Optimal Capacity of Axially Loaded Bent Pile

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ABSTRACT:

The behaviour of bent pile, with consequent initial stresses, subjected to axial load is studied. Finite difference method is used to find the variation of pile stress parameter with respect to slenderness parameter. It is shown that too rigid pile and also too flexible pile fail to derive benefit from the surrounding soil. On the other hand, moderately rigid or flexible pile is more beneficial. Design charts and formulae are given to select pile cross section which has optimal axial capacity.

Key Words:

Pile, bent pile, hinged-hinged ends, initial stresses, axial load, design charts, design formulae, optimal capacity

Introduction

Foundations in weak soils such as soft clays, loose silts and loose sands pose two serious problems: the first, their low bearing strength and the second, their excessive settlement characteristics. In such soils, engineers often prefer piles to support heavy engineering structures with a view to transfer the loads to firm strata or rock below.

In general piles are proportioned to resist axial loads. Long piles have small natural out-of-straightness defects (initial curvature), acquired during manufacturing and/or handling, not visible to the naked eye; the pile being free of initial stresses. During installation, although the pile toe may be on plumb line, the out-of-straightness defects in the pile get magnified under hammer blows particularly after the toe has reached stiffer layers of soil. If the pile can undergo elastic recovery, the initial stresses acquired during driving will disappear or else they remain in the pile. The initial curvature, to a smaller extent without initial stresses, and to a greater extent with initial stresses, causes reduction in axial capacity of the pile as opposed to the ideal straight pile (Broms 1963).

The strength of a free standing initially curved column decreases continously as the slenderness increases, whereas for a pile, surrounded by soil, bent during installation with consequent initial stresses, the strength increases first with increase in slenderness until a peak is reached and then the strength decreases with further increase in slenderness (Rao and Mazindrani 1981).

In this paper, the variation of pile stress parameter with respect to slenderness parameter is presented in design charts. These charts are sueful for optimal design of pile foundations.

Brief Historical Review

The behaviour of initially curved piles is analogous to the bucking of curved columns. In fact, the effect of the surrounding soil is to provide lateral bracing to the pile and therefore to increase its load carrying capacity. Westergaard and Osgood (1928) presented an analysis of initially curved column using fourier series. They concluded that it is sufficiently accurate to represent the initial curvature of the column by a simple half sine wave. It was also concluded that the axial capacity of an initially curved column is very much reduced as opposed to a perfectly straight column owing to its initial out-of-straightness.

Glick (1948) analysed initially curved pile with hinged-hinged ends and with no initial stresses, the pile having undergone elastic recovery. The surrounding soil was assumed to be elastic with no adhesion. He solved the governing differential equation using fourier series to represent the deformation of the pile. He found that the terms excepting the first in the fourier series have little influence on pile bending under axial load and concluded that the initial curvature of the pile could be represented with sufficient accuracy by a simple half sine wave. Following Timoshenko (1930), he recommended that the maximum initial out-of-straightness at center of pile to be taken as equal to 1/400 to 1/1000 times the length of pile. He also found that the axial capacity of an initially curved pile is very much reduced as opposed to a perfectly straight pile owing to its initial out-of-straightness defects.

The effect of the soil reactions on bending moments in the pile was considered by Walter (1951). This solution and its assumptions are similar to Glick's solution. It was concluded that the soil reactions have a neutralising effect on bending moments in the pile. The effect of nonlinearity of the soil response on the pile behaviour was studied by Gibson (1952). He used similar assumptions as Glick. He found that the stiffness of the ground is the decisive factor in ensuring the stability of piles, the ultimate lateral resistance of the ground is generally not important. Again he recommended the use of half sine wave for representing the imperfections in the pile.

A semi-empirical analysis for curved piles was presented by Johnson (1962). In this analysis the pile was assumed to be hinged at ends and soil behaviour was taken to be perfectly elastic. Johnson concluded that the reduction in capacity for toe bearing piles is greater than that for similarly bent shaft bearing piles.

Unless the pile toe has penetrated sufficiently into the stiff layer at the bottom and the pile top is embedded enough in the pile cap, it is difficult to obtain conditions corresponding to fixed-fixed ends. Thus, Glick (1948) and later Walter (1951), Gibson (1952) and Broms (1963) assumed hinged-hinged ends mainly to produce a conservative design formula. A translating end condition at the toe of the pile is not practical unless the pile toe after having penetrated a very soft soil layer is simply resting on an inclined stratum of hard rock. At the top, the pile cap resting on ground usually has enough resistance from the soil underlying and adjacent to it to prevent it from lateral movement. For this reason hinged-hinged end conditions are assumed in the present study.

One of the numerical methods which is suitable to the analysis of curved pile is the finite difference method. This method is quite simple for calculating buckling loads of the pile and its deflection. Furthermore, the convergence of the finite difference method for the initially curved pile is found to be fast. Rao and Mazindrani (1981) used this method to solve the initially curved pile. They found the deflection vector by use of computer. With the deflection vector in hand, the maximum bending stress was found. With yield stress as criterion of design, the axial capacity of initially curved pile was calculated.

The two main defects in most previous research are:

1- The pile having been pushed off plumb line during installation, the possibility that the pile did not undergo elastic revovery was ignored,

2- The neutralising moments from lateral soil reactions were not taken into account in the analyses.

The present solution has given due consideration to these aspects. In this paper using finite difference method, solution to the bent pile, with consequent initial stresses, assuming that the pile could not undergo elastic recovery, is obtained. Based on the results two design formulae are recommended to select pile section which has optimal axial capacity.

Formulation

The deformed pile, embedded in perfectly elastic clay soil, subjected to axial load and an elemental pile length dx at depth x are shown in Fig. 1. From the equilibrium of the pile element, the differential equation of the deformed pile is derived (Mazindrani, et al. 1977) as:

$$EIY^{IV} + 2(EI)^{I}Y^{III} + (EI)^{II}Y^{II} + P(Y^{II} + Y_0^{II}) + 2P^{I}(2P^{I}(Y^{I} + Y_o^{I}) + K_hY = 0 \qquad (1)$$

In Eq. 1, P is the axial load on pile at depth x and K_h is the horizontal soil modulus, assumed constant with depth such as in pre-loaded cohesive soils. Due to skin resistance, it is assumed that the axial load varies with the depth. If P_0 is the axial load at top of the pile and α is the constant of axial load variation, the axial load P at depth x is assumed (Reddy and Valsangkar 1970) as:

$$P = P_0(1 - \alpha x/l) \qquad (2)$$

In Eq. 2, l is the total length of the pile. It is assumed that the initial curvature of the bent pile with consequent initial stresses is a half sine wave difined by:

$$Y_0 = a_0 \operatorname{Sin} \pi x/l \qquad (3)$$

In Eq. 3, Y_0 is the initial out-of-straightness of pile at depth x taken as induced entirely during installation (although a portion of it is due to causes such as manufacturing and/or handling) and a_0 is the initial off set at mid height of the pile taken as 1/400 times the length of the pile, being the upper limit of the range proposed by Timoshenko (1930).

Usually the rigidity of pile cross section is constant and therefore terms involving $(EI)^{I}$ and $(EI)^{II}$ will vanish. Considering the fact that the terms involving rate of change of axial load do not play as significant a part as the terms involving axial load (Johnson 1962), Eq. 1 can be simplified as:

$$EIY^{IV} + P(Y^{II} + Y_0^{II}) + K_hY = 0 \qquad (4)$$

Eq. 4 can be written in non-dimensional form. In order to change Eq. 4 to new form, some parameters need to be defined. Let T be relative stiffness factor, k be radius of gyration, A be area of cross-section of pile, σ be axial stress at depth x, z be depth parameter, a_0 be taken as 1/400 times length of the pile, then the following equations can be written:

$$T = \sqrt[4]{EI/K_h} \qquad (5)$$

$$Z = x/T \qquad (6)$$

$$Z_{max} = l/T \qquad (7)$$

$$\sigma = P/A \qquad (8)$$

$$I = Ak^2 \qquad (9)$$

$$d^4Y/dz^4 + (\sigma/E)(T/k)^2\, d^2Y/dz^2 + Y = (\pi^2/400)(\sigma/E)(T/k)^2 (T/Z_{max}) \operatorname{Sin}\pi Z/Z_{max} \qquad (10)$$

Eq. 10 is a non-dimensional form of Eq. 4. The maximum total stress in the pile should be limited. Maximum total stress $(\sigma_{total})_{max}$ is to be equal to or less than the yield stress of the pile material, σ_{yield}. If r is taken to be the ratio of section modulus divided by area of pile section, then the criterion of pile design can be written as:

$$(\sigma + (M+M_0)/Ar)_{max} \leqslant \sigma_{yield} \qquad (11)$$

While the initial stresses in the pile are considered, those in the soil are assumed to be negligible, the mechanism being as shown in Fig. 2. It may be seen that the pile has been bent during installation and will have a tendency, in the absence of the driving hammer load, to recover elastically by moving laterally to the right which is prevented by the soil on that side (concave side) of the pile. While the soil reaction pressures on the concave side of the pile will be greater than those corresponding to earth pressure at rest, the soil reaction pressures on the convex side of the pile will be lesser. The net soil reaction pressures shown in Fig. 2(b) will then oppose the movement of the pile to the right towards the soil. However, when under permanent load, the pile moves laterally to the left; the net soil reaction pressures shown in Fig. 1(b) will oppose the movement. This is the reason why initial stresses in the soil due to deflections y_0 are not considered in the analysis.

Numerical Analysis

Differential Eq. 10 with yield criterion of Eq. 11 is solved numerically. Finite difference method along with Gauss-Siedel iterations is used to solve the governing differential equation of the pile. A computer program is written for NCR-DMV microcomputer to obtain solution. Computational steps of the program are as follows:

1- Reading and Printing number of pivots, number of iterations; soil and pile data: T, E, Z_{max}, k, r, α and σ_{yield}.
2- Assume axial stress at top, compute axial stress at various pivotal points.
3- Compute induced deflections at various points and obtain total deflections.
4- Find curvatures at various points and obtain bending stress.
5- Compute total stresses at various points.
6- Pick up the maximum total stress.
7- Compare the maximum total stress with the yield stress of the pile. If the maximum total stress is equal to or greater than the yield stress of the pile, print slenderness parameter and stress parameter. If not go to the 2nd step.

Optimal Design Criteria

The following observations are made from a study of the results in design charts of Fig. 3 to 14. These design charts are for steel piles with yield stress equal to 250 N/mm^2. For given set of other parameters, the axial capacity of initially bent pile decreases with increasing relative stiffness factor T, that is, when the surrounding clay soil is soft with low horizontal soil modulus, the pile becomes relatively rigid in comparison to soil losing its interactive ability with the soil, that is, the pile does not feel the surrounding soil, and hence resulting in low strength of pile.

Increasing Z_{max} and keeping other parameters constant, upto optimal T/K values, results in increased strength of pile. This is because, for moderate piles, in which the induced deflections and hence the moments are smaller, when the pile becomes relatively slender due to greater length, that is larger Z_{max}, the induced deflections are relatively larger resulting in larger neutralising moments from the soil reactions thus increasing the pile capacity. However, beyond optimal T/k values increasing Z_{max} has the effect of reducing pile capacity. This is due to the fact that with large T/k values the pile is already slender with large induced deflections and consequent moments and increasing the length of such a pile would mean further increasing the induced deflections and moments and these are so large that the neutralising moments from soil reactions become small in comparison, thus resulting in in-effective soil-pile interaction giving reduced strength of pile.

Strength of fully shaft bearing piles (α = 1) is observed to be larger than correspondingly bent fully toe bearing piles (α = 0). Similar behaviour of bent piles (dog-leg piles) was reported by Johnson (1962).

For optimal design of initially bent piles with Z_{max} values about 5.0, it is recommended that $50 \leqslant T/k \leqslant 60$. For piles with Z_{max} values in the range of 10.0 to 15.0 it is recommended that $35 \leqslant T/k \leqslant 45$. These recommendations result, in the following two design formulae:

a) - For Z_{max} about 5.0, use the following equation:

$$\sqrt[4]{E/K_h} = (50 \text{ to } 60)\sqrt{k/\sqrt{A}} \tag{12}$$

b) - For Z_{max} in the range of 10.0 to 15.0, use the following equation:

$$\sqrt[4]{E/K_h} = (35 \text{ to } 45)\sqrt{k/\sqrt{A}} \tag{13}$$

For optimal design, using Eqs. 12 and 13, $\sqrt[4]{E/K_h}$ being constant for a given soil and pile material, pile section is chosen such that its k and A values satisfy the above equations.

Example

In order to show the application of aforementioned design criteria, an example in sloved. The results will be compared with the solution of ideal straight pile.

Column footing of a high-rise building carrying 3000 KN is to be supported by steel columns driven to firm rock through a 10 m bed of soft clay with $K_h = 0.8\ N/mm^2$. Design a suitable section of H-pile and obtain the required number of piles.

Referring to structural steel tables (Eisenhuttenleute 1969), choose IPBV 100 (120mm x 106mm) with the following characteristics:

$A = 5320\ mm^2, K_y = 27.4\ mm, W_y = 75000\ mm^3, I_y = 3990000\ mm^4$

Left hand side of Eq. 13 with $E = 200000\ N/mm^2$ for steel is:

$\sqrt[4]{E/K_h} = \sqrt[4]{200000/0.8} = 22.361$

Right hand side of Eq. 13 becomes:

$(35 \text{ to } 45)\sqrt{k/\sqrt{A}} = (35 \text{ to } 45)\sqrt{27.4/\sqrt{5320}}$

$= 21.45 \text{ to } 27.59$ O.K.

$T = \sqrt[4]{EI/K_h} = \sqrt[4]{2x10^5 x399x10^4/0.8} = 999.4mm$

$= 1000.0\ mm \quad T/k = 999.4/27.4 = 36.47$

$Z_{max} = l/T = 10000/999.4 = 10.006 = 10.0$

Assume $\alpha = 0.0$

$r = W_y/A = 75000/5320 = 14.1\ mm$

Interpolating from Figs. 7 and 8 gives:

$\sigma_0/250 = 0.826$

$\sigma_0 = 206.5\ N/mm^2$

Ultimate capacity of each pile 206.5 x 5320/1000 = 1098.6 KN with factor of safety 2, safe capacity of each pile = 549.3 KN

Number of piles required = 3000 / 549.3 = 5.46

Use 6 piles

Compare with the ideal straight pile formula (Whitaker 1976):

$$P_{cr} = (\pi^2 EI/l^2)(m^2 + \lambda/m^2) \tag{14}$$

Where $m = \sqrt[4]{\lambda}$ rounded off to the next higher number and $\lambda = K_h l^4 / \pi^4 EI$

$= 0.8(10000)^4/(399x10^4 x2x10^5 x \pi^4) = 102.92$

$m = \sqrt[4]{102.92} = 3.185$ rounded off to 4.

$P_{cr} = ((\pi^2 x2x10^5 x399x10^4)/(10^4 \text{ x } 10^4 x1000))(16 + 102.92/16)$

$P_{cr} = 1732.61$ KN as against 1098.6 KN from the present study.

Summary and Conclusions

Based on non-dimensional differential equation of the pile and numerical solution of this equation, the behaviour of initially curved pile is studied. Numerical results are shown in the charts of Figs. 3 to 14. The pile is assumed to develop initial out-of-straightness defects with consequent initial stresses and the bending moments from the lateral soil reactions are considered.

It is clear that the accuracy of the solution is increased by dividing the pile into larger number of parts and by increasing the maximum number of iterations in Gauss-Siedel procedure. However, most of the cases which are studied converge with nine internal pivotal points and ten iterations. The results are shown in the charts which show the variation of stress parameter with respect to slenderness parameter. These charts show that the pile capacity increases as slenderness increases to some extent till the bending moments from soil reactions help. Afterwards the capacity of pile decreases as the slenderness is further increased.

These charts also show that too rigid piles and also too flexible piles fail to derive benefit from the surrounding soil. In contrast, moderately rigid or flexible piles are more beneficial. The design charts along with the given formulae can be used to select pile cross section which has the optimal axial capacity.

Acknowledgements

The authors express their thanks to the authorities of Ferdowsi University of Mashhad, Iran, for supporting the course of this study and making available various facilities during this work. Useful comments of our colleagues, professors A. Haerian and F. Irani, are acknowledged.

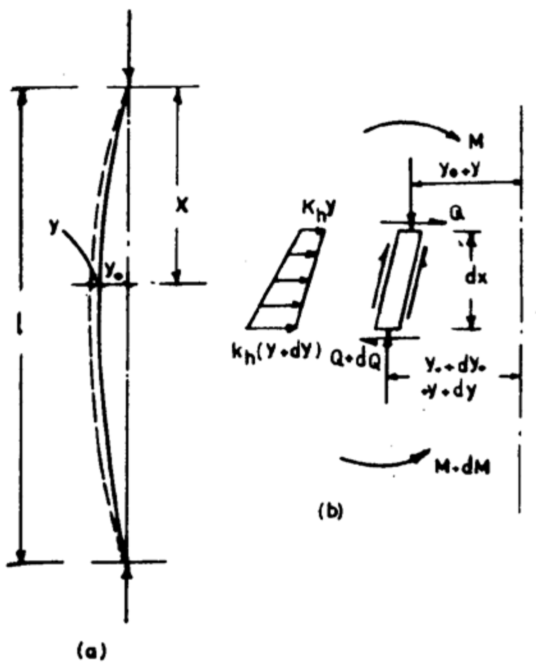


Fig. 1 - Bent pile subjected to axail load

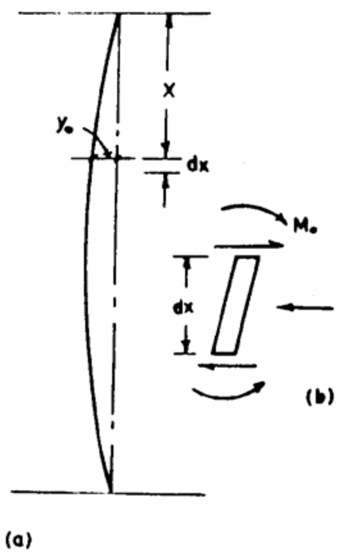
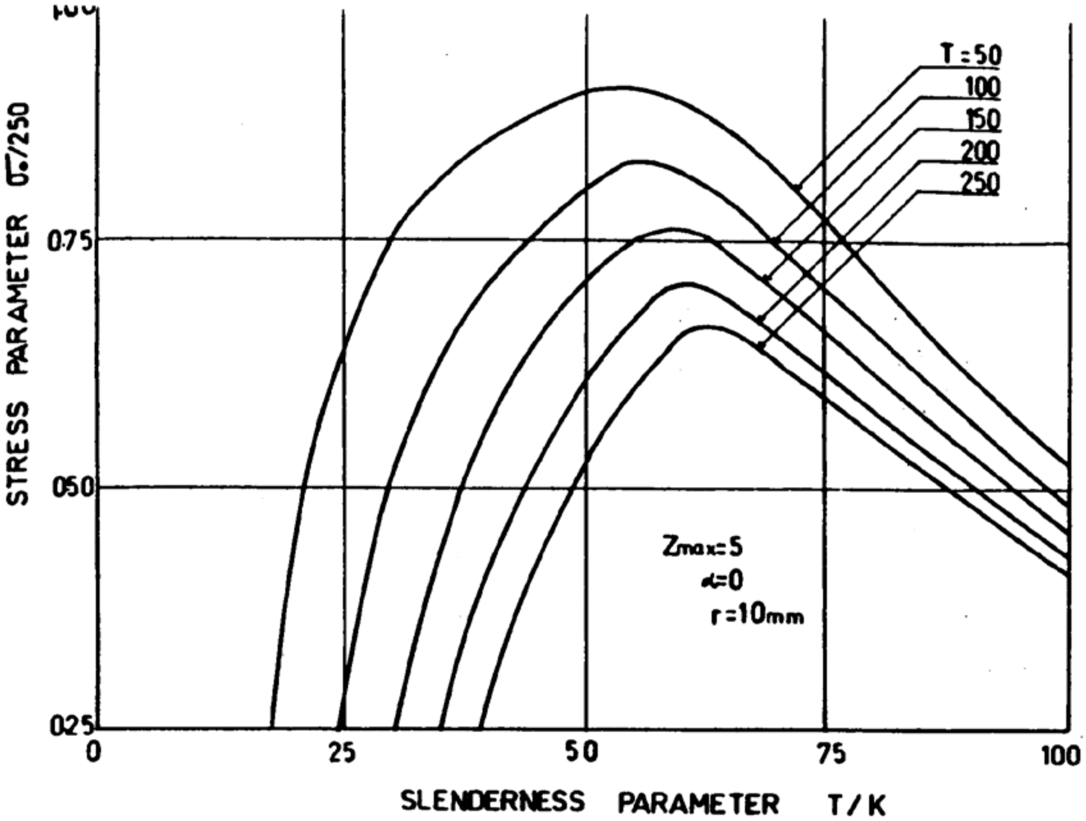


Fig. 2 - Initial stress in the pile



T=50
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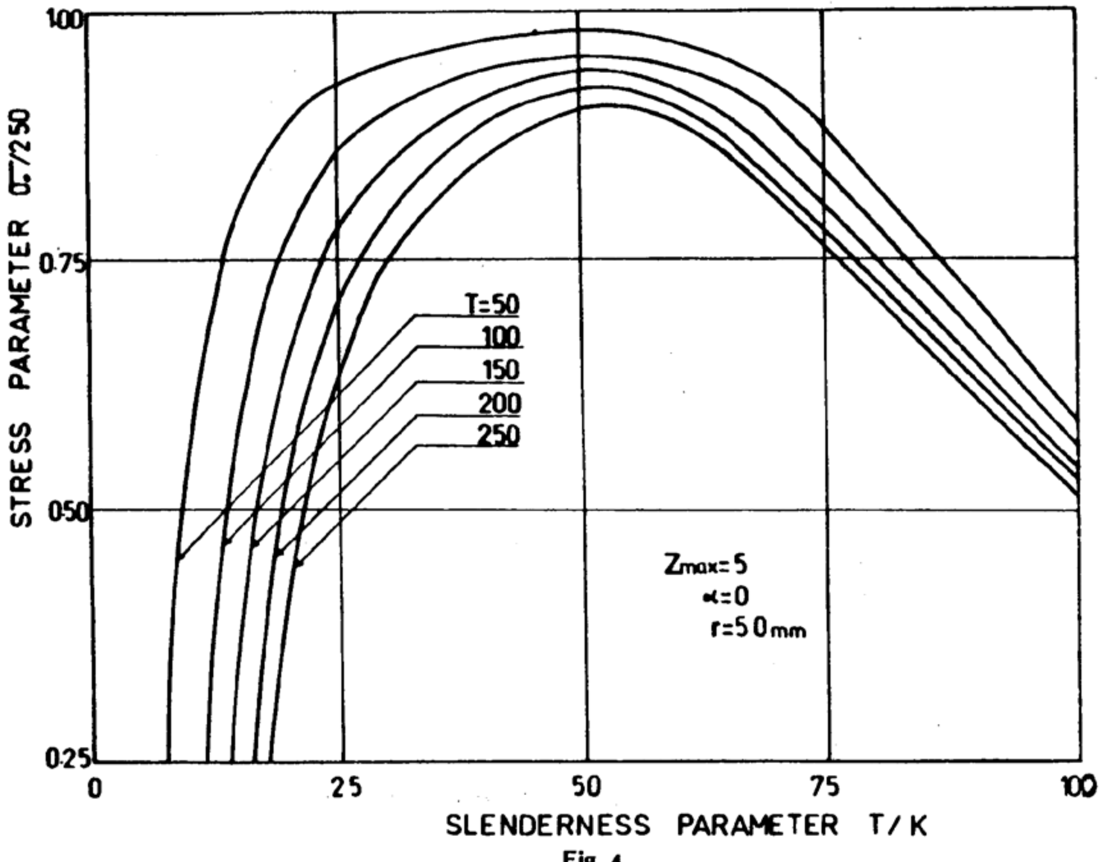
150
200
250
Zmax=5
α=0
r=10mm
STRESS PARAMETER σ/250
SLENDERNESS PARAMETER T/K

Fig. 3



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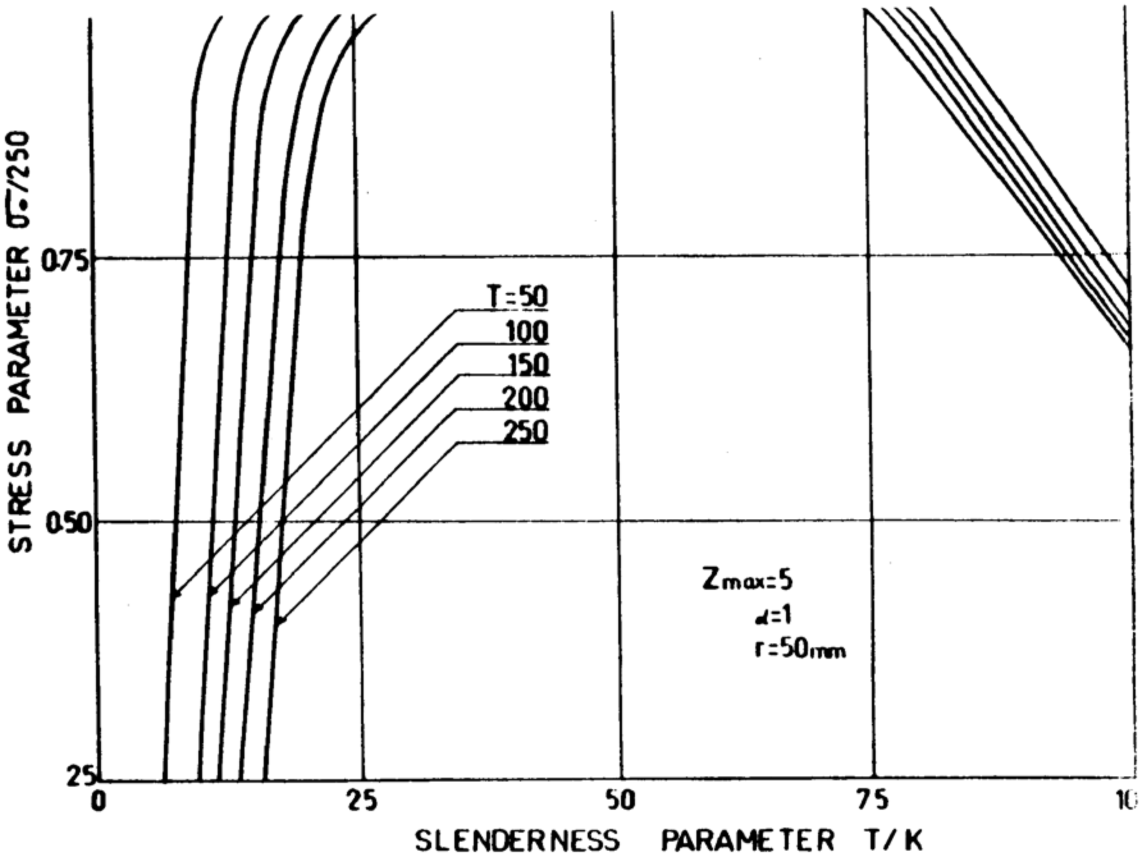
250
Zmax=5
α=0
r=50mm
STRESS PARAMETER σ/250
SLENDERNESS PARAMETER T/K

Fig. 4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0
25
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150
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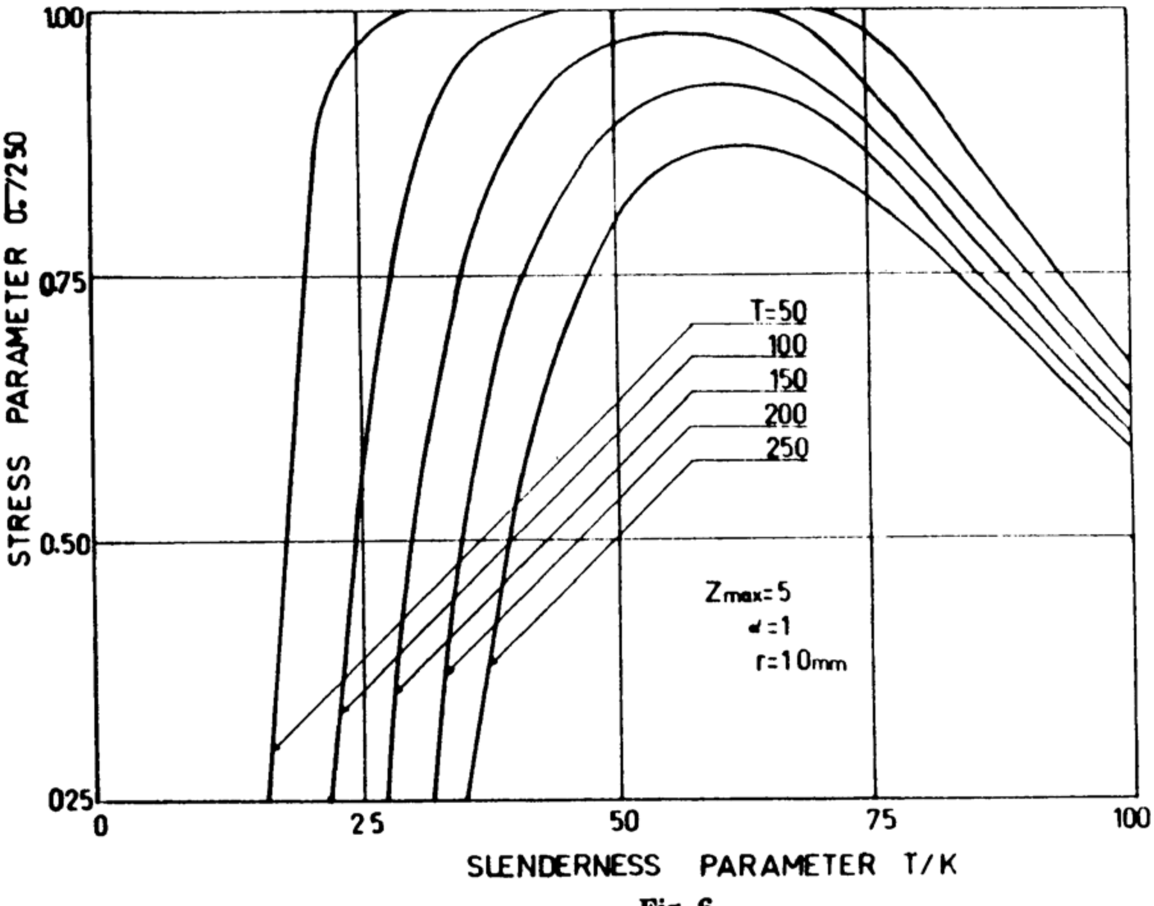
α=1
r=50mm
SLENDERNESS PARAMETER T/K
Fig. 5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0
25
50
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100
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100
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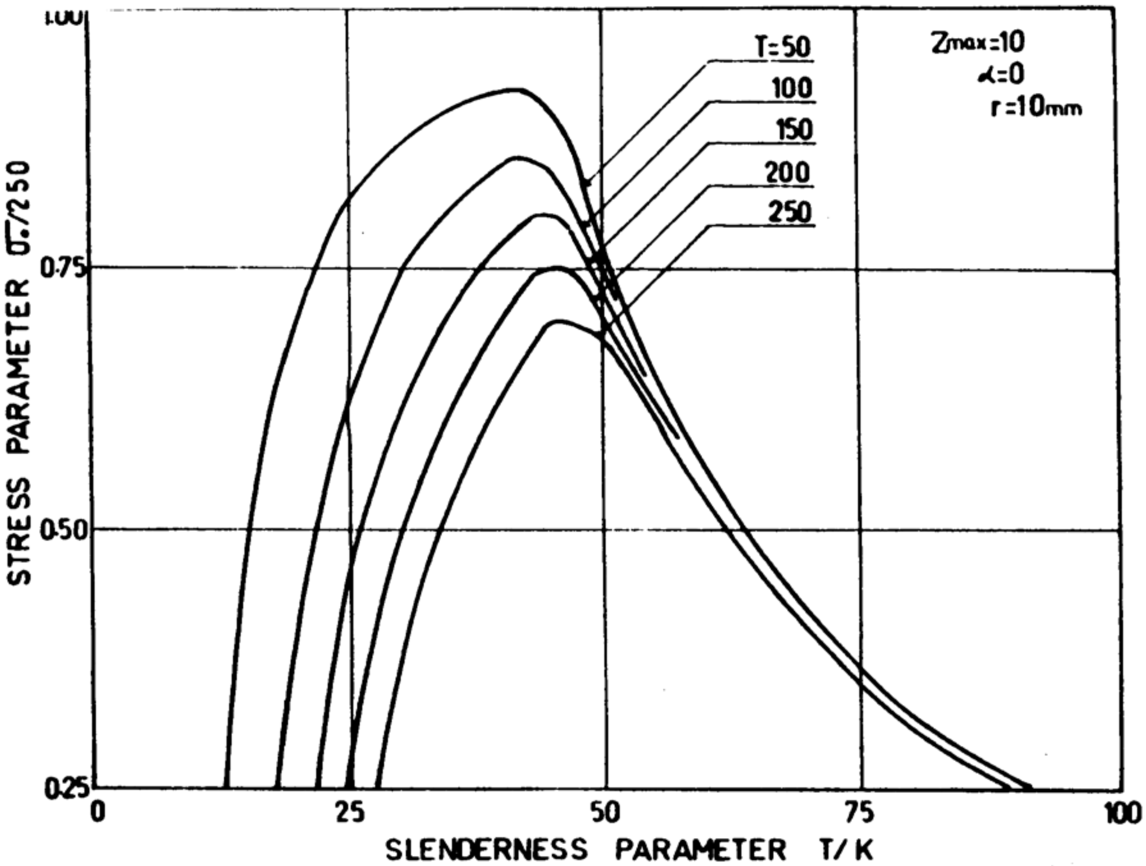
250
Zmax=5
α=1
r=10mm
SLENDERNESS PARAMETER T/K
Fig. 6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0.50
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0
25
50
75
100

Fig. 7

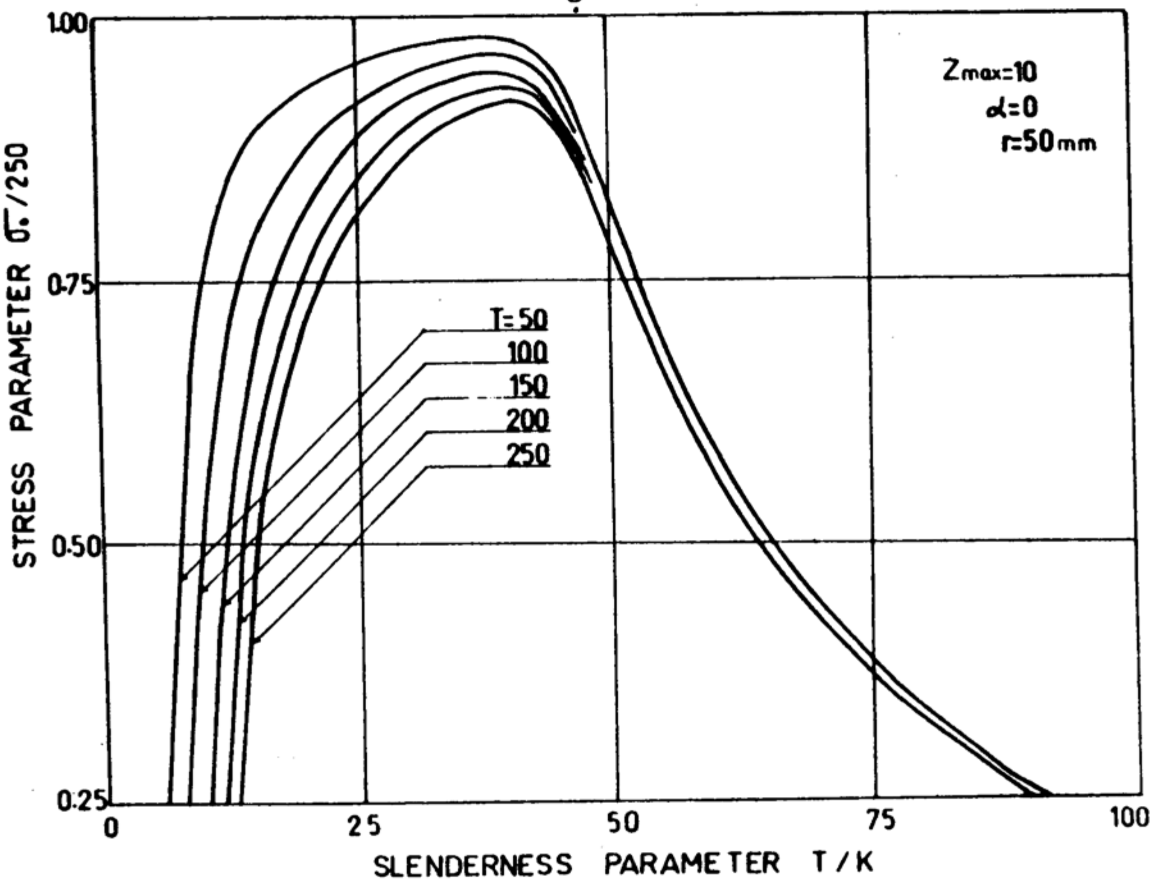
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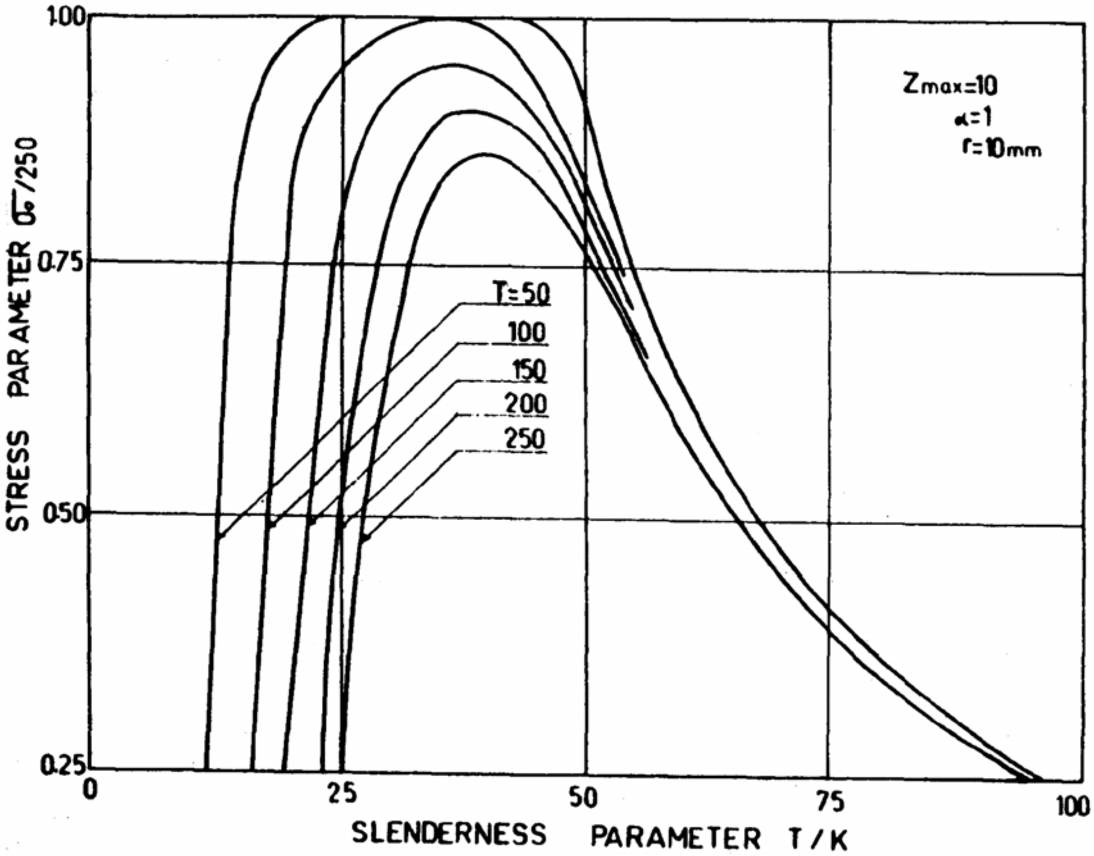
Fig. 8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r=10mm
T=50
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250

Fig. 9

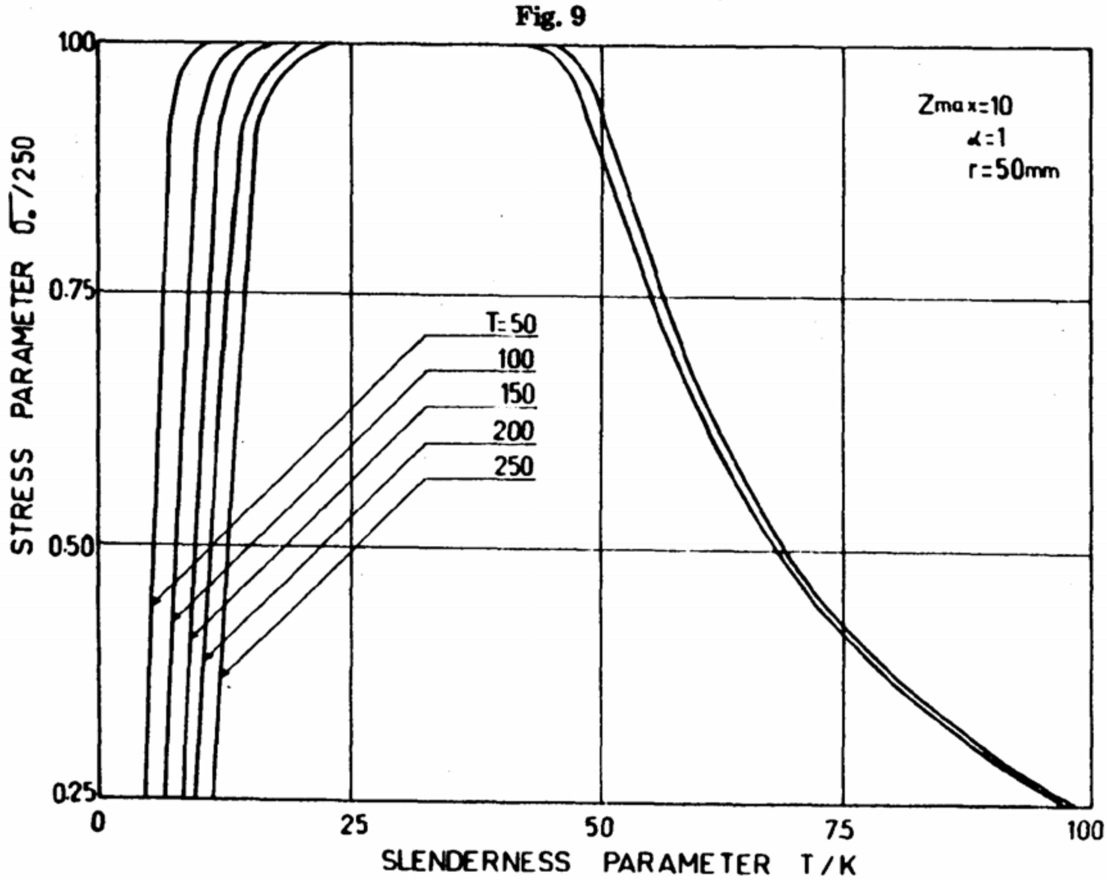
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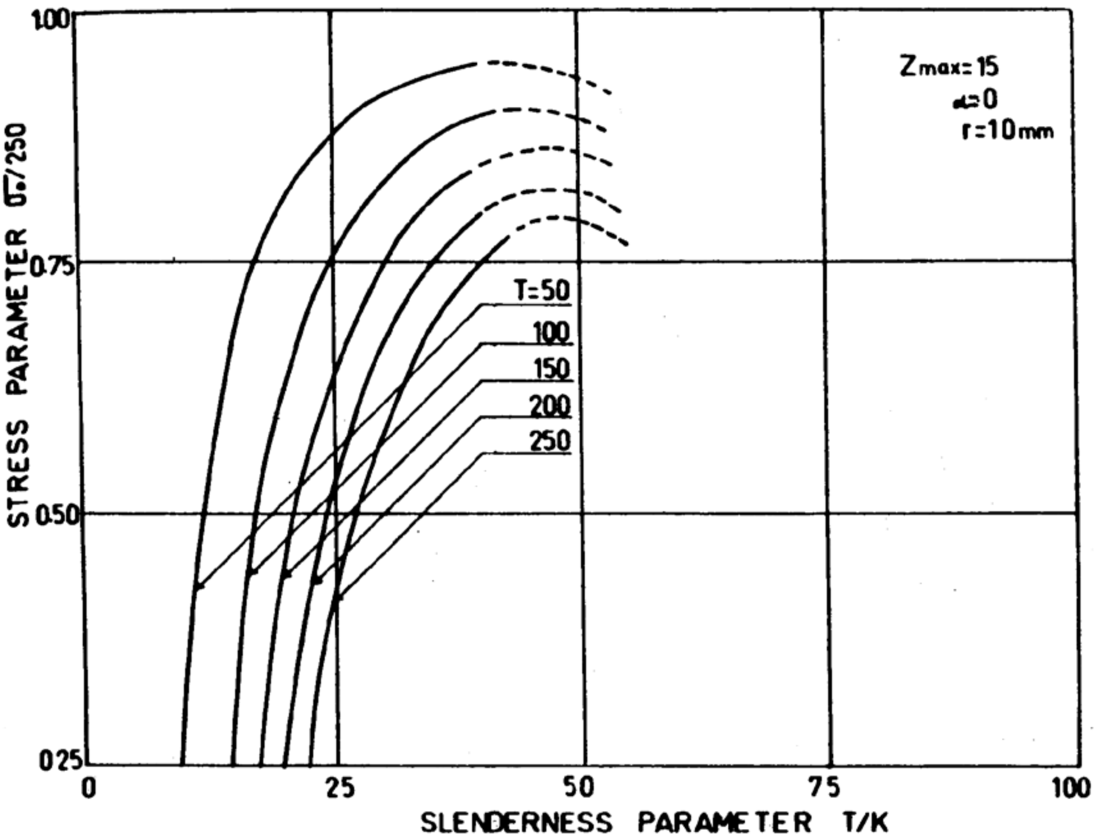
Fig. 10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SLENDERNESS PARAMETER T/K
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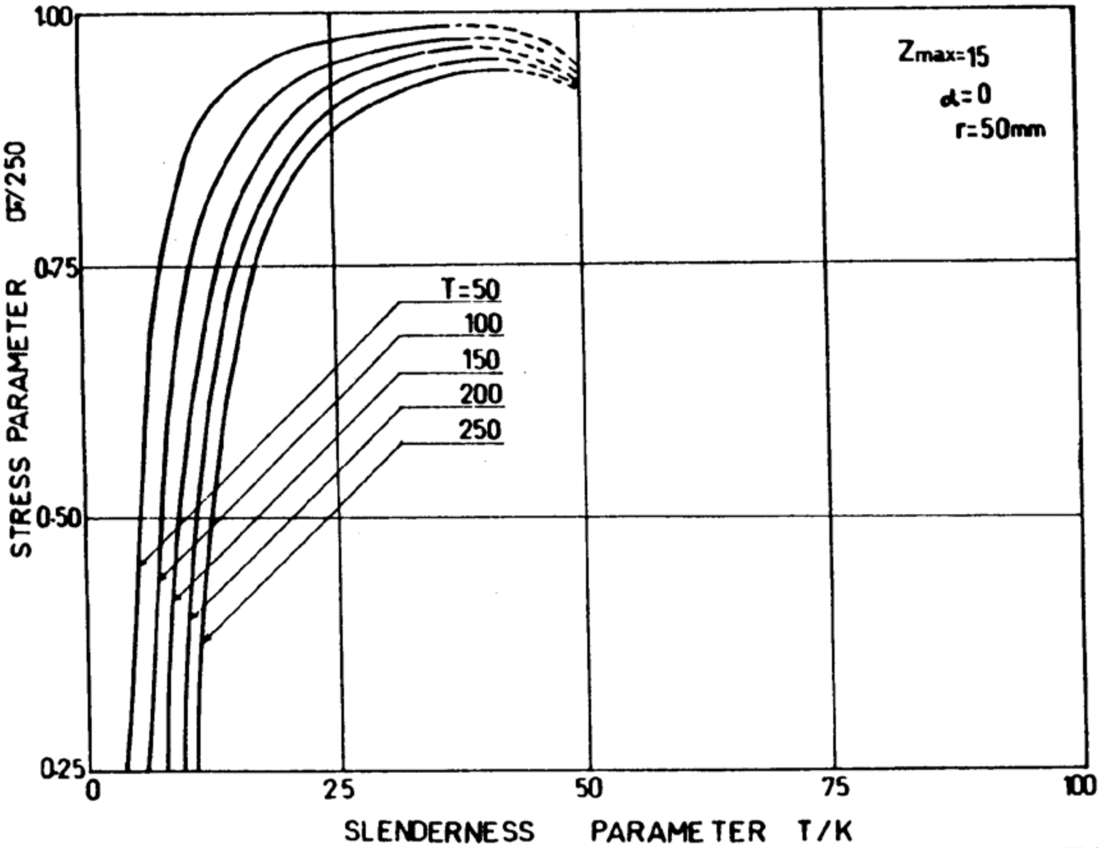
25
50
75
100

Fig. 11.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200
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STRESS PARAMETER σ/250
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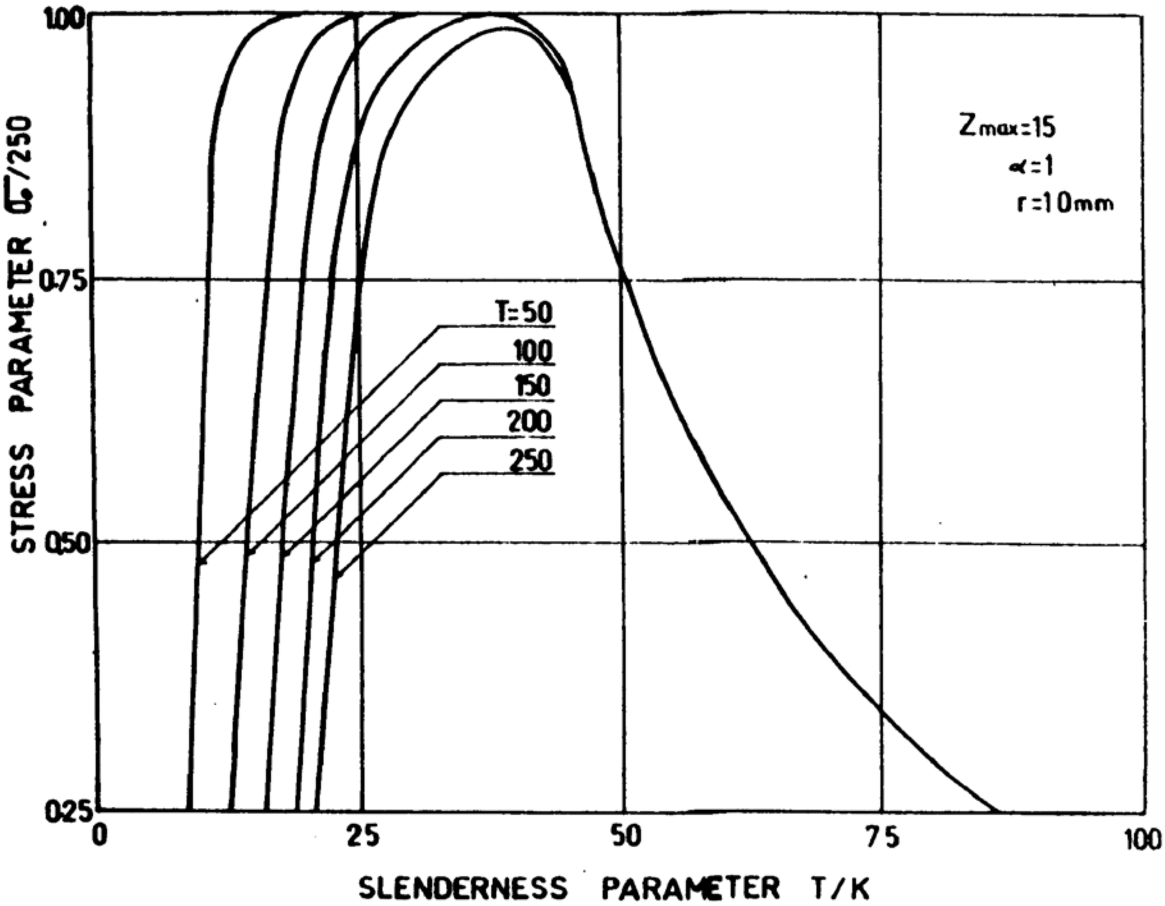
100

Fig. 12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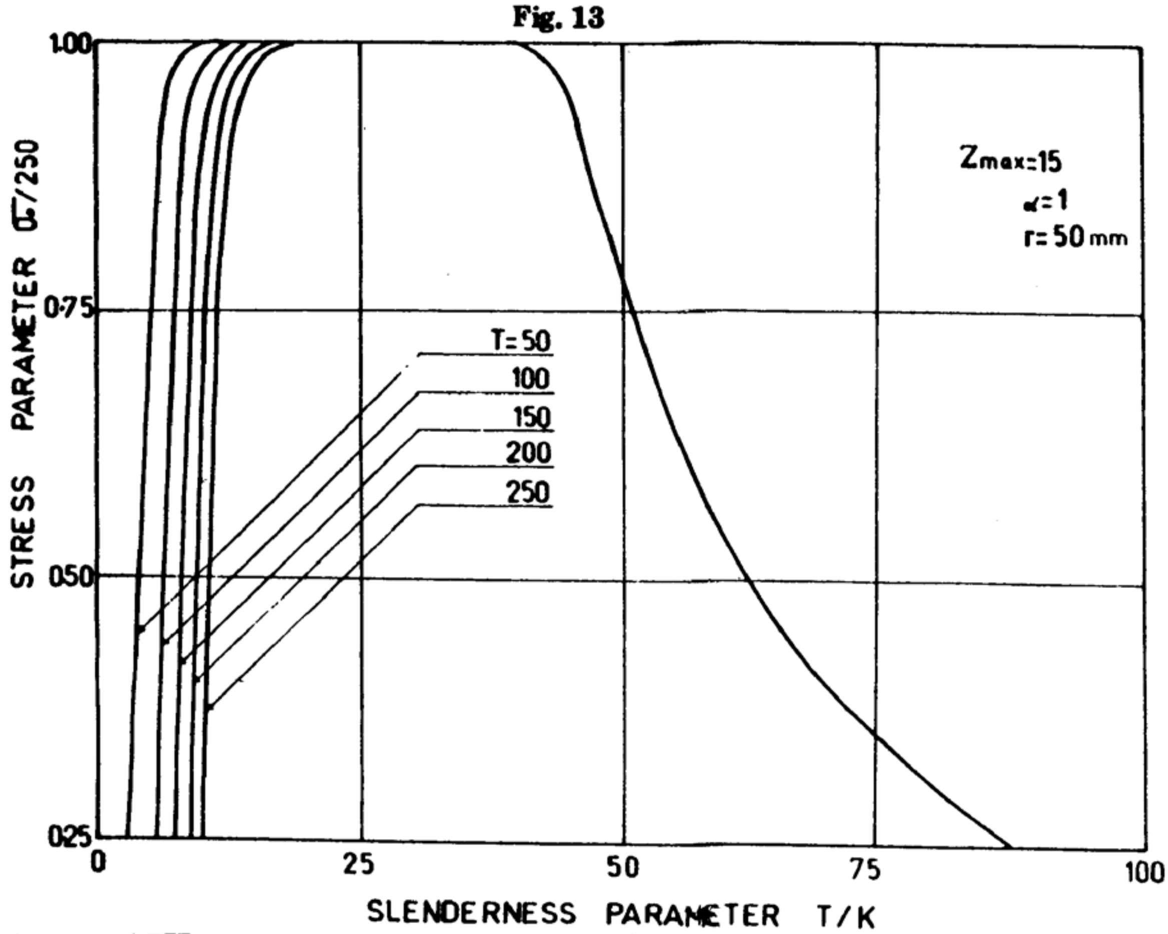
SLENDERNESS PARAMETER T/K

Fig. 13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150
200
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0
25
50
75
100
SLENDERNESS PARAMETER T/K

Fig. 14

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Notation

The following symbols are used in this paper:

a_0 = initial offset at mid height of bent pile,

A = area of pile section,

E_{steel} = modulus of elasticity of steel,

EI = flexural stiffness of pile at depth x,

$(EI)^{I}$, $(EI)^{II}$ = first and second derivatives of EI with respect to x,

I = moment of inertia of pile section at depth x,

I_y = moment of inertia about y-axis,

k = radius of gyration of pile section,

k_h = horizontal soil modulus,

k_y = radius of gyration about y-axis,

l = length of pile,

M = induced moment in the pile at depth x,

M_0 = initial moment in the pile at depth x,

P = axial load on pile at depth x,

P^{I} = first derivative of P with respect to x,

P_{cr} = buckling load of straight pile,

P_0 = axial load on top of pile,

r = ratio of section modulus to area of section of pile,

T = relative stiffness factor of pile-soil system,

W_y = section modulus about y-axis,

x = depth to any point along pile,

Y = induced deflection in the pile under axial load P at depth x,

Y^{I}, Y^{II}, Y^{III}, Y^{IV} = first, second, third and fourth derivatives of Y with respect to x,

Y_0 = initial deflection of bent pile at depth x,

Y_o^{I}, Y_o^{II} = first and second derivatives of Y_0 with