Gable Frame Analysis Using Iteration Procedure

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Abstract

A formulation for calculating end moments of gable frame members which are primarily subjected to bending is presented. The resultant equations can be solved by Gauss–seidel iteration. Computational procedure is given, and based on that, sample problems are solved. The method presented in this paper can be applied to the single span as well as multispan gable frames. In addition, the procedure is convenient for ordinary calculator user and also it can be utilized for computer programming.

1– Introduction:

The moment distribution method was developed by Hardy Cross. He solved the joint–equilibrium equations by using Gauss seidel iteration (9). This method is easy and quick and also it is one of the most commonly used analytical procedures in structural engineering (6).

The Kani's method is similar to moment distribution. This method utilizes the Gauss– Seidel iteration to solve the slope– deflection equations without explicitly writing these equations. By iterating the unknown member end– moments, Kani's method will result in any desired accuracy in the member end moments. The Kani's method is easy and quick and also has a built– in error– elimination capability(7). This means that computational errors in the moment iteration will automatically disappear in subsequent operations. For these advantages, the method is very useful for manual analysis of the rectangular rigid frame structures (8).

The moment distribution has a lengthy procedure in analyzing the frame structures with several degrees of side– sway. However, Kani's method can easily handle the rectangular frame with multiple degrees of side– sway. Manual analysis of gable frames mostly uses the moment distribution or slope deflection methods(3). These methods are usually lengthy and have no built in error– elimination capability.

Although, Kani's method is available for rectangular frames, the author is not aware of a similar procedure applied to the gable frames. In this paper, analysis of gable frames using iteration procedure, similar to Kani's method, will be presented. It is assumed that structure behaves linearly elastic and has small deformation. Here the effect of bending only will be considered. The formulation, steps of calculation and numerical examples will also be given.

The method presented here can be applied to the single span as well as multispan gable frames. Furthermore, this method is convenient for ordinary calculator, and because the procedure has cyclic nature,it can very easily be adapted to computer programming.

2- A short historical background:

The concept of framework analysis was started in nineteenth century by the efforts of Maxwell, Castigliano and Mohr, among others. In that era, structural analysis was usually based on force method in which the member forces were chosen as unknowns. Although force method is easy to use for small problems, it needs solution of a large number of algebraic equations for large indeterminate frames.

Maney in United States and Ostenfeld in Denmark presented the basic ideas of framework analysis based on displacement parameters in about 1915 (6). Hardy Cross introduced the moment distribution method in 1930 (3). At that time, larger frameworks which were hard to treat by force method were solved by moment distribution. The moment distribution method was a great tool for structural analysis for many years. It is still a good tool for manual solution of small frame–works.

An excellent extension of the slope- deflection method was presented by Gasper Kani of Germany(4). This method has the simplicity of moment distribution method and can be applied to rectangular frames. Kani's method is able to analyze the simple frame with or without joint sway. It has the advantages of simplicity, speed and a built– in error– elimination capability so that computational errors automatically disappear in subsequent operations.

In early 1950s, digital computers were available in the market. Speed and accuracy of computer made it a very common tool of structural analysis. Matrix structural analysis and finite element method which use the computer were introduced for analysis of any type of structure. At the present time, most of struc– tural analysis methods for complex structure use finite element.

In the era of computers, classical methods still have their values. These methods are useful for manual analysis of structure. Furthermore, these methods are very helpful for visualizing structural behavior. In this paper an iteration procedure similar to Kani's method, which is adaptable to gable frame analysis, is presented.

3– Formulation:

Gable frames are usually used in one story building with long span and high rise. One of the uses of this type of structure is in industrial buildings such as factories. These structures are used in different shapes. Some gable frames under different loadings are shown in figures (1) to (4).

Each bay of gable frame consists of columns and ceiling portions. Member end–moments of these parts are induced from joint rotation and joint translation. For the ceiling portion, the cap part, and columns, the load– displacement relations are established. From these relations, the iteration equations which are able to analyze the structure will be found. The result satisfies the equilibrium and compatibility conditions.

In the process of applying the interation equa– tions, the fixity of the column ends will be enforced. As a result, the equilibrium, compatibility and boun– dary conditions required forvalid structural solutions are simultaneously satisfied. All of the equations derived are based on the assumption that the structure behaves linearly elastic and the resultant deformations are small.

3–1– Stiffness matrices:

Column stiffness entries are well known and are available in the textbooks on computer analysis of

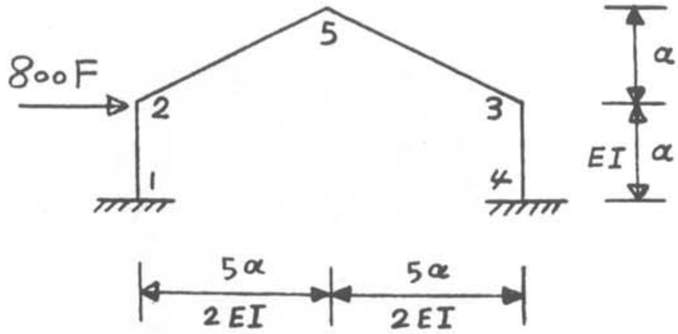

Figure (1) – Gable Frame Under Concentrated Load

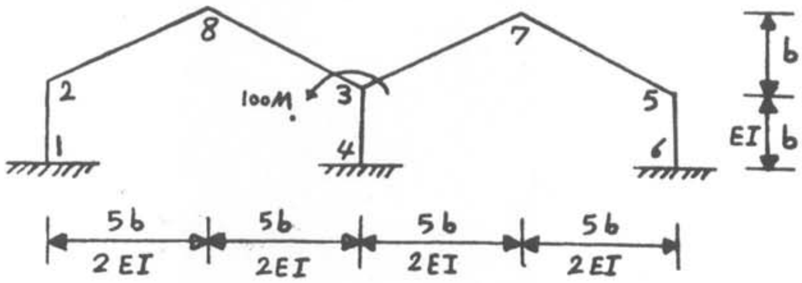

Figure (2) – Gable Frame Under Concentrated Moment

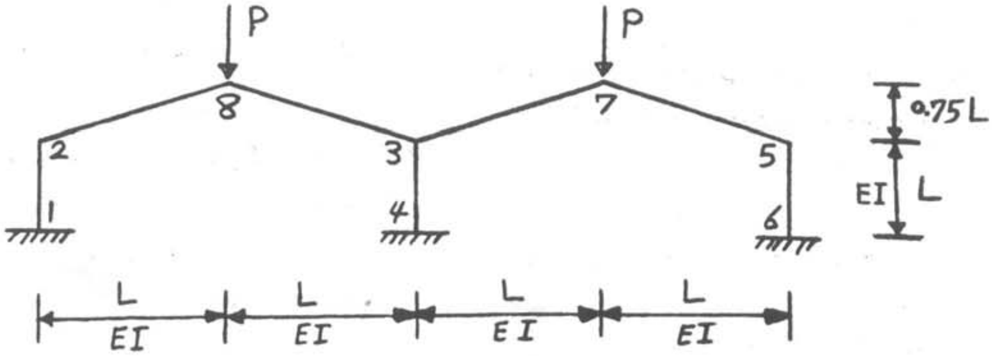

Figure (3) – Two – Span Gable Frame Under Concentrated Loads

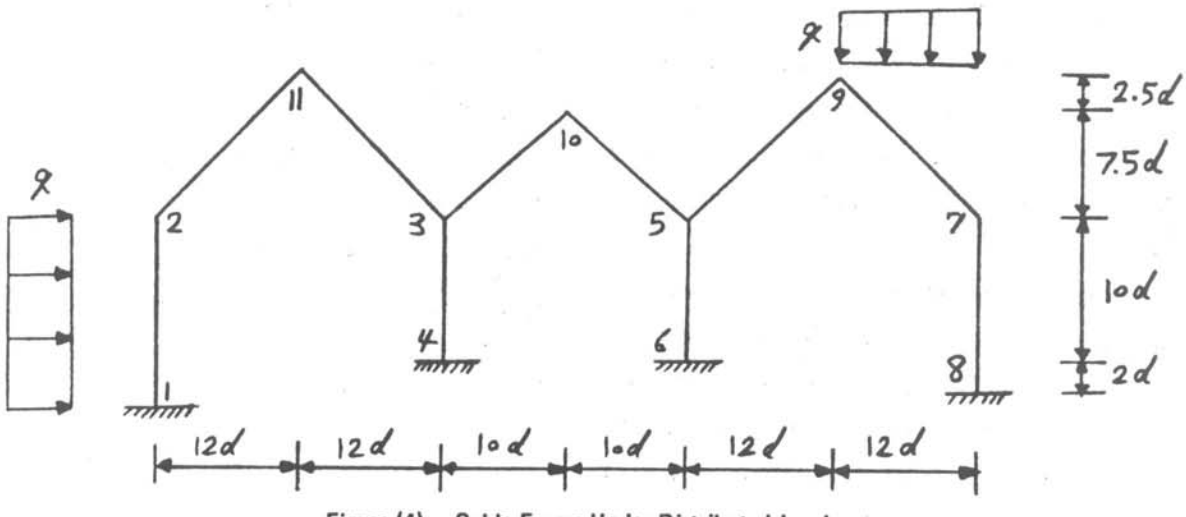

Figure (4) – Gable Frame Under Distributed Loads

structures (5). A general column is shown in figure(5). In this figure D1 to D4 shows the displacement components. Neglecting the axial effect, the stiffness of columns with flexural rigidity EI and length L is given as follows:

$$
\begin{aligned}
S_{11} &= 12EI/L^3 \\
S_{12} &= 6EI/L^2 \\
S_{13} &= -S_{11} \\
S_{14} &= S_{12} \\
S_{22} &= 4EI/L \\
S_{23} &= -S_{12} \\
S_{24} &= 2EI/L \\
S_{33} &= S_{11} \\
S_{34} &= -S_{12} \\
S_{44} &= S_{22}
\end{aligned}
\qquad (1)
$$

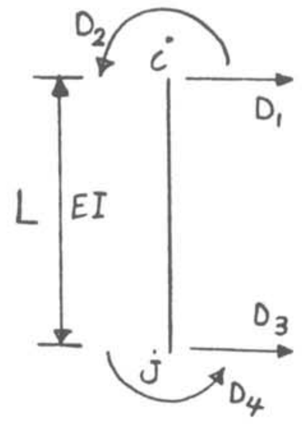

Figure (5) – A General Column

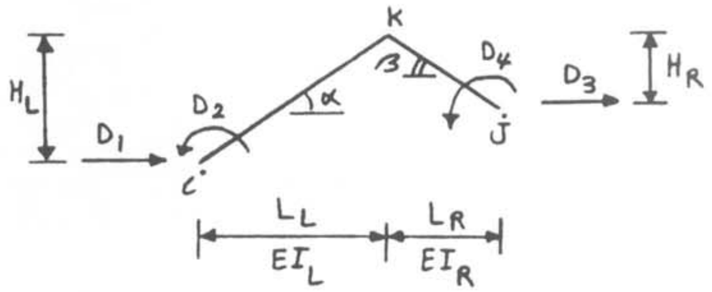

Figure (6) – A Ceiling Portion of Gable Frame

A general ceiling portion of the gable frame with four degrees of freedom is shown in figure (6). The vertical component at joints i and j does not exist due to inextension of the columns at these joints. The stiffness matrix of ceiling portion of the frame is required for the derivation of iteration equations . Several methods can be utilized to obtain the entries of stiffness matrix for ceiling portion of the gable frames. In this paper, the simplest method which is based on the definition of stiffness coefficient is used. According to definition of stiffness for each degree of freedom, there are the forces that must be applied at all degrees of freedom to produce a unit displacement at that degree of freedom.

One degree of freedom at each time is released and unit displacement at this degree of freedom is imposed on the ceiling portion of the structure. As a result four structures are required to be analyzed for finding all entries of stiffness matrix. Different methods can be utilized for these analyses. In this paper, slope deflection method is used. First the displacement and then unknown forces which are stiffness coefficients are found. The results after simplifying can be presented as following equations:

$$
\begin{aligned}
S_{11} &= S'''(6+4L'+4/L'+S'/L'+L'/S')/[L_L(\tan\alpha+\tan\beta)] \\
S_{12} &= -S'''(1+2L'+L'/S') \\
S_{13} &= -S_{11} \\
S_{14} &= S''(2+L'+S') \\
S_{22} &= [(3+4S')/(1+S')]\,(EI/L)_{ik} \\
S_{23} &= -S_{12} \\
S_{24} &= -[1/(1+S')]\,(EI/L)_{jk} \\
S_{33} &= S_{11} \\
S_{34} &= -S_{14} \\
S_{44} &= -(4+3S')\,S_{24}
\end{aligned}
\qquad (2)
$$

Equations (2) define the stiffness matrix of the ceiling portion of the gable frame. It is clear that out of sixteen entries of stiffness matrix only six independent entries exist. In order to simplify the equations, new terms are used which are defined as follows:

$$
\begin{aligned}
L' &= L_R/L_L \\
S' &= (EI/L)_{jk} / (EI/L)_{ik} \\
S'' &= [3\mathrm{COS}\beta\,(EI)_{jk}] / [L_R^2(1+S')(\mathrm{Tan}\,\alpha + \mathrm{Tan}\beta)] \\
L &= L_L + L_R
\end{aligned} \qquad (3)
$$

The set of equations(2) can be further simplified if the symmetry of the ceiling portion is considered. In the case that the ceiling portion of structure has a vertical axis of symmetry the entries of stiffness matrix can be written as follows:

$$
\begin{aligned}
S_{11} &= 48EI\mathrm{COS}^3\alpha / (L^3\,\mathrm{SIN}^2\alpha) \\
S_{12} &= -12EI\mathrm{COS}^2\alpha / (L^2\mathrm{SIN}\,\alpha) \\
S_{13} &= -S_{11} \\
S_{14} &= S_{12} \\
S_{22} &= 7EI\mathrm{COS}\,\alpha / L \\
S_{23} &= -S_{12} \\
S_{24} &= -S_{22}/7 \\
S_{33} &= S_{11} \\
S_{34} &= S_{12} \\
S_{44} &= S_{22}
\end{aligned} \qquad (4)
$$

Two points should be noted at this stage. The first point is concerning the independence of the entries of stiffness matrix for the symmetric ceiling. From equations(4), it is clear that only three independent entries of stiffness matrix exist. The second point is that all of the stiffness matrices written so far are symmetric and the unwritten stiffness entries can be found from the given parts.

3–2– Equivalent loads:

Gable frames are subjected to the joint loads as well as the member loads. On the other hand, the iteration equations are written for the joints. Therefore, equivalent joint loads for the given member loads should be determined.

The equivalent joint loads for any type of loading applied to the columns of the frame are given in the textbooks of computer methods in structural analysis. In this section the equivalent loads for the ceiling portion of gable frame are presented. In order to find these loads, a ceiling portion under general loading is assumed. The end joints of this structure, as shown in figure (7), are fixed and action is to be calculated.

Several methods can be utilized for calculating fixed end actions. In this paper, slope deflection method is used to find displacement and fixed end actions. The fixed end actions are presented in terms of fixed end moment of members ik and jk in the following equations:

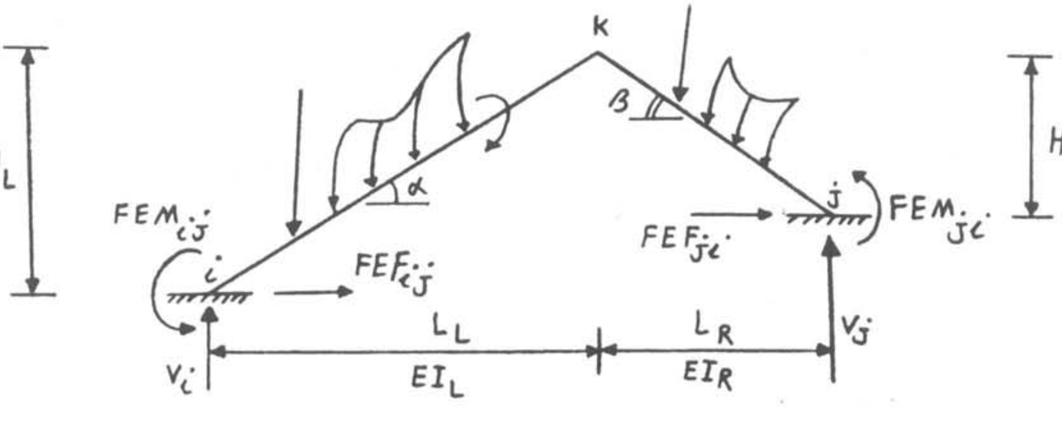

Figure (7) – A Ceiling Portion Under General Loads

$$FEM_{ij} = FEM_{ik} - 0.5\ (\ FEM_{ki} + FEM_{kj}\ )\ /\ (\ 1+S'\ ) \quad (5)$$

$$FEM_{jl} = FEM_{JK} - 0.5S(\ FEM_{ki} + FEM_{kj}\ )/(\ 1+S'\ ) \quad (6)$$

The subscript of fixed end moments in the above equations shows the moment due to loads of member ik and kj when these members are fixed at both ends. Using the equilibrium equations for structure of figure (7), the end forces FEF_{ij} and FEF_{ji} are obtained. These values are expressed in terms of vertical and horizontal resultant forces of applied load to the ceiling portion, F_v and F_H, moments of applied loads in member ik with respect to k, M_L, and moments of applied loads in member jk with respect to k, M_R. The final forms of end forces after simplification are given in the following equations:

$$FEF_{ij} = [1/\ (\ Tan\alpha + Tan\beta\ )]\ \{-F_v - F_H Tan\beta + [1/(1+S')](\ FEM_{ki} + FEM_{kj}\ )(1.5/L_L + (\ 1-0.5S'\ )/L_R] - (\ FEM_{ki} + FEM_{ik} + M_L\ )/L_L - (\ FEM_{ki} - FEM_{jk} - M_R\ )/L_R\} \quad (7)$$

$$FEM_{jl} = -FEF_{ij} - F_H \quad (8)$$

The sign convention for joint forces is assumed to be positive if loads are applied in the directions of $\rightarrow$, $\uparrow$, and $\circlearrowleft$. After calculating the joint equivalent load, these values are added to the corresponding concentrated loads applied at the joints. If p_i and M_i are applied at joint i then the resultant joint loads at this joint, FEP_i and FEM_i, can be found from following equations:

$$FEM_i = \Sigma_i FEM \quad - M_i \quad (9)$$

$$FEP_i = \Sigma_i FEF \quad - P_i \quad (10)$$

3– 3– Iteration equations:

The effects of rotation and translation of a joint on the induced moment are presented in the form of iteration equations. In the first part of this section the rotation equation is derived from figure (6) for which the ceiling portion stiffness is written. Moments and forces at joints i and j of ceiling portion are written as follows:

$$P_{ij} = FEF_{ij} + S_{11} + D_1 + S_{12} D_2 + S_{13} D_3 + S_{14} D_4 \quad (11)$$

$$M_{ij} = FEM_{ij} + S_{21} D_1 + S_{22} D_2 + S_{23} D_3 + S_{24} D_4 \quad (12)$$

$$P_{ji} = FEF_{ji} + S_{31} D_1 + S_{32} D_2 + S_{33} D_3 + S_{34} D_4 \quad (13)$$

$$M_{ji} = FEM_{ji} + S_{41} D_1 + S_{42} D_2 + S_{43} D_3 + S_{44} D_4 \quad (14)$$

Moments due to rotation are denoted by single prime and moments due to translation are denoted by double prime. Using stiffness coefficients which relate these moments to joint displacements, the following equations can be written:

$$M'_{ij} = S_{24} D_2 \quad (15)$$

$$M'_{ji} = S_{42} D_4 \quad (16)$$

$$M''_{ij} = S_{14} D1 \quad (17)$$

$$M''_{ji} = S_{32} D_3 \quad (18)$$

The joint force and moment can be written in terms of rotation and translation moments. This can be done, if the displacements from equation (11) to (18) are omitted. The results after this operation are in the form of following equations:

$$P_{ij} = FEF_{ij} + (S_{11}/S_{14})M''_{ij} + (S_{12}/S_{24})M'_{ij} + (S_{13}/S_{23}) M''_{ji} + (S_{14}/S_{24})M'_{ji} \quad (19)$$

$$M_{ij} = FEM_{ij} + (S_{12}/S_{14})M''_{ij} + (S_{22}/S_{24})M'_{ij} + M''_{ji} + M'_{ji} \quad (20)$$

$$P_{ji} = FEF_{ji} + (S_{13}/S_{14})M''_{ij} + (S_{23}/S_{24})M'_{ij} + (S_{33}/S_{23}) M''_{ji} + (S_{34}/S_{24})M'_{ji} \quad (21)$$

$$M_{ji}=FEM''_{ji}+M'_{ij}+(S_{34}/S_{23})M''_{ji}+(S_{44}/S_{24})M'_{ji} \qquad (22)$$

All of members meeting at a general joint , i , should satisfy the compatibility condition. In other words, the displacement for any member must be the same. Using equations (20) , (15) and (9) along with compatibility condition for joint i, the following equation which is the iteration equation for the rota– tion of the joint results:

$$M'_{ij}=R_{ij}(FEM_i+\sum_j M'_{ji}+\sum_j A_{ij}M''_{ij}+\sum_j M''_{ji}) \qquad (23)$$

This equation can be used iteratively to find the moment due to rotation of any joint. In equation (23) some parameters are used which can be defined in the following equations:

$$R_{ij}=-(S_{24}/\Sigma S_{22})_{ij} \qquad (24)$$

$$A_{ij}=(S_{12}/S_{14})_{ij} \qquad (25)$$

In the second part of this section, the iteration equation for the translation of joint is presented. Again the compatibility condition for a general joint, i, is considered. Using this condition along with equations (10), (17) and (19), results in the iteration equation *for translation effect of the joint as follows:*

$$M''_{ij}=D_{ij}(FEP_i+\sum_j B_{ij}M'_{ij}+\sum_j C_{ij}M'_{ji}+\sum_j E_{ij}M''_{ji}) \qquad (26)$$

In above equation some parameters are used, and some others will be used later. These parameters are defined in the following equations :

$$D_{ij}=-(S_{14}/\Sigma S_{11})_{ij} \qquad (27)$$

$$B_{ij}=(S_{12}/S_{24})_{ij} \qquad (28)$$

$$C_{ij}=(S_{14}/S_{24})_{ij} \qquad (29)$$

$$E_{ij}=(S_{13}/S_{23})_{ij} \qquad (30)$$

$$F_{ij}=(S_{22}/S_{24})_{ij} \qquad (31)$$

$$G_{ij}=(S_{11}/S_{14})_{ij} \qquad (32)$$

Iteration equation for translation effects, that is, equation (26), can be solved iteratively for any joint to give the corresponding moments. Finally, the effect of rotations and translation can be superimposed to find the end actions of members. This can be done by utilizing equations (19), (20) and (28) to (32). The results are presented in the following equations:

$$P_{ij}=FEF_{ij}+G_{ij}M''_{ij}+B_{ij}M'_{ij}+E_{ij}+C_{ij}M'_{ji} \qquad (33)$$

$$M_{ij}=FEM_{ij}+A_{ij}M''_{ij}+F_{ij}M'_{ij}+M''_{ji}+M'_{ji} \qquad (34)$$

4– Numerical procedure:

All of the aforementioned computational efforts are employed to find the end moment of the gable frame members. The following steps summarize the numerical procedure to perform the gable frame analysis:

1– Find the required parameters such as A_{ij}, B_{ij}, ...etc. from equations (24), (25), (27) to (32).

2– Calculate the fixed end moments in columns and ceilings. Equations (5), (6), (7) and (8) can be used for this purpose.

3– Add up the fixed end actions of all members to find the joint actions. Equations (9) and (10) are used in this step.

4– Solve the set of equations (23) and (26) by performing Gauss – Seidel iteration. This step must by repeated until the required accuracy is reached.

5– Determine the end actions of all members by utilizing equations (33) and (34).

6– Calculate the joint displacements. if it is required, Equations (15) and (16) can be used for this purpose.

It is clear that iteration can be performed first on either rotation moment or translation moment. This means that the use of corresponding iteration equation does not affect the final results. It is better to start with the one which has greater effect. Furthermore, the choice of starting joint does not affect the final results. However, for faster convergence, it is better to start at the joint with the greater joint actions..

5– Illustrative examples:

In order to demonstrate the use of the computational procedure, four examples are solved in this section. The first example is a structure under the concentrated joint load. A twospan gable frame under concentrated moment is solved in the second example. Another two – span gable frame subjected to concentrated roof loads is analyzed in the third example. Finally, the fourth example shows the ability of this procedure in analyzing the frame with three spans subjected to distributed member force.

5–1– Example one:

Figure (1) shows a gable frame which is solved as a first example. Beams and columns are prismatic members and their properties along with the applied load are given in this figure. It is aimed to find the end moment of the members.The effect of concentrated joint load is presented in this example.

In order to demonstrate the use of iterative equations clearly, it is suggested that the required parameters and calculated values be presented in a table. Table (1) shows these values. This table clearly indicates the calculated values in each cycle. THe results, that is, end actions of members and joint displacements, are shown in this table. It should be noted that after sufficient iteration, the final results are comparable with the exact solution (1).

5–2– Example two

A two–span gable frame is given in figure (2). It is required to calculate end moments of all members. The property of the frame members and appiled concentrated joint moment are given in the figure. In this example the ability of the method is tested for concentrated joint moment. Table (2) shows all parameters and calculated values in each cycle. The results presented in this table are comparable with the exact solution (1).

5–3– Example three:

Analyze the frame shown in figure (3). This problem is available on page 296, in the textbook of "Structural Analysis" by Ghali and Neville (2). The answers of this problem are checked with results of the presented method.

Required parameters and calculated values for this example are shown in Table (3). The results are comparable with the answers supplied by reference (2). This example proves that the method is able to consider roof loads applied to gable frames.

5–4– Example four:

Distributed loads on column and roof of gable frames are considered in this example. A three – span gable frame which is shown in figure (4) is solved. Dimensions of gable frame and load properties are given in this figure and the flexural rigidity of all members is assumed to be constant EI. Table (4) presents the solution for example four. This example shows the ability of the method to analyze multispan gable frame.

6 – Convergence Factors:

For design purpose, the solution need not be very accurate. By suing this method, approximate solution will be achieved in the first few iterations. In order to have an accurate solution the iteration must be continued until the rotation and translation moments do not differ by more than acceptable errors in two subsequent iterations.

In the previous section, th ability of the method in solving gable frames was demonstrated by numerical examples. The iteration procedure was started with zero initial values. As a matter on fact, initial values need not always be zero and the iteration can start with any initial values. A better choice of initial values will result in faster convergence.

The rate of convergence towards the exact solution in this method depends upon type of structure and loading condition. In general, gable frame with higher stiffness has shorter iteration procedure than the gable frame with flexible elements. It is clear that the iteration can be started from any joint. Faster convergence towards the exact solution is obtained if the starting point has higher joint actions. The path of iteration does not change the result, therefore the joint with higher joint actions should have higher priority of iteration for faster convergence.

Better initial estimation of moments and choice of the next values in iteration process will shorten the analysis. One way to estimate the moments in any cycle, is to find the direction and variation of these values. This can be done by the first two or three iterations.

7 – Error study:

According to the points presented in the last section several factors will affect the required number of cycles of ireration for a solution with a desired accuracy. In order to have a feeling of the rate of convergence some error studies are presented in this section. This study is based on the relative deviation error of member end moment. The difference between the exact and calculated values divided by the exact value is an indication of this error for any member and moment.

The first three example problems are chosen for this study. Table (5) presents the error in each cycle of gable frame for example one. Error study of gable frames in examples two and three are presented in table (6) and (7) respectively. From these studies, it is concluded that, with no fast convergence plan, the rate of convergence of the method is good, comparable with similar methods such as moment distribution and kani's method.

8– Summary and Conclusion:

In this paper, a formulation for calculating end moments of gable frame members which are primarily subjected to bending is presented. It was assumed that structure behaves linearly elastic and has small deformation. As a mommon sign convention, counterclockwise end moment and rotation of members are considered positive.

The method presented here can be applied to gable frame composed fo prismatic members subjected to any type of loading. Furthermore, this procedure is convenient for hand calculation and because its formulae have cyclic nature, they can be adapted to computer programming very easily. The ability of the method has been demonstrated with four example problems. Analysis with this procedure is easy and quick and can be continued up to any desired accuracy. This method has a built– in error– elimination capability and the error in the member end– moment will disappear automatically by iteration procedure.

11 – Tables:

JOINT		2		3	
MEMBER		21	23	32	34
A		1	– 1	– 1	1
B		3/a	6/a	6/a	3/a
C		3/a	–6/a	–6/a	3/a
E		2/a	–2/a	–2/a	2/a
R		–0.3722	0.0365	0.0365	–0.3722
D		–0.418a	–0.082a	–0.082a	–0.418a
FEM_i		0.		0.	
FEP_i		–800/a		0.	
CYCLE 1	M'	0.	0.	2.393	–24.41
	M''	334.41	65.58	10.75	54.83
2	M'	–104.96	10.29	4.485	–45.74
	M''	349.41	68.52	21.12	107.71
3	M'	–114.09	11.18	6.927	–70.64
	M''	469.14	92.	29.63	151.10
4	M'	–153.99	15.10	8.50	–86.72
	M''	491.58	96.41	37.20	189.69
5	M'	–164.11	16.09	10.03	–102.32
	M''	542.09	106.31	42.49	216.67

Tagle (1) – Solution of Example One

Table (1) – Continued

6	M′	−181.77	17.82	11.02	−112.39
	M″	560.56	109.93	47.02	239.77
7	M′	−189.34	18.56	11.89	−121.34
	M″	584.61	114.65	50.15	255.73
8	M′	−198.04	19.42	12.48	−127.34
	M″	579.07	117.09	52.74	268.92
9	M′	−202.95	19.90	12.98	−132.37
	M″	609.47	119.52	54.56	278.21
10	M′	−207.52	20.34	12.32	−135.88
	M″	617.18	121.04	56.03	285.68
11	M′	−210.50	20.64	13.60	−138.71
	M″	623.88	122.35	57.08	291.05
12	M′	−213.0	20.88	13.80	−140.75
	M″	628.46	123.25	57.90	295.27
13	M′	−214.75	21.05	13.95	−142.34
	M″	632.17	123.98	58.51	298.36
14	M′	−216.15	21.19	14.07	−143.51
	M″	634.84	124.50	58.98	300.75
MOMENT		202.5Fa	−200Fa	−11.8Fa	13.7Fa
FORCE		621.2F	174F	−174F	171F
ROTATION		$-109.5Fa^2/EI$	$-109.5Fa^2/EI$	$-73Fa^2/EI$	$-73Fa^2/EI$

Table (2) – Solution of Example Two

JOINT		2		3			5	
MEMBER		21	23	32	34	35	53	56
A		1	-1	-1	1	-1	-1	1
B		$^{3}/_{b}$	$^{6}/_{b}$	$^{6}/_{b}$	$^{3}/_{b}$	$^{6}/_{b}$	$^{6}/_{b}$	$^{3}/_{b}$
C		$^{3}/_{b}$	$-^{6}/_{b}$	$-^{6}/_{b}$	$^{3}/_{b}$	$-^{6}/_{b}$	$-^{6}/_{b}$	$^{3}/_{b}$
E		$^{2}/_{b}$	$-^{2}/_{b}$	$-^{2}/_{b}$	$^{2}/_{b}$	$-^{2}/_{b}$	$-^{2}/_{b}$	$^{2}/_{b}$
R		−0.3722	0.0365	0.029	−0.269	0.029	0.0365	−0.3722
D		−0.418b	−0.082b	−0.0704b	−0.358b	−0.0704b	−0.082b	−0.418b
FEM_i			0.		−100M.			0.
FEP_i			0.		o.			0.
CYCLE 1	M'	0.	0.	−2.91	29.64	−2.91	−0.11	1.08
	M''	0.	0.	0.	0.	0.	−1.43	−7.29
2	M'	1.08	−0.11	−2.99	30.55	−2.99	−0.48	4.89
	M''	−7.29	−1.43	−4.3	−21.82	−4.3	−2.39	−12.2
3	M'	4.89	−0.48	−3.46	35.30	−3.46	−0.66	6.80
	M''	−12.2	−2.39	−5.	−25.51	−5.	−3.49	−17.80
4	M'	6.80	−0.66	−3.6	36.71	−3.6	−0.87	8.93
	M''	−17.80	−3.49	6.08	−31.	−6.08	−4.11	−20.97
5	M'	8.93	−0.87	−3.74	38.19	−3.74	−0.99	10.13
	M''	−20.97	−4.11	−6.61	−33.72	−6.61	−4.69	−23.92

Table (2) – Continued

6	M'	10.13	–0.99	–3.83	39.09	–3.83	–1.09	11.21
	M''	–23.92	–4.69	–7.06	–36.03	–7.06	–5.04	–25.73
7	M'	11.21	–1.09	–3.90	39.78	–3.90	–1.16	11.89
	M''	–25.73	–5.04	7.37	–37.59	–7.37	–5.34	–27.25
8	M'	11.89	–1.16	–3.95	40.28	–3.95	–1.22	12.45
	M''	–27.25	–5.34	–7.60	–38.76	–7.60	–5.54	–28.25
9	M'	12.45	–1.22	–3.98	40.46	–3.98	–1.25	12.83
	M''	–28.25	–5.54	–7.76	–39.60	–7.76	–5.69	–29.04
10	M'	12.83	–1.25	–4.01	40.90	–4.01	–1.28	13.12
	M''	–29.04	–5.69	–7.88	–40.21	–7.88	–5.80	–29.59
MOMENT		–3.3M.	2.6.M	29M	41.6M.	29M	2.9M.	–3.3M.
FORCE		–19.6 M./b	21M./b	–21 M./b	42 M./b	–21 M./b	21 M./b	–19.8 M./b

Table (3) – Solution of example Three

JOINT		2		3			5	
MEMBER		21	23	32	34	35	53	56
A		1	−1	−1	1	−1	−1	1
B		$3/L$	$8/L$	$8/L$	$3/L$	$8/L$	$8/L$	$3/L$
C		$3/L$	$-8/L$	$-8/L$	$3/L$	$-8/L$	$-8/L$	$3/L$
E		$2/L$	$-8/3L$	$-8/3L$	$2/L$	$-8/3L$	$-8/3L$	$2/L$
R		−0.2941	−0.0588	0.0417	−0.2083	0.0417	0.0588	−0.2941
D		−0.2922L	−0.1558L	−0.11L	−0.2064L	−0.11L	−0.1558	−0.2922L
FEM_i		0.			0.			0.
FEP_i		2/3L			0.			−2/3L
CYCLE 1	M'	0.	0.	−0.004	0.022	−0.004	−0.002	0.010
	M''	−0.195	−0.104	−0.030	−0.057	−0.030	0.086	0.160
2	M'	0.037	−0.007	−0.002	0.009	−0.002	0.003	−0.016
	M''	−0.229	−0.122	−0.018	−0.035	−0.018	0.092	0.172
3	M'	0.037	−0.007	−0.001	0.007	−0.001	0.004	−0.020
	M''	−0.228	−0.122	−0.012	−0.023	−0.012	0.101	0.189
4	M'	0.035	−0.007	−0.001	0.004	−0.001	0.005	−0.023
	M''	−0.223	−0.119	−0.008	−0.015	−0.008	0.104	0.195
5	M'	0.033	−0.007	0.	0/003	0.	0.005	−0.025
	M''	−0.217	−0.116	−0.005	−0.010	−0.005	0.106	0.199

Table (3) – Continued

6	M'	0.031	−0.006	0.	0.002	0.	0.005	−0.026
	M''	−0.214	−0.114	−0.003	−0.006	−0.003	0.107	0.201
7	M'	0.030	−0.006	0.	0.001	0.	0.005	−0.027
	M''	−0.211	−0.113	−0.002	−0.004	−0.002	0.108	0.203
8	M'	0.030	−0.006	0.	0.001	0.	0.005	−0.027
	M''	−0.209	−0.112	−0.001	−0.003	−0.001	0.109	0.204
9	M'	0.029	−0.006	0.	0.	0.	0.005	−0.028
	M''	−0.208	−0.111	−0.001	−0.002	−0.001	0.109	0.205
MOMENT		−0.149PL	0.151PL	−0.115PL	0.	0.116PL	−0.149PL	0.149PL
FORCE		−0.329P	0.339P	−0.339P	0.	0.339P	−0.339P	0.327P
ROTATION		$0.013PL^2/_{EI}$	$0.013PL^2/_{EI}$	0.	0.	0.	$-0.013PL^2/_{EI}$	$-0.013PL^2/_{EI}$

Table (4) – Solution of Example Four

JOINT		2		3			5			7	
MEMBER		21	23	32	34	35	53	56	57	75	78
A		1	–1	–1	1	–1	–1	1	–1	–1	1
B		1/4d	3/5d	3/5d	3/10d	4/5d	4/5d	3/10d	3/5d	3/5d	1/4d
C		1/4d	–3/5d	–3/5d	3/10d	–4/5d	–4/5d	3/10d	–3/5d	–3/5d	1/4d
E		1/6d	–1/5d	–1/5d	1/5d	–4/15d	–4/15d	1/5d	–1/5d	–1/5d	1/6d
R		–0.299	0.0674	0.0354	–0.2212	0.0442	0.0442	–0.2212	0.0354	0.0674	–0.299
D		–3.863d	–1.781d	–0.7879d	–2.461d	–1.313d	–1.313d	2.461d	–0.7879d	–1.781d	–3.863d
FEM_i		–12.			0.			–3.		–15.	
FEP_i		–6/d			0.			3.6/d		–3.6/d	
CYCLE 1	M'	3.59	–0.69	0.35	–2.21	0.44	–0.01	0.06	–0.01	–0.98	5.11
	M''	23.18	10.68	1.36	4.24	2.26	–3.47	–6.50	–2.08	5.66	12.27
2	M'	–0.66	0.13	0.28	–1.76	0.35	0.13	–0.64	0.10	–0.58	3.02
	M''	23.18	10.68	1.08	3.38	1.80	–3.02	–5.66	–1.81	4.56	10.08
3	M'	–0.56	0.11	0.33	–2.04	0.41	0.13	–0.65	0.10	–0.64	3.31
	M''	25.01	11.53	1.37	4.29	2.29	–2.69	–6.05	–1.62	5.22	11.33

Tabe (4) – Continued

4	M'	−0.95	0.18	0.35	−2.19	0.44	0.17	−0.84	0.13	−0.59	3.06
	M''	25.29	11.65	1.51	4.72	2.52	−2.48	−4.64	−1.49	5.23	11.36
5	M'	−1.04	0.20	0.37	−2.30	0.46	0.18	−0.91	0.14	−0.58	3.03
	M''	25.65	11.82	1.62	5.06	2.70	−2.34	−4.39	−1.40	5.33	11.56
6	M'	−1.14	0.22	0.38	−2.37	0.47	0.19	−0.97	0.15	−0.57	2.98
	M''	25.83	11.90	1.68	5.26	2.80	−2.25	−4.22	−1.35	5.37	11.64
7	M'	−1.19	0.23	0.39	−2.41	0.48	0.20	−1.	0.16	−0.57	2.95
	M''	25.95	11.96	1.72	5.39	2.87	−2.20	−4.12	−1.32	5.40	11.71
8	M'	−1.23	0.23	0.39	−2.44	0.49	0.20	−1.02	0.16	−0.56	2.94
	M''	26.03	12.	1.75	5.46	2.91	−2.16	−4.06	−1.30	5.41	11.74
MOMENT		$11.57qd^2$	$-11.5qd^2$	$7.75qd^2$	$0.59qd^2$	$-8.29qd^2$	$4.13qd^2$	$-6.11qd^2$	$2.qd^2$	$-17.6qd^2$	$17.6qd^2$
FORCE		−1.97qd	1.96qd	−1.96qd	0.36qd	1.58qd	−1.58qd	−1.12qd	2.69qd	−2.69qd	2.69qd

Table (5) – Solution Errors for Example One

JOINT		2		3	
MEMBER		21	23	32	34
EXACT MOMENT		202.5Fa	−202.5Fa	−13.7Fa	13.7Fa
CYCLE					
1	M_{cal}	334Fa	−52Fa	38Fa	6Fa
	ERROR	−65%	74%	377%	56%
2	M_{cal}	139Fa	−115Fa	26Fa	16Fa
	ERROR	31%	43%	289%	−17%
3	M_{cal}	241Fa	−134Fa	25Fa	10Fa
	ERROR	−19%	34%	282%	27%
4	M_{cal}	184Fa	−156Fa	15Fa	16Fa
	ERROR	9%	23%	209%	−17%
5	M_{cal}	214Fa	−166Fa	10Fa	12Fa
	ERROR	−6%	18%	173%	12%
8	M_{cal}	201Fa	−188Fa	−4Fa	14Fa
	ERROR	0.%	7%	70%	−2%
12	M_{cal}	202Fa	−198Fa	−10Fa	13.7Fa
	ERROR	0.%	2%	27%	0.%
13	M_{cal}	202.6Fa	−199Fa	−11Fa	13.7Fa
	ERROR	0.%	2%	20%	0.%

Table (6) – Solution Errors for Example Two

JOINT		2		3			5	
MEMBER		21	23	32	34	35	53	56
EXACT MOMENT		–3.3M.	3.3M.	29.3M	41.4M.	29.3M.	3.3M.	–3.3M.
CYCLE								
1	M_{cal}	0.	–2.9M	20.3M	59.3M	18.8M	–0.7M	–5.1M
	ERROR	100%	188%	31%	–43%	36%	121%	–54%
2	M_{cal}	–5.1M	–5.1M	23.7M	39.2M	22.4M	–1.5M	–2.4M
	ERROR	–54%	254%	19%	5%	23%	145%	27%
3	M_{cal}	–2.4M	–2.7M	26.4M	45.1M	25.1M	–0.3M	–4.2M
	ERROR	27%	182%	10%	–9%	14%	109%	–27%
4	M_{cal}	–4.2M	–1.5M	27.1M	42.4M	26.3M	0.6M	–3.1M
	ERROR	–27%	145%	7%	–2%	10%	82%	6%
5	M_{cal}	–3.1M	–0.11M	27.8M	42.7M	27.1M	1.3M	–3.7M
	ERROR	6%	103%	5%	–3%	7%	60%	–12%
6	M_{cal}	–3.7M	0.7M	28.2M	42.2M	27.7M	1.8M	–3.3M
	ERROR	–12%	79%	4%	–2%	5%	45%	0%
7	M_{cal}	–3.3M	1.5M	28.5M	42M.	28.2M	2.2M	–3.5M.
	ERROR	0%	54%	3%	–1%	4%	33%	–6%
8	M_{cal}	–3.5M	2M	28.7M	41.8	28.5	2.5M	–3.3M
	ERROR	–6%	40%	2%	–1%	3%	24%	0%

Table (7) – Solution Errors for Example Three

JOINT		2		3		5		
MEMBER		21	23	32	34	35	53	56
EXACT MOMENT		−0.149PL	0.149PL	−0.115PL	0.	0.115PL	−0.149PL	0.149PL
CYCLE								
1	M_{cal}	−0.159PL	0.069PL	−0.043PL	−0.014PL	−0.144PL	−0.106PL	0.181PL
	ERROR	−31%	54%	62%		−25%	29%	−21%
2	M_{cal}	−0.155PL	0.153PL	−0.099PL	−0.017PL	0.162PL	−0.135PL	0.140PL
	ERROR	−4%	−3%	14%		−9%	9%	6%
3	M_{cal}	−0.154PL	0.160PL	−0.107PL	−0.010PL	0.126PL	−0.142PL	0.149PL
	ERROR	−3%	−7%	7%		−9%	5%	0%
4	M_{cal}	−0.152PL	0.159PL	−0.112PL	−0.007PL	0.122PL	−0.145PL	0.148PL
	ERROR	−2%	−7%	3%		−6%	3%	1%
5	M_{cal}	−0.151PL	0.157PL	−0.113PL	−0.004PL	0.120PL	−0.147PL	0.149PL
	ERROR	−1%	−5%	2%		−4%	1%	0%
6	M_{cal}	−0.150PL	0.154PL	−0.114PL	−0.003PL	0.119PL	−0.148PL	0.149PL
	ERROR	0%	−3%	1%		−3%	1%	0%
7	M_{cal}	−0.150PL	0.153PL	−0.115PL	−0.002PL	0.118PL	−0.149PL	0.149PL
	ERROR	0%	−3%	0%		−3%	0%	0%
8	M_{cal}	−0.150PL	0.152PL	−0.115PL	−0.001PL	0.117PL	−0.149PL	0.149PL
	ERROR	0%	−2%	0%		−2%	0%	0%

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12—Notation:

The following symbols are used in this paper:

A_{ij} , B_{ij} , C_{ij} , D_{ij} , E_{ij} , F_{ij} , G_{ij} , L' , S' , S'' , R_{ij} =defined parameters,

D_i, to D_4 =displacement components,

EI= flexural rigidity ,

FEF_{ij}=fixed end force at i ,

FEM_{ij}=fexed end moment at i ,

FEM_i=resultant joint moment at i ,

FEP_i=resultant joint load at i ,

F_H=horizontal resultant of applied load to the ceiling portion ,

F_v = vertical resultant of applied load to the ceiling portion.

L' , L_L , L_R=lengths ,

M_L = moments of applied load in member ik with respect to k,

M_R =moments of applied loads in member jk with respect to k ,

M_i=applied moment at joint i ,

M_{ij}=member moment (ij) at i ,

M'_{ij}= moments due to rotation

M''_{ij}=moments due to translation ,

P_i=applied load at joint i ,

P_{ij}=member force (ij) at i ,

S_{ij}=stiffness entries ,

α , β =angles.