

# Cutting Tool Reliability in Just-In-Time Production Decisions

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**Abstract.** In this paper, the effect of tool reliability and tool replacement on just-in-time (JIT) production decisions was analyzed. It is proposed that the JIT planning decision be jointly made with the tool replacement decision on a machining centre. A model has been developed to simultaneously address the JIT planning, tool reliability and tool replacement problems. The objective is to determine job sequence, tool spare requirement and corresponding replacement intervals so that the total *expected* processing cost can be minimized. A tabu search algorithm is developed to solve the problem. An example problem is included to illustrate the proposed procedure, followed by discussions about the impact of tool reliability on total earliness and tardiness and total production cost.

## 1. Introduction

Just-In-Time production is one of the important production planning goals. In the literature pertinent to JIT production planning, the processing times have mostly been assumed to be given and deterministic. It is also assumed that tools are completely reliable throughout the production period. Similarly, with a few exceptions (i.e., Avci and Akturk, 1996, Liu, et al, 1997), tool replacement policies have been studied assuming that job processing sequence is known in advance. In addition, fixed intervals of tool replacement are usually suggested in literature. In reality, the processing time of a particular operation is often probabilistic in nature due to the probabilistic tool life distribution. The time to completion of the operations is directly affected by tool reliability and tool replacement decision. On the other hand, the tool replacement decision also depends on which set of operations are scheduled on the machine under consideration. Furthermore, the workpiece material has also direct impact on tool reliability. Therefore, the JIT production decisions made based on deterministic processing time and the tool replacement decisions made with a presumed processing sequence may not be applicable in real applications.

In view of the above, the JIT production planning is investigated with consideration of tool reliability in this paper. A model is developed to provide solutions to both JIT scheduling and tool replacement problems. Due to the combinatorial nature of the proposed model, a tabu search approach is suggested to solve the combined decision problem. The formulation of the problem and development of the proposed solution procedure are given in the following sections.

## 2. Problem Statement and Formulation

In an automated manufacturing environment, each automated machining centre is capable of carry out a number of operations for various parts. Each part has a fixed operation sequence specified by a set of required tools and its own distinct due date. Once a part is mounted on the machine, unless a tool fails during operation, it will stay on the machine until all of its operations are completed. If an in-process tool failure occurs, both the tool and the part become defective. For each cutting tool, the tool life distribution is affected by the workpiece material to be processed by the tool. Also, since each tool may be used for several different parts, the effective tool life depends not only on one workpiece but several parts that are to be processed by the tool. However, the set of parts to be processed by a particular tool is not pre-determined but rather an output of the planning decision. Each tool may or may not be replaced at the beginning of each operation depending on the trade-off between the tool cost and the cost of possible defective parts. Also, to achieve the JIT goal it is desirable to process each job as close to its due date as possible. Furthermore, the tool spare level also affect the overall process reliability. The more tools of each type we have, the more reliable the production process will be. However, excessive tool spare leads to high cost, which is obviously undesirable. The planning objective is therefore to find the best part sequence, tool spare level and tool replacement interval so that the total *expected* processing cost is minimized.

## 2.1 Notation

The following notation will be used in this paper:

$I$	tool type index
$j, k$	job index
$l$	position index
$s$	job sequence index
$a_j/b_j$	penalty cost per unit time of tardiness/earliness
$c/C$	setup/ machining cost per unit time
$E_j/T_j$	earliness/tardiness of job $j$
$d_j$	due date of job $j$
$G(s)$	total cost of sequence $s$
$h_{ij}(t)$	hazard rate when tool $I$ is used for job $j$
$I_j$	a set of tools required for job $j$
$\mu(s, l)$	the job associated with the $l$ th position in sequence $s$
$m_{ij}$	a tool replacement indicator; if tool $I$ is replaced before processing job $j$ , $m_{ij}=1$ ; otherwise, $m_{ij}=0$
$M_i$	available number of spares for tool $I$
$\Pi_{ij}$	cumulative dollar value on job $j$ up to the time when the operation performed by tool $I$ is completed
$\pi_j$	raw material cost of job $j$
$Q_i$	cost of tool type $I$
$R_{ij}/r_{ij}$	reliability of tool $I$ operating on job $j$ if it is/is not replaced before processing job $j$
$Y_{ij}$	machining time spent by tool $I$ on job $j$

## 2.2 The Model

To minimize the total expected processing cost, the JIT planning and tool replacement problems are formulated as an integrated model given as follows:

$$\begin{aligned} & \underset{s}{\text{Min}} \{G(s)\} = \\ & \underset{s}{\text{Min}} \left\{ \sum_{l=1}^J \left[ \sum_{i \in I_{\mu(s,l)}} \min(Zr_{i\mu(s,l)}, ZR_{i\mu(s,l)}) + ct_{\mu(s,l-1)\mu(s,l)} \right] + a_{\mu(s,l)} T_{\mu(s,l)} + b_{\mu(s,l)} E_{\mu(s,l)} \right\} \end{aligned} \quad (1)$$

subject to:

$$\sum_{l=1}^J m_{i\mu(s,l)} \leq M_i \quad \forall s, i \in \mathcal{A}_{\mu(s,l)} \quad (2)$$

$$\sum_{k=1}^l \left( \sum_{i \in I_{\mu(s,l)}} Y_{i\mu(s,l)} + ct_{\mu(s,k-1)\mu(s,k)} \right) - d_{\mu(s,l)} \leq T_{\mu(s,l)} \quad \forall s, l \quad (3)$$

$$d_{\mu(s,l)} - \sum_{k=1}^l \left( \sum_{i \in I_{\mu(s,l)}} Y_{i\mu(s,l)} + ct_{\mu(s,k-1)\mu(s,k)} \right) \leq E_{\mu(s,l)} \quad \forall s, l \quad (4)$$

$$E_{\mu(s,l)} \geq 0 \quad \forall s, l \quad (5)$$

$$T_{\mu(s,l)} \geq 0 \quad \forall s, l \quad (6)$$

Where

$$Zr_{i\mu(s,l)} = (Q_i + \Pi_{i\mu(s,l)})(1 - r_{i\mu(s,l)}) + (CY_{i\mu(s,l)} r_{i\mu(s,l)}) \quad \forall s, l, i \in \mathcal{A}_{\mu(s,l)} \quad (7)$$

$$ZR_{i\mu(s,l)} = (Q_i + \Pi_{i\mu(s,l)})(1 - R_{i\mu(s,l)}) + (CY_{i\mu(s,l)} R_{i\mu(s,l)}) + Q_i \quad \forall s, l, i \in \mathcal{A}_{\mu(s,l)} \quad (8)$$

$$\Pi_{i\mu(s,l)} = \pi_{\mu(s,l)} + \sum_{n=1}^{O(i,\mu(s,l))} CY_{n\mu(s,l)} \quad \forall s, l, i \in \mathcal{A}_{\mu(s,l)} \quad (9)$$

$$R_{i\mu(s,l)} = \exp \left( - \int_0^{Y_i} h_{i\mu(s,l)}(t) dt \right) \quad \forall s, l, i \in \mathcal{O}_{\mu(s,l)} \quad (10)$$

$$r_{i\mu(s,l)} = \left( \prod_{q \in \tau_i(l)} R_{iq} \right) R_{i\mu(s,l)} \quad \forall s, l, i \in \mathcal{O}_{\mu(s,l)} \quad (11)$$

$$m_{i\mu(s,l)} = \begin{cases} 1 & \text{if } ZR_{i\mu(s,l)} < Zr_{i\mu(s,l)} \\ 0 & \text{otherwise} \end{cases} \quad \forall s, l, i \in \mathcal{O}_{\mu(s,l)} \quad (12)$$

As shown above, the objective function consists of three cost components: setup cost  $ct_{\mu(s,l-1),\mu(s,l)}$  which is deterministic and sequence-dependent, tool replacement cost  $Zr_{i\mu(s,l)}$  (or  $ZR_{i\mu(s,l)}$ ), and earliness/tardiness penalty cost represented by the last two terms in the objective function. The earliness and tardiness costs are probabilistic and dependent on the job sequence as well as historical decision of tool replacements. Constraint (2) ensures that the required tools should not exceed the available tool spares. Constraints (3) and (4), in conjunction with constraints (5) and (6), determine the amount of tardiness or earliness of each job in the sequence. If tool  $i$  is replaced before processing job  $\mu(s,l)$ , the sum of the expected tool cost, defective job cost and machining cost is  $Zr_{i\mu(s,l)}$  as shown in Equation (7); otherwise the sum is  $ZR_{i\mu(s,l)}$  given by Equation (8). These costs for each tool-job combination are computed or updated, using Equations (9) to (11). The above model can not be solved efficiently using commercial software due to its combinatorial nature. To provide quick and good solutions, a tabu search algorithm will be developed. A description of the proposed solution procedure is given in the following section.

### 3. Proposed Solution Procedure

In general the sequencing problem with sequence-dependent processing times or costs on a conventional machine is equivalent to the travelling salesman problem (TSP) (Bellmore and Nemhauser, 1968) -- a well-known NP-complete problem (Rinnooy Kan, 1976). The optimal solutions of such a problem can be feasibly obtained only for small problems. The total expected cost of the JIT production is not only sequence-dependent but position-dependent as well. This is because the costs of tools, possible defects, and process all depend on which position a part has been scheduled in a sequence. This obviously makes it more challenging to solve the problem optimally, if not entirely infeasible for real planning problems. For this reason, we propose a tabu search approach to provide good and quick solutions.

Tabu search, first introduced by Glover (1990), is an optimization technique used to solve combinatorial optimization problems. Tabu search starts with an initial feasible solution and moves stepwise towards a good solution. Such a move, however, may not necessarily be an improving one. A distinct characteristic of tabu search is its ability to avoid local optima by using a list of forbidden moves (tabu list). During the search process the best solution found is recorded and updated. This process continues until either significant improvement of the solution cannot be achieved or the maximum allowable search time has been reached.

The tabu search algorithm for solving our proposed model is summarised as follows:

#### Step 1 Initialization

Read input data and specify the maximum allowable search time and the length of tabu list.  
Find a starting sequence and compute the associated production cost.

#### Step 2 Search

- Generate a feasible neighbour sequence for the initial sequence. If the sequence is not in the current tabu list, calculate the objective value and update the best sequence in the neighbourhood; otherwise, discard this neighbouring solution. Repeat this for all feasible neighbouring solutions.
- Make a move, i.e., switch from the previous initial sequence to a new one and update tabu list.
- If an improvement is observed, update the best solution.
- If the maximum allowable search time is over, stop. Otherwise, update the number of moves made so far in the current phase, go to step 3; otherwise, go to the beginning of this step.

#### Step 3 Diversification (re-initiate a search phase)

Clear tabu list and diversify the search path by choosing one of the following strategies: (a) Restart from the last job sequence, (b) Restart from the best job sequence obtained so far, or (c) Restart from a randomly selected job sequence. Go to step 2.

The tool spare level can be determined by solving the problem with different tool spare levels. The spare level that yields lowest total expected cost with acceptable process reliability will be selected.

#### 4. Illustrative Example

The proposed tabu search algorithm has been applied to sequence 30 jobs on a machining centre with 20 types of tools. In the following examples, tool lives are assumed to follow Weibull distribution. It should be pointed out that any other appropriate tool life distributions can be used in the computer program. Obviously, the scale parameter  $\alpha$  and shape parameter  $\beta$  of the Weibull distribution are affected by the tool-part combinations. In this example, it is assumed that the two parameters are known for each tool-part combination and are distributed in the following ranges:

$$\alpha = [74, 1245] \quad \text{and} \quad \beta = [0.531, 1.680]$$

Both machining cost and setup cost are set at \$2/minutes. The information about machining times, due dates and earliness/tardiness penalties is given in Table 1 and Table 2. Due to the space limitation, the rest of the input data including setup times, and raw material and tool costs have been omitted. The detailed information can be found in (Kolahan, 1999). To determine the actual spare requirements, the available copies for each tool type is set at a high number (8 copies) though the actual required tool copies could be much less. The results for the best search parameters and 15 minutes of search time are presented in Table 3 which lists the final job sequence, deviations from the due dates, actual spare consumption for each tool type, and the tool replacement intervals. Based on these results the job sequence and tool replacement decisions can be made simultaneously. The examination of the effect of tool spare level on the total expected cost and process reliability is not included in this paper.

**Table 1** Machining times (min.) for the example problem

Tool type	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Job No.																				
1	---	2.0	---	---	---	1.5	---	3.0	---	---	---	1.0	---	---	---	---	---	3.5	1.2	1.8
2	---	---	3.5	2.5	---	---	---	---	3.0	---	---	---	---	3.5	2.5	---	1.2	---	3.0	---
3	2.5	1.3	---	2.5	2.0	---	---	---	---	3.5	3.0	---	1.2	---	---	---	2.5	---	---	1.0
4	3.5	---	2.5	---	---	1.5	2.3	---	---	1.0	---	---	0.6	2.5	---	---	---	---	---	---
5	---	4.0	3.0	---	2.0	---	---	---	5.0	---	2.0	---	---	---	1.0	1.5	---	---	---	---
6	---	---	---	1.8	---	---	4.0	2.0	---	---	---	2.5	3.5	---	---	---	1.0	0.7	---	---
7	---	2.5	---	---	---	1.0	---	2.5	---	---	---	3.0	---	---	---	---	---	3.5	1.0	0.5
8	---	---	2.0	2.2	---	---	---	---	3.0	---	---	---	---	3.5	2.5	---	1.0	---	3.0	---
9	2.0	6.0	---	1.0	1.5	---	---	---	---	3.0	3.0	---	1.0	---	---	---	2.0	---	---	2.0
10	3.2	---	2.8	---	---	1.0	2.5	---	---	1.5	---	---	2.0	2.2	---	---	---	---	---	---
11	---	3.5	2.0	---	2.5	---	---	---	1.0	---	2.5	---	---	---	4.0	1.5	---	---	---	---
12	---	---	---	4.2	---	---	3.5	2.5	---	---	---	2.3	1.3	---	---	---	2.5	0.5	---	---
13	---	2.0	---	---	---	1.5	---	2.5	---	---	---	0.8	---	---	---	---	---	3.4	1.0	2.0
14	---	---	3.6	2.4	---	---	---	---	3.0	---	---	---	---	1.2	2.4	---	1.2	---	3.0	---
15	---	1.4	0.5	---	2.3	1.8	---	---	---	3.5	3.0	---	1.5	---	---	---	2.5	---	---	1.5
16	---	1.1	---	2.5	---	---	1.4	2.2	---	---	1.8	---	---	1.0	2.0	---	---	---	---	---
17	---	4.0	2.5	---	2.5	---	---	---	1.2	---	2.5	---	---	---	0.5	1.8	---	---	---	---
18	---	---	---	3.0	---	---	3.5	2.5	---	---	---	2.0	1.5	---	---	---	2.5	3.0	---	---
19	---	1.4	---	---	---	1.0	---	2.5	---	---	---	3.0	---	---	---	---	---	0.6	1.2	1.8
20	---	---	1.7	2.5	---	---	---	---	3.0	---	---	---	---	3.5	1.4	---	3.5	---	3.0	---
21	2.4	5.5	---	2.1	1.6	---	---	---	---	1.0	0.7	---	3.8	---	---	---	2.4	---	---	1.0
22	3.5	---	1.4	---	---	1.0	2.5	---	---	2.0	---	---	3.0	2.6	---	---	---	---	---	---
23	---	1.0	1.0	---	2.2	---	---	---	0.5	---	2.5	---	---	---	4.0	1.8	---	---	---	---
24	---	---	---	0.5	---	---	4.0	2.0	---	---	---	2.5	3.8	---	---	---	2.2	3.0	---	---
25	---	2.8	---	---	---	1.0	---	2.2	---	---	---	0.7	---	---	---	---	---	3.5	1.0	2.0
26	---	---	4.0	2.0	---	---	---	---	3.0	---	---	---	---	3.0	2.5	---	1.0	---	2.0	---
27	2.0	1.5	---	2.5	2.0	---	---	---	---	0.5	3.0	---	3.5	---	---	---	2.5	---	---	1.0
28	3.0	---	2.2	---	---	1.5	2.1	---	---	1.7	---	---	5.0	2.0	---	---	---	---	---	---
29	---	4.0	3.0	---	2.0	---	---	---	1.0	---	2.5	---	---	---	4.0	1.5	---	---	---	---
30	---	---	---	4.5	---	---	3.5	2.0	---	---	---	2.5	1.0	---	---	---	2.5	3.0	---	---

"---" means no operation is assigned

**Table 2** Due dates and cost data

Job	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$d_j$ (min) 72	360	300	30	270	330	432	390	180	210	228	420	210	465	315	
$a_j$ (\$/min)	5.0	4.0	4.0	1.5	6.5	4.0	6.0	4.5	5.0	4.5	1.5	2.0	4.0	3.5	1.0
$b_j$ (\$/min)	5.0	3.0	3.0	1.5	1.5	4.0	3.0	2.0	3.0	1.0	1.0	0.5	3.0	0.5	1.0
Job	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
$d_j$ (min) 150	210	282	480	405	30	120	240	480	360	345	60	90	240	144	
$a_j$ (\$/min)	8.5	2.5	3.0	8.0	4.0	5.5	6.0	5.5	4.5	7.0	5.0	5.0	5.5	3.5	5.0
$b_j$ (\$/min)	0.5	1.0	0.5	1.0	1.5	2.0	2.0	2.0	1.0	4.0	2.0	1.0	1.5	2.5	1.5

**Table 3** Solution for the example problem ( $M_i = 8$ )

Tool type	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Seq <sup>(a)</sup>	ET	Replacement intervals																		
4	16	0	---	0	---	---	0	0	---	---	0	---	---	0	0	---	---	---	---	---
21	-6	0	0	---	0	0	---	---	---	---	0	0	---	0	---	---	---	0	---	0
27	21	0	---	0	0	---	---	---	---	0	0	---	0	---	---	---	0	---	---	0
1	0	---	0	---	---	---	0	---	0	---	---	---	0	---	---	---	---	---	0	0
28	0	0	---	0	---	---	0	0	---	---	0	---	---	0	0	---	---	---	---	---
22	12	1	---	0	---	---	1	0	---	---	1	---	---	1	1	---	---	---	---	---
16	29	1	---	0	---	---	1	0	---	---	1	---	---	0	1	---	---	---	---	---
30	3	---	---	---	0	---	---	0	1	---	---	---	0	0	---	---	---	0	1	---
10	53	1	---	1	---	---	0	1	---	---	1	---	---	0	1	---	---	---	---	---
9	0	0	---	0	0	---	---	---	---	0	0	---	0	---	---	---	0	---	---	0
17	12	---	1	0	---	1	---	---	---	0	---	1	---	---	---	0	0	---	---	---
13	-4	---	0	---	---	---	0	---	0	---	---	---	0	---	---	---	---	---	0	0
23	11	---	0	0	---	1	---	---	---	0	---	0	---	---	---	0	0	---	---	---
29	-8	---	0	0	---	0	---	---	---	0	---	0	---	---	---	1	1	---	---	---
5	3	---	0	1	---	0	---	---	---	0	---	0	---	---	---	1	0	---	---	---
18	-5	---	---	---	1	---	---	0	1	---	---	---	0	0	---	---	---	0	1	---
11	-77	---	0	0	---	0	---	---	---	1	---	0	---	---	---	0	0	---	---	---
3	-26	1	0	---	0	0	---	---	---	---	0	0	---	0	---	---	---	1	---	0
6	-14	---	---	---	1	---	---	1	0	---	---	---	1	0	---	---	---	0	0	---
26	-18	---	---	1	0	---	---	---	---	0	---	---	---	---	1	1	---	1	---	1
25	-16	---	1	---	---	---	1	---	0	---	---	---	1	---	---	---	---	---	0	1
2	-36	---	---	0	0	---	---	---	---	0	---	---	---	---	1	0	---	0	---	0
8	-25	---	---	0	0	---	---	---	---	0	---	---	---	---	1	0	---	0	---	0
20	-31	---	---	0	0	---	---	---	---	1	---	---	---	---	0	1	---	0	---	1
7	-19	---	0	---	---	---	0	---	0	---	---	---	0	---	---	---	---	---	1	0
14	-5	---	---	1	0	---	---	---	---	1	---	---	---	---	1	1	---	1	---	1
19	-3	---	0	---	---	---	0	---	0	---	---	---	1	---	---	---	---	---	0	1
24	-22	---	---	---	1	---	---	1	0	---	---	---	0	1	---	---	---	1	0	---
12	-102	---	---	---	0	---	---	1	1	---	---	---	0	0	---	---	---	0	1	---
15	-226	1	1	---	0	0	---	---	---	---	1	1	---	0	---	---	---	0	---	0
Spares <sup>(b)</sup>		6	3	5	3	1	3	4	3	3	4	2	3	2	7	5	1	4	4	5

"---" no operation is assigned  
 "1" tool replacement is required  
 "0" no tool replacement is required

<sup>(a)</sup> : Total expected cost = \$5085  
<sup>(b)</sup> : Tool spare cost = \$654  
 ET: Tardiness or earliness of jobs

## 5. Conclusion

The tool reliability has been incorporated into the JIT production decisions. A model for the combined

JIT scheduling and tool replacement problem with probabilistic tool life distribution has been formulated in this paper. To solve this problem a tabu-search approach has been proposed. The outcome of the model provides simultaneous solutions to the part sequencing and tool replacement problems. The computational results indicate that the tool spare level has a saturation point. This is evident from the actual spare requirements shown in Table 3. In addition, unlike the policies suggested in most tool replacement literature, tool replacement intervals are not fixed in our computational results. They may be different in terms of both cutting time and number of completed jobs.

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