \Sigma_b S + U_b^{\mathrm{H}} Z = \sqrt{\frac{\rho}{t}} \Sigma_b S + Z' \tag{7}$$

Because U_b is a unitary matrix, all elements of Z' are Gaussian and *i.i.d.*

Due to block diagonal form of Σ_b , with reception of R in (7), it is striagthforward to detect S_b^i (transmitted subblock associated with subchannel Σ_i) independently.

IV. ONE DIMENTIONAL BSVD (1-D BSVD)

In this section, we focus on one specific form of BSVD that uses the combination of Alamouti ST coding and eigen-beamforming related to the first block of Σ_b . We suppose that Σ_1 in (4) is a 2×2 matrix (the dimensions of other blocks are not important). While H is decomposed as indicated in (3), transmitted signal is given as

$$X = V_b(:,1:2)S \tag{8}$$

where S has k Alamouti submatrices as bellow:

$$S = \begin{pmatrix} x_1 & -x_2^* & \cdots & x_{2k-1} & -x_{2k}^* \\ x_2 & x_1^* & \cdots & x_{2k} & x_{2k-1}^* \end{pmatrix} \tag{9}$$

where $k = n/2$ and $x_i, i = 1,...,n$ are transmitted symbols chosen from a complex signal constellations. It is obvious that the transmission rate of S per each channel is one. At the receiver, Y in (1) is multiplied by the two first columns of U_b .

$$R = U_b^{\mathrm{H}}(:,1:2)Y = \sqrt{\frac{\rho}{N}} U_b^{\mathrm{H}}(:,1:2) U_b \Sigma_b V_b^{\mathrm{H}} X$$

$$= \sqrt{\frac{\rho}{N}} \Sigma_b(1:2,1:2)S + U_b^{\mathrm{H}}(:,1:2)Z$$

$$= \sqrt{\frac{\rho}{N}} \Sigma_b(1:2,1:2)S + Z'' \tag{10}$$

In this case, the system decoder consists of one Alamouti decoder that must merely run k times. The elements of Z'' are Gaussian and *i.i.d* variables, because $U_b(;,1:2)$ is a unitary matix.

We call this method as one dimensional block SVD (1-D BSVD) because of using one block of channel matrix which has the maximum Frobenius norm for transmitted data. We mention that this method maximizes the received SNR of the first 2×2 block, because Σ_1 has the maximum Frobenius norm. Moreover, with beamforming, it is feasible to prevent the effect of other blocks that have less Frobenius norm (energy).

V. COMPARISONS AND SIMULATION RESULTS

We consider a MIMO system with $M = N = 4$ and 8 . The elements of the flat channel matrix $H \in \mathbf{C}^{M \times N}$ are complex Gaussian variables with zero mean and unit variance. The utilized signal constellations are QPSK and 16-QAM.

To have a fair comparison, we chose OSTBCs and a QOSTBCs for 4 and 8 transmit antennas. For 4 transmit antennas and the OSTBC with rate 0.75 we choose [3]

$$S_1 = \begin{pmatrix} x_1 & -x_2^* & -x_3^* & 0 \\ x_2 & x_1^* & 0 & x_3^* \\ x_3 & 0 & x_1^* & -x_2^* \\ 0 & -x_3 & x_2 & x_1 \end{pmatrix} \tag{11}$$

For QOSTBC with rate one and for 4 transmit antennas we select [4]

$$S_2 = \begin{pmatrix} x_1 & -x_2^* & -x_3^* & x_4 \\ x_2 & x_1^* & -x_4^* & -x_3 \\ x_3 & -x_4^* & x_1^* & -x_2 \\ x_4 & x_3^* & x_2^* & x_1 \end{pmatrix} \tag{12}$$

For 8 transmit antennas and OSTBC with rate 0.5 we choose [3]

$$S_3 = \begin{pmatrix} x_1 & -x_2^* & -x_3^* & 0 & -x_4^* & 0 & 0 & 0 \\ x_2 & x_1^* & 0 & x_3^* & 0 & x_4^* & 0 & 0 \\ x_3 & 0 & x_1^* & -x_2^* & 0 & 0 & x_4^* & 0 \\ 0 & -x_3 & x_2 & x_1 & 0 & 0 & 0 & -x_4^* \\ x_4 & 0 & 0 & 0 & x_1^* & -x_2^* & -x_3^* & 0 \\ 0 & -x_4 & 0 & 0 & x_2 & x_1 & 0 & x_3^* \\ 0 & 0 & -x_4 & 0 & x_3 & 0 & x_1 & -x_2^* \\ 0 & 0 & 0 & x_4 & 0 & -x_3 & x_2 & x_1^* \end{pmatrix} \tag{13}$$

Finally, for 8 transmit antennas and a QOSTBC with rate one we pick up[6]

$$S_4 = \begin{pmatrix} x_1 & -x_2 & -x_3^* & -x_4^* & -x_5^* & -x_6^* & -x_7 & -x_8 \\ x_2 & x_1 & -x_4^* & x_3^* & -x_6^* & x_5^* & x_8 & -x_7 \\ x_3 & x_4 & x_1^* & -x_2^* & -x_7^* & -x_8^* & x_5 & x_6 \\ x_4 & -x_3 & x_2^* & x_1^* & -x_8^* & x_7^* & -x_6 & x_5 \\ x_5 & x_6 & x_7^* & x_8^* & x_1^* & -x_2^* & -x_3 & -x_4 \\ x_6 & -x_5 & x_8^* & -x_7^* & x_2^* & x_1^* & x_4 & -x_3 \\ x_7 & -x_8 & -x_5^* & x_6^* & x_3^* & -x_4^* & x_1 & x_2 \\ x_8 & x_7 & -x_6^* & -x_5^* & x_4^* & x_3^* & -x_2 & x_1 \end{pmatrix} \tag{14}$$

In the proposed 1-D BSVD scheme for 4 transmit antennas, 4 symbols are transmitted in 4 time intervals; thus the transmission rate is one. Whereas for S_1 , three symbols are sent in 4 time intervals, hence the transmission rate is 0.75 that is less than that of the 1-D BSVD scheme. For S_2 the transmission rate is similar to 1-D BSVD and equals one.

In the similar way, for 8 transmit antennas in 1-D BSVD, eight symbols are transmitted in 8 time intervals; thus the transmission rate is one. However, four and eight symbols are transmitted in 8 time intervals by S_3 and S_4 codes, respectively thus the transmission rates are 0.5 and one, respectively.

As noted before, the ML detection in 1-D BSVD scheme is straightforward due to utilizing the Alamouti coding; whereas the detection processes of the OSTBCs and the QOSTBCs (while $M = N \geq 3$) are more complex [2], [3] and [7].

The simulation results have been obtained for 1000 channel realizations. Fig.1 and Fig.2 show the performances of the proposed scheme, the OSTBC and the QOSTBC for QPSK and 16-QAM modulation schemes, respectively. In these figures, the number of the transmitted antennas is four and S_1 in (11) and S_2 in (12) are used for the OSTBC and the QOSTBC, respectively. As seen in Fig.1 and Fig.2, the 1-D BSVD scheme outperforms the OSTBC (S_1) and the QOSTBC (S_2) at least by 1dB and 2dB, respectively. Note that the transmission rate of S_1 is 0.75, whereas the transmission rate of the proposed scheme is one.

In Fig.3 and Fig.4, the performances of the proposed 1-D BSVD, the OSTBC and the QOSTBC schemes are shown for QPSK and 16-QAM modulation methods, respectively, when the number of transmit antennas is eight. The shown results of the OSTBC and the QOSTBC in Fig.3 and Fig.4 are obtained based on the S_3 in (13) and the S_4 in (14), respectively. As seen in simulation results , the 1-D BSVD scheme outperforms both the OSTBC and the QOSTBC in all SNR ranges, and by increasing SNR, its performance grows better. The main reason of attaining better performances in the 1-D BSVD scheme is that the transmitted power is directed into the subspace of H that has the most gain.

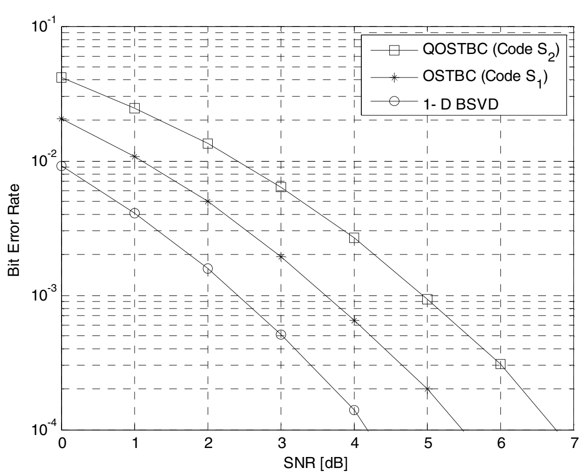

Fig. 1. Comparison between 1-D BSVD, OSTBC (Code S_1) and QOSTBC (Code S_2) for $M = N = 4$ and QPSK constellation.

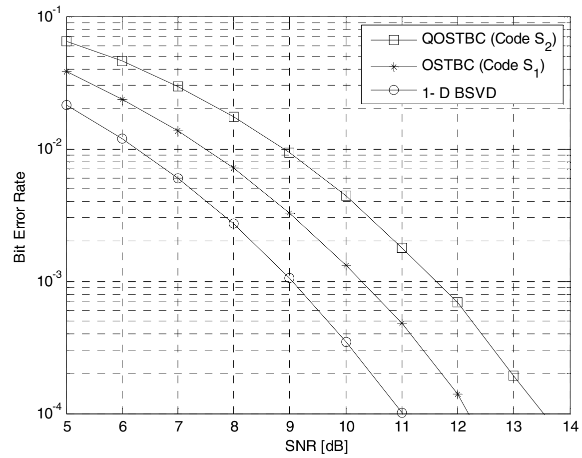

Fig. 2. Comparison between 1-D BSVD, OSTBC (Code S_1) and QOSTBC (Code S_2) for $M = N = 4$ and 16-QAM constellation.

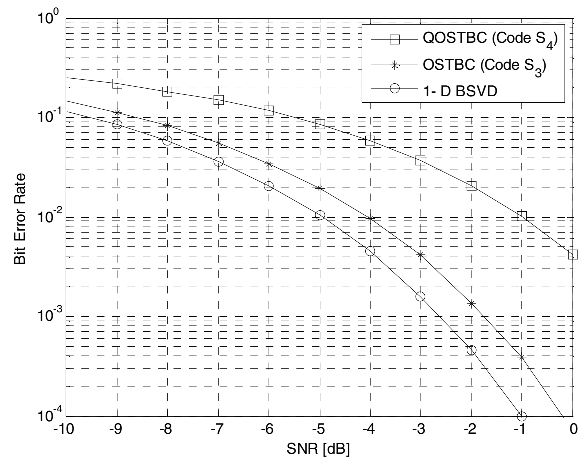

Fig. 3. Comparison between 1-D BSVD, OSTBC (Code S_3) and QOSTBC (Code S_4) for $M = N = 8$ and QPSK constellation.

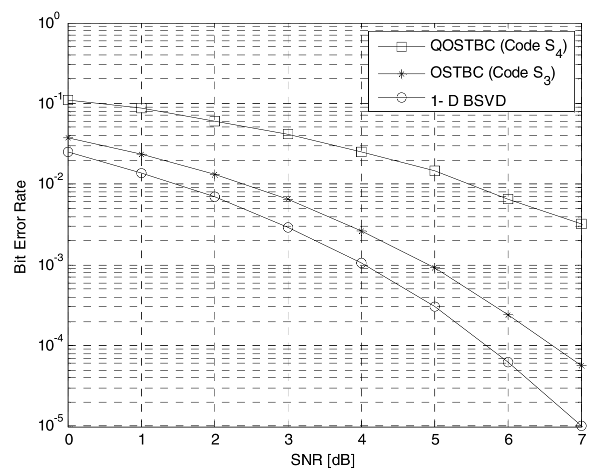

Fig. 4. Comparison between 1-D BSVD, OSTBC (Code S_3) and QOSTBC (Code S_4) for $M = N = 8$ and 16-QAM constellation.

VI. CONCLUSIONS

A joint beamforming and space-time coding scheme has been proposed in this paper for multiple-input and multiple-output (MIMO) systems based on block singular value decomposition (BSVD). In this scheme, the MIMO channel is divided into some independent MIMO subchannels. Based on beamforming, the transmitted power is directed into the dominant MIMO subchannel in the proposed scheme that has been called 1-D BSVD. Due to using low complex decoding procedure and also having transmitting rate one, Alamouti space-time coding has been employed in the dominant MIMO subchannel. The performances of the proposed joint beamforming and space-time coding scheme have been compared with the performances of the OSTBC and the QOSTBC under different situations by computer simulations. The BER performances have been indicated that the 1- D BSVD outperforms both the OSTBC and the QOSTBC in all SNR ranges.

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