

## A GENETIC ALGORITHM FOR STRUCTURAL OPTIMIZATION

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### ABSTRACT

The optimized shape of trusses are found by presenting a new selection and reproduction operator. The stress, buckling and displacement constraints, consistent with design code, are used. Design variables are discrete size variables (member areas) and continuous shape variables (nodal coordinates). The proposed method, is fast and has a stable convergence. In addition, it results in an optimized structure with low weight.

**Keywords:** genetic algorithms, reproduction operator, optimal truss, size variables, shape optimization, continuous and discrete variables, buckling constraints.

### 1. INTRODUCTION

In all of genetic algorithms three basic operators: reproduction, crossover and mutation are presented [1]. By reproduction operators, it is decided that a string should survive or not. In addition, how many of that string, should be placed in the mating pool, to produce the next generation of the strings. Decision is done based on the fitness of any string by different methods. In fact, fitness shows the ability to survive and reproduction in the next generations. In the structural optimization problems, the fitness function is a combination of objective function and constraints [2-4].

Selection and reproduction operator, in addition to final influences the fluctuation of optimized solutions in successive generations, and effects the optimized result. Because of the importance of this operator in genetic search strategies, researchers have looked for new operator of this type, so that by using it, optimization process has little fluctuation and high convergence. On the other hand, a structure with less weight will be gained. Thus, it is intended to use the method of the ranking selection and a mathematical relationship for computing the number of any string reproduction. The important point is that the number of reproductions should be in such way, so that, unordinary strings will not dominate the population.

In this paper, at first, the mathematical model of truss shape optimization and the AISC constraints are discussed [5-7]. Then, the basis and specialities of genetic computational steps are presented. After that, some examples of truss structures are solved and their convergence history is compared to other methods. Here, the optimization problem has both continuous (shape) and discrete (size) variables. In addition, based on the design code rules, the optimized solution is found under the stress, buckling and displacement constraints.

## 2. MATHEMATICAL MODEL FOR TRUSS SHAPE OPTIMIZATION

The optimization problem of truss shape that its design variables are a combination of discrete variables of member cross section areas  $\{A\}$  and continuous nodal coordinates variables  $\{X\}$ , is formulized as:

$$\text{Minimize:} \quad W(A, X) = \sum_{i=1}^m \rho_i A_i L_i = \sum_{i=1}^m \rho_i A_i \left[ \sum_{j=1}^3 (x_{ij}^a - x_{ij}^b)^2 \right]^{0.5} \quad (1)$$

$$\text{Stress constraint:} \quad \{\sigma^L\} \leq \{\sigma\} \leq \{\sigma^U\} \quad (2)$$

$$\text{Displacement constraint:} \quad \{D^L\} \leq \{D\} \leq \{D^U\} \quad (3)$$

$$\text{Nodal coordinates constraint:} \quad \{X^L\} \leq \{X\} \leq \{X^U\} \quad (4)$$

In these relationships,  $\{A\} = \{A_1, A_2, \dots, A_m\}^T$  is the size variable vector that is selected from the discrete variables list. In addition  $\{X\} = \{X_1, X_2, \dots, X_n\}^T$  is the shape variable vector that can have any continuous quantity. In these relationships, objective function,  $W(A, X)$  is the total truss weight and  $\rho_i, A_i, L_i$  are  $i$  the member specific weight, cross section area and member length, respectively. Also,  $X_{ij}^a$  is the  $j$ th coordinate of the node of member  $i$ .  $m$  and  $n$  are the number of members and joint coordinates, respectively. Also,  $\{\sigma^L\}, \{\sigma^U\}, \{D^L\}, \{D^U\}, \{X^L\}$  and  $\{X^U\}$  are the lower and upper bounds of the member stress vector  $\{\sigma\}$ , displacement vector  $\{D\}$  and nodal coordinates vector  $\{X\}$ . On the other hand, nodal displacements and member stresses are found by structural analysis from the following equations:

$$[K] \{D\} = \{P\} \quad (5)$$

$$\{\sigma\} = [T] \{D\} \quad (6)$$

In the presented equations,  $[K]$  is the stiffness matrix of the structure,  $\{P\}$  is the nodal forces and  $[T]$  is a transformation matrix that relates the nodal displacements to the member stresses. For the member under axial compression, the allowable stress depends on the slenderness ratio  $\lambda_i$ . The slenderness ratio is a function of the member length  $L_i$  radius of gyration  $r_i$  and effective length factor  $K_i$ . This dependence is written as:

$$\lambda_i = \frac{KL_i}{r_i} \quad (7)$$

Tensile,  $\{\sigma_i^U\}$ , and compressive,  $\{\sigma_i^L\}$ , allowable stresses based on AISC code are found by:

$$\bar{\sigma}_i = \begin{cases} \sigma_i^L & \sigma_i < 0 \\ \sigma_i^U = 0.6F_y & \sigma_i \geq 0 \end{cases} \quad (8)$$

$$\sigma_i^L = \begin{cases} \left(1 - \frac{\lambda_i^2}{2C_c}\right) F_y & \lambda_i < C_c \\ \frac{5}{3} + \frac{3\lambda_i}{8C_c} - \frac{\lambda_i^3}{8C_c} & \\ \frac{12\pi^2 E}{23\lambda_i^2} & \lambda_i \geq C_c \end{cases} \quad (9)$$

In these relationships,  $E$  is the modulus of elasticity,  $F_y$  is the yield stress of steel and  $C_c$  is the slenderness ratio dividing elastic and inelastic buckling zones.  $C_c$  is calculated as follows:

$$C_c = \sqrt{2\pi^2 E / F_y} \quad (10)$$

The genetic methods can not solve constrained optimization problems directly, so that the problem should be transformed to a unconstrained one. To do this, methods such as penalty function or Lagrangian multipliers should be used. If penalty function technique is used, enough precision should be employed for the penalty quantity for the constraints violation. It can be simply found that how penalty is acceptable. A large penalty can break many designs and slow down the process [8]. Here, the modified and normalized objective function is presented as:

$$\phi_i = \frac{W_i}{W_{\max}} + R_p \sum_{k=1}^m \Gamma \quad (11)$$

$$\Gamma = \max \left\langle \frac{g_k}{\bar{g}}, 0 \right\rangle \quad (12)$$

In eq (11),  $W_i$  is its design weight,  $W_{\max}$  is the maximum weight among a generation of designs and  $R_p$  is the penalty function coefficient. In eq (12),  $g_k$  is the stress and displacement constraints and  $\bar{g}$  is the bounds of the constraints.

### 3. GENETIC ALGORITHM FOR OPTIMIZATION

#### 3.1 Coding and decoding of design variables

In genetic methods, representing a design (design variable set) has a specific importance. These representations relate the real problem to the genetic algorithm. It should be added that this process works on coded variables. There are many types of representation, for example, representing the string with binary numbers, floating point and etc. Each design variable is made of a substring of 0 and 1. Then, by adding of top and bottom of these substrings, a design is built. The used variables in the genetic techniques are in fact discrete ones, but, with a specific precision continuous variables can be entered in the problem. Now, coding and decoding of any type of variables, discrete and continuous, are discussed.

### 3.1.1 Continuous design variable

1- Coding: If the desired precision in continuous design variables is  $\varepsilon$ , the minimum length (l) of substring should be:

$$l \geq \log_2 \left( \frac{x^U - x^L}{\varepsilon} \right) \quad (13)$$

In this equation,  $x^U$  and  $x^L$  are upper and lower bounds of the continuous variable  $x$ .

2- Decoding: First, binary substring is transformed to a decimal I with no sign. Then, the physical quantity X is calculated as :

$$x = x^L + I \cdot \frac{x^U - x^L}{2^l - 1} \quad (14)$$

### 3.1.2 Discrete design variables

1. Coding: the length of substring is dependent on the number of discrete design variables. The following relationship is valid:

$$S_1 = \text{Number of discrete variables} \quad (15)$$

2. Decoding: to decode the strings, the unsigned decimal equivalent of binary string is calculated, and then it will be mapped to the physical value of  $x$ .

### 3.2 Fitness Function

Based on the Darwinian survival of the fittest theory, fitness is a qualitative value that measures the ability of reproduction of living creatures. In the genetic algorithms, fitness is used to allocate the reproduction ability of tested designs. Thus, a type of measurement, "goodness" or "quality" is what should be maximized. In other words, strings with higher fitness, have more chance to be selected as father and mother to produce the next generation. On this basis, fitness function is found as :

$$F = \phi_{\max} - \phi \quad (16)$$

In this equation,  $\phi_{\max}$  is modified objective function, that avoids F from becoming negative.

In genetic schemes, search progresses to find strings with high fitness. Therefore, the maximization problem of the fitness function is considered. On the other hand, in nature, any chromosome has a fitness that is qualitative and non-negative. On this basis, to transform the minimization problem of the weight of structure to a maximization problem of fitness, the following equation is usually used [9]:

$$F_i = C_{\max} - W_i \quad (17)$$

Here,  $F_i$  is the fitness function and  $W_i$  is the weight of structure in the  $i$ th design, and  $C_{\max}$  is a sufficiently large number to prevent fitness become negative. In the investigation by Wu and Chou, fitness function was presented as :

$$F_i = \phi_{\max} - \phi_i \quad (18)$$

Here,  $\phi_i$  is the modified objective function of the  $i$ th design, and  $\phi_{\max}$  is the maximum modified objective function of the generation.

### 3.3 Population size

One of the important factors influencing suitable execution of the genetic method is “population size”. The number of strings in a generation (iteration) is called population size. The greater this factor, the more will be the number of design points in a cycle of search and less will be the probability of falling in the local optimal points. In other words, population size will prevent the search to be enclosed in a subspace. Thus, it gives the genetic techniques the ability of multimodal optimization functions. Population size should be an even number and be constant in the optimization process. Of course, if population size gets big, fitness function evaluation will increase that will increase the number of analysis in any cycle of search process. Therefore, cost and time of calculations will increase. It is not usual to take population size more than 100 in the genetic method [2].

### 3.4 Selection and reproduction operator

Reproduction process is based on search of production of “better” (with higher fitness) members (designs) and deletion of “worse” (with low fitness) members [10]. Strings are selected as parents for proliferation and production of new generation, based on their fitness. The law of selection is: the best member of a generation produces more, average member remains pair and the worst member disappears. There are two types of selection: the roulette wheel and the ranking selection.

In the roulette wheel method, a selection probability is allocated to any member of population, based on its fitness. If  $F_i$  is the fitness value of this member, its selection probability is:

$$P_{si} = \frac{F_i}{\sum_{i=1}^{np} F_i} \quad (19)$$

In this equation,  $np$  is the population size. The number of reproductions of the  $i$ th string can be found as :

$$nr_i = (np)P_{si} = \frac{F_i}{F_{avg}} \quad (20)$$

Here,  $F_{avg}$  is the average fitness of the population.

In the ranking selection, the fitness of strings are used indirectly for selection and reproduction. The selection process is such that, first all of the population strings are sorted in ascending or descending order, based on their fitness, then, their selection probability is calculated based on their rank in this group. If a group of strings with population size  $np$  are sorted in ascending fitness of  $F_1, \dots, F_{np}$ , then, selection probability is found by:

$$P_{si} = \frac{i}{\sum_{i=1}^{np} i} = \frac{i}{\frac{np(np+1)}{2}} = \frac{2i}{np(np+1)} \quad (21)$$

To distinguish number of reproductions of this string,  $p_{si}$  probability is multiplied by population size, so the number of reproductions of  $i$  th string equals to :

$$nr_i = (np)P_{si} = \frac{2i}{np+1} \quad (22)$$

The advantages of ranking selection are :

1. The ability to prevent the predominance of unordinary members, so that early convergence is avoided.
2. Wandering among approximately equal values and even standstill in population is avoided.

These are aspects of the genetic methods called survival of the fittest. In other words, better members produce more so that their desirable specialties can be transferred to their children.

The algorithm of selection and reproduction used in this paper is as follows:

First step -strings are descendingly sorted based on their fitness.

Second step -the rank (R) of any string in population is calculated.

Third step -the generation index (N) of  $i$ th string is found as :

$$N_i = (np - R_i + 1)^3 \quad (23)$$

Fourth step -the sum of generation indices of the last step is calculated as :

$$\sum_{i=1}^{np} N_i = \frac{(np)^2 (np+1)^2}{4} \quad (24)$$

Fifth step. the number of reproduction of the  $i$ th string is found as :

$$nr_i = (np) \frac{N_i}{\sum_{i=1}^{np} N_i} = \frac{4N_i}{np(np+1)^2} \quad (25)$$

Selection of suitable crossover operator is dependent on the problem domain and string representation of the variables. Usually, in traditional genetic algorithms, unipoint or two points operators have been used. However, some recent researches have shown that operators with more crossover points, like three or four points, are much more powerful.

### 3.5 Mutation operator

This operator has an important role in genetic methods as a guard. In these techniques, mutation seldom happens, and that is a reflection of mutation in real world. Some of numbers (0, 1) in a

particular situation in all strings of a population may be deleted in the reproduction and crossover process. Recovering of these situations, is not possible only by use of reproduction and crossover operators. To fix this by the mutation process, some of binary numbers in strings change with mutation propability of  $P_n$ . The mutation probability should be calculated precisely. If it has a low value, reproduction of good schemas will be prevented, and calculation will be lowered to a random search method. Jenkins proposes a mutation propability of  $P_n = 0.05$  [8].

### 3.6 Schema theory

First, some terms used in the schema theory are described. Then, the influence of the three operators, reproduction, crossover and mutation, on spreading a special type of schema in sequential generations is considered. Here, strings are shown as binary numbers. This assumption does not damage the generality of the problem. Any of the 0 or 1 numbers in a string is called a gene. A schema  $H$  is composed of three symbols 0, 1 and \*. \* shows a number 0 or 1 in a specific situation. For example,  $H = * 1 1 * 0 *$  is a schema of length 7. The string  $A = 0111000$  is one example of schema  $H$ , because genes of string  $A$  is compatible with constant situations 2,3 and 5 of schema. In a binary string of length  $l$ , there are  $3^l$  schemas. Generally, for  $k$  symbols, there are  $(k+1)^l$  schemas. In a population with  $np$  strings, there exists maximum number of  $np \times 2^l$  schemas, because any string shows  $2^l$  schemas [1].

It is assumed that in a given time interval  $t$ , the number of special schemas  $H$  existed in population  $A(t)$ , are written as  $m = m(H, t)$ . The number of expected reproductions of a special schema  $H$  in the next generation, under simultaneous action of reproduction, crossover and mutation operators, is given by the next equation. It is reminded that the multiplication of very small numbers are neglected.

$$m(H, t+1) \geq m(H, t) \left[ \frac{F(H)}{\bar{F}} \right] \left[ 1 - (P_c) \frac{\delta(H)}{l-1} - O(H)(P_m) \right] \quad (26)$$

In this equation,  $F(H)$  is the average fitness of strings that schema  $H$  presents in time  $t$ . Also,  $\bar{F} = \sum F_i / np$  is the average fitness of the total population.  $P_c$  and  $P_m$  are the crossover and mutation probability on strings, respectively. Generally, the smaller the defining length of schema, the lower its order, and higher its average fitness, it will increase in the next generations. This growth is done exponentially. For the importance of this conclusion, it is called "the schema theorem" or "the fundamental theorem of genetic algorithms". This theorem tells what happens in genetic search process and proves that search process is not random at all [9].

### 3.7 Convergence criteria

A critical and very important part of optimization process is ascertaining when search for optimized solution should be stopped. The convergence criteria can have great influence on workability and dependability of optimization process. One of stopping criterion is the number of iterations. In genetic search, the new strings of a generation are better than the last generation, and after some generations, depending on the reproduction, crossover and mutation operators, and also, the number of problem variables and the number of integers that variables can allocate, population gets full of strings with high fitness that have only a little difference with the best member of that generation's fitness. It is evident that in such a state, the average fitness gets

close to the best member's fitness. Rajeev and Krishnamurthy have proposed that search should be stopped when the fitness of 85 percent of existant strings in the population becomes similar [4]. Grierson and Pak use this convergence criteria: "search process in genetic methods goes as far as in specific numbers of simultaneous generations, there is no improvement in the most feasible design or when a defined degree of homogeneity is found in design groups".

In addition to these convergence criterion, a criteria is defined as :

$$\left| \frac{F_{\max} - F_{\text{avg}}}{F_{\max}} \right| < \varepsilon \quad (27)$$

In this relationship,  $F_{\max}$  and  $F_{\text{avg}}$  are equal to maximum and average fitness in the present iteration, respectively, and  $\varepsilon$  is the desired precision.

### 3.8 The algorithm

Now, the steps of the discussed genetic method is presented:

**First step -Data:** The geometric and mechanical properties of the structure are entered. If the size variables are discrete, they should be entered, too. For continuous variables, shape or size, the upper and lower limits and desired precision are needed. Population size, crossover and mutation probabilities, crossover type, penalty function coefficient and maximum number of iterations are other data needed.

**Second step - Initial population production:** Binary string of necessary length, which is a function of number of design variables and their precision, are produced randomly. The number of these strings is equal to population size.

**Third step – Decoding:** In this step, the strings produced from the previous step are decoded and the real values of design variables are calculated.

**Fourth step – Fitness function evaluation:** All designs (strings) in the population are analyzed and stresses, displacements and their weights are calculated. Then, using external penalty function, fitness function of any string is found.

**Fifth step - Reproduction operator:** In this step, using proposed selection and reproduction operator, number of reproductions of any string is determined and the same number of strings are put in the mating pool.

**Sixth step - Crossover operator:** Depending on crossover probability, necessary number of strings, two by two, are selected randomly and based on their type, crossover takes place on them.

**Seventh step - Mutation operator:** In this step, the nature of a percentage of total binary numbers in population, which depends on mutation probability, is changed. In other words, a number of 0 or 1s are selected randomly and are changed from 0 to 1 or from 1 to 0.

**Eighth step - Convergence criteria:** Here, the criteria to stop the search is controlled. If this criteria is reached, it goes to the next step, else, goes back to the third step.

**Ninth step -Reporting the results and end of process:** The results of genetic search that contain design variables and the optimum weight of structure, and also, the time elapsed in process, are printed in the output files.

## 4. NUMERICAL WORKS



In this section, some optimization problems, having size and shape variables, are analyzed. These examples have been selected such that they are being useful and practical. To decrease design variables and also make them applicable, approximate linking of variables have been used. Some of the shape variables, nodal coordinates, are linking to each other. In grouping these variables, in addition to keeping symmetry with respect to one or two axes, suitable operation of structure, and beautiful architecture are considered too. The upper and lower limits have been chosen so that the nodes can not move on each other. The cross section of the members are selected as double angle that are presented in table (8). In this table, the minimum radius of gyration of any section, that is used in calculation of allowable compressive stress, is given. The cross sections are grouped so that the diagonal, upper and lower members are placed in different groups. Such groups are inserted in related tables. In all of examples, the modulus of elasticity,  $E$ , density,  $\rho$ , and yield stress of steel,  $F_y$ , are as follows:

$$E = 2.07 \times 10^5 \text{ MPa}, \rho = 7860 \text{ kg/m}^3, F_y = 2.38 \times 10^2 \text{ MPa}$$

Because of applicability aspect of structures, constraints of optimization are the design rules of AISC. The constraints are: Stress, buckling and nodal displacements. Stress and buckling constraints are used for all members. On this basis, their stress is checked so that it does not exceed the allowable value. If stress in one or some members is exceeded from the allowable stress, this constraint is not valid and the design is not acceptable. To pull the design to the feasible space, external penalty function has been used. In this method, designs are transferred to the acceptable zone of design space by allocating suitable penalties for the invalid constraints. If a member is compressive, buckling phenomena is controlled based on design code. Also, key nodal points, for instance, the nodes that loads act on them, or the free end nodes in cantilevers, that have the largest and most critical displacements are constrained to a maximum value in all directions. In fact, with this constraint, the required stiffness of the structure is provided. In this paper, the penalty function coefficient is increased 5 units after 20 iterations. Selection of this coefficient is simple by normalizing the objective and constraints functions.

#### *4.1 Two dimensional truss bridge with 26 members*

In this example, the optimum shape of a two dimensional truss with 26 members, shown in Figure 1, is found. In this structure, the lower joints are loaded. These joints should have constant situation. The optimization problem includes stress, buckling and nodal displacement constraints. The allowable stresses are found from the AISC code. Here, displacement of nodes number 3, 5, 7, 9, 11 and 13 are restrained to 0.05 meters. The members of the truss are located in four groups as presented in table 1. The cross sectional area of the members are selected from double angles of table (8).

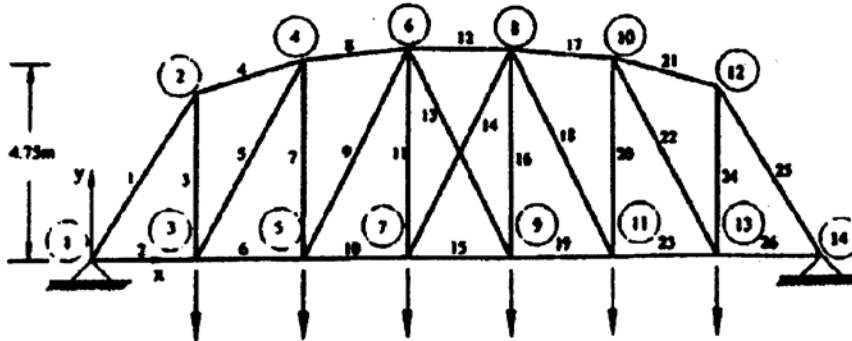


Figure 1 Two dimensional truss bridge with 26 members

Table 1 Member cross section groups

Group	Member Number
1	2, 6, 10, 15, 19, 23, 26
2	1, 4, 8, 12, 17, 21, 25
3	3, 7, 11, 16, 20, 24
4	5, 9, 13, 14, 18, 22

The nodes 2,4, 6,8, 10 and 12 can move along y axis. Shape design variables, using the structure symmetry, has upper and lower bounds in meters as:

$$4.75 \leq X_1 = y_6 = y_8 \leq 5$$

$$4 \leq X_2 = y_{45} = y_{10} \leq 4.75$$

$$2 \leq X_3 = y_2 = y_{12} \leq 4$$

The maximum difference of upper and lower shape variables equal 2 meters. If precision of 0.01 is needed for these variables, minimum length of shape and size variables are 8 and 6, respectively. This optimization problem includes four independent variables of size and 3 of shape. So, the length of any string equals 48.

The final optimized solution for this structure, using all of four methods, are presented in table 2. The best solution is found using proposed method, having weight of 2588 kilograms. After that, the methods of 7, 2 and 5 are in orders. The difference between the proposed method solution with the best one of other methods 7 with the minimum weight of 2671 kg is 3.1 percent. In this example, 5 gives the worst optimized solution. One reason is the selection of a nonsuitable weight normalization factor that should be entered in that method. Numerical experience is needed for any problem to find a suitable value for this factor. This, damages the automation of the method and is one of difficulties for this technique.

Table 2 Final solution for truss bridge with 26 members

Variable	Ref. [2]	Ref. [7]	Ref. [5]	Proposed
$A_1(\text{cm}^2)$	15.06	15.06	16.26	15.06
$A_2$	60.00	60.00	60.00	55.00
$A_3$	23.8	21.00	22.00	22.00
$A_4$	50.20	50.20	50.80	50.80
$X_1(\text{m})$	4.75	4.75	4.808	4.750
$X_2$	4.00	4.00	4.006	4.00
$X_3$	2.25	2.563	2.484	2.500
Weight (kg)	2680.0	2670.9	2707.02	2587.5
Time (Sec)	171.03	167.41	161.81	168.35

The solution fluctuation of any generation is shown in Figure 2. Rapid convergence and little fluctuation in the proposed algorithm and method of ref [2]) and [7]) is evident in the figure. In these methods, the fluctuation has been omitted completely after 25th iteration, but in ref [5]) method, this exists till the end of the process.

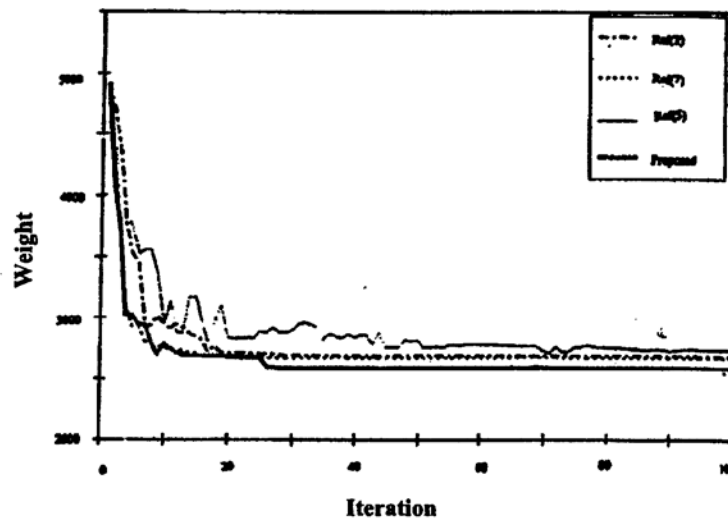


Figure 2 Convergence history for optimized solution of truss bridge with 26 members

#### 4.2 Space truss with 34 members

In this example, a space truss with 34 members, shown in Figure 3, is studied. The structure loading is presented in table 3. Optimization of the shape of this truss is performed under stress, buckling and nodal displacement constraints. The stress in all members should not be more than allowable ones. In nodes 9, 10, 11 and 12, displacement in z direction is restrained to 0.05 meters, and also, to 0.02 meters in x and y directions. The structure members are divided to 7 groups -as presented in table 4 -and their cross section will be chosen from double angle profiles of table 8.

Table 3 Loading or space truss with 34 members

Node	P <sub>x</sub> KN	P <sub>y</sub> KN	P <sub>z</sub> KN
9	0	10	-25
10	0	0	-25
11	10	0	-25
12	10	10	-25

Table 4 Groups of members for space truss with 34 members

Group	Member Number
1	1, 2, 3, 4
2	5, 6, 7, 8, 9, 10, 11, 12
3	13, 14, 15, 16
4	17, 18, 19, 20
5	21, 22, 23, 24, 25, 26, 27, 28
6	29, 30, 31, 32
7	30, 34

The displacement of nodes 1,2,3 and 4 in x and y directions, and also, of nodes 5,6,7 and 8 in x, y and z directions are selected as shape design variables. Because of symmetry of the structure with respect to xz and yz planes, the variables are divided into 5 groups. These groups and their upper and lower limits are as:

$$1 \leq X_1 = x_1 = x_2 = -x_3 = -x_4 \leq 4$$

$$1 \leq X_2 = y_2 = y_3 = -y_1 = -y_4 \leq 4$$

$$1 \leq X_3 = x_5 = x_6 = -x_7 = -x_8 \leq 4$$

$$1 \leq X_4 = y_6 = y_7 = -y_5 = -y_8 \leq 4$$

$$2 \leq X_5 = z_5 = z_6 = z_7 = z_8 \leq 6$$

The maximum difference between upper and lower limits of shape variables is in the fifth group ( $X_5$ ) and equals to 4. If precision of 0.01 is used, the length of substrings showing these variables will be 9. The total number of independent design variables in this example equals to 12. Here, to make the optimization problem unconstrained, a penalty function coefficient of 10 is used. The weight normalization factor for [5] scheme will be equal to 1500.

The optimization results of the four discussed methods are presented in table 5. As it is evident in the table, the best solution belongs to the proposed method giving an optimized structure with the weight of 1327.5 kilograms. Next is [2] method, with minimum weight of

1385.4 kg and [5] with weight of 1397.8 kg and at the last, [7] technique, with an optimized structure having weight of 1398 kg. It is reminded that the optimum structure dimensions in xy plane will be 2 x 2 meters and these dimensions are constant through the height of structure. In other words, the optimized structure is the one being prismatic in the height.

Table 5 The optimized solution for space truss with 34 members

Variable	Ref. [2]	Ref. [7]	Ref. [5]	Proposed
$A_1(\text{cm}^2)$	21.60	21.60	21.60	16.26
$A_2$	13.80	16.26	16.26	13.82
$A_3$	9.68	9.68	9.68	9.68
$A_4$	11.64	11.38	11.64	11.64
$A_5$	15.06	11.64	11.64	15.06
$A_6$	9.68	9.68	11.38	9.68
$A_7$	1.000	9.68	9.68	9.68
$X_1(\text{m})$	1.000	1.000	1.041	1.094
$X_2$	1.000	1.000	1.000	1.000
$X_3$	1.000	1.000	1.059	1.000
$X_4$	1.000	1.000	1.029	1.000
$X_5$	3.578	4.563	4.266	3.594
Weight (kg)	1385.4	1398.0	1397.8	1327.5
Time (Sec)	259.64	255.63	238.81	254.58

The convergence history in any generation is shown in Figure 4. In this problem the solution fluctuations have been reduced in ref [5]) method.

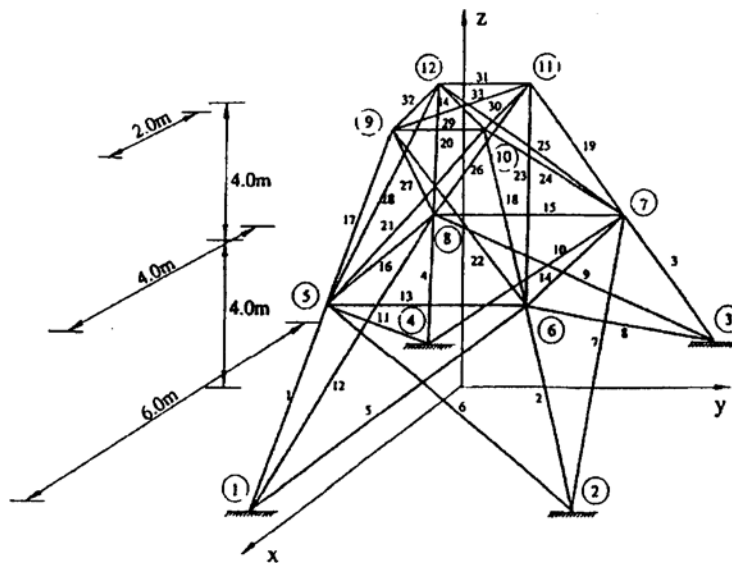


Figure 3 Convergence history of optimized solution for space truss with 34 members

## 5. CONCLUSIONS

In this method, variations of the optimized weight of the structure in sequential iterations has less fluctuations. It proceeds monotonously to a design with smaller weight.

Studying the proposed relationship for selection and reproduction operators, it was seen that strings with less fitness than the average total fitness is omitted soon and they lose the opportunity of being placed in mating pool. In other words, they will not participate in generating the strings of the next generation. In addition, the population characteristics will not be broken in any generation and good schemas will remain from the previous generations. Therefore, convergence to the optimized solution, specially in large populations that have population size greater than 50, will increase significantly.

Based on the experience of the previous investigation and also the writers numerical works, crossover type is effective in search process. Three points, four points and uniform crossovers having percentages of 10, 20, 30, 40 and 50 have been studied in this investigation, and among them, three points crossover has shown to be the best. So, this type of crossover has been used for the analysis.

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