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Robust and adaptive control with application to mechanical systems

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***Abstract* - In this paper a robot with n degrees of freedom is considered. The dynamical equations can be obtained by the Newton-Euler or the Lagrange method. Many control algorithms exist in the literature. There are robust control algorithms and adaptive control algorithms. In this paper a combination of robust control and adaptive control is proposed. This method gives a better performance than only robust and only adaptive methods. Two robust control methods are studied in this paper: variable structure control and H infinity control. A real time identification is used and the controller is adapted in real time. We have Lyapunov stability or input-output stability under some assumptions. The method can be applied to robotics. The method can also be applied to many other systems for example electrical motors and machine tools. Simulation results are given. This method can be used in automatic production lines.**

***Keywords* - Robust control, adaptive control, Lyapunov stability, H infinity, robotics**

I. INTRODUCTION

In this paper a robot with n degrees of freedom is considered. The dynamical equations are given by n coupled nonlinear differential equations. Robust control and adaptive control have been subject of a lot of research in the past three decades [2], [3], [4], [5], [12], [17], [18], [19], [20], [21]. Robust control and adaptive control can be used for different systems. A combination of robust control and adaptive control is proposed in this paper. This method gives a better performance than only robust and only adaptive methods.
Two control methods are studied in this paper: variable structure method and feedback linearization.
In variable structure control a Lyapunov like function is used. The state reaches the sliding surface and slide on it. The sliding mode is invariant with respect to parameter variations.
In feedback linearization the system is globally linearized and the effect of the model plant mismatch can be considered as a nonlinear perturbation. The stability of the closed loop system can be studied by the input-output stability method and the small gain theorem.
In robust control time variations are compensated in some limits but if the parameter changes are larger than some limits the performance is deteriorated and the closed loop system can become unstable. A real time identification can be used to adapt the controller.

II. DYNAMICS OF THE SYSTEM

A robot with n degrees of freedom is considered. See figure 1 .

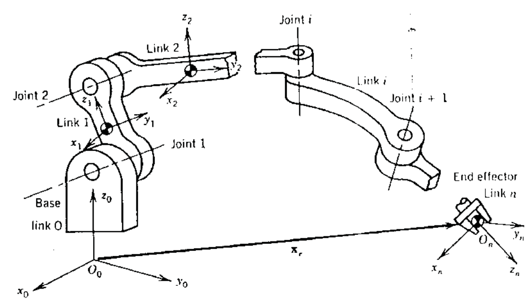

Figure 1 A robot with n degrees of freedom

The Newton-Euler method or the Lagrange method can be used to obtain the dynamical model of the system. The dynamical equations of the system are given by :

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau \tag{1}$$

Where

$q = \begin{pmatrix} q_1 \\ \vdots \\ q_n \end{pmatrix}$ is the vector of joint angles,

$\tau = \begin{pmatrix} \tau_1 \\ \vdots \\ \tau_n \end{pmatrix}$ is the vector of joint torques,

$M(q)$ is the inertia matrix,
$C(q,\dot{q})$ is the vector of centrifuge and Coriolis torques and
$G(q)$ is the vector of gravitation .

The method can be used for robots with revolute joints and with prismatic joints. In this case q_i is the angle for the revolute joint and the distance for the prismatic joint. An example of such a robot is given in the figure 2 .

Many control algorithms exist in the literature for the control of robot manipulators [1], [2], [3], [4], [5], [6], [7], [9], [22], [23] . Two control methods have been studied in this paper : feedback linearization and variable structure control .

III. FEEDBACK LINEARIZATION

First we consider the ideal case where there is no model plant mismatch. In that case we have:

$$\hat{M} = M \qquad (2)$$

$$\hat{C} = C \qquad (3)$$

$$\hat{G} = G \qquad (4)$$

The differential equations given by (1) are coupled and non linear. Let

$$\hat{M}(q).v + \hat{C}(q,\dot{q}) + \hat{G}(q) = \tau \qquad (5)$$

By putting (5) in (1) we have:

$$M(q).\ddot{q} + C(q,\dot{q}) + G(q) = \hat{M}(q).v + \hat{C}(q,\dot{q}) + \hat{G}(q)$$

After some simplifications we have

$$\ddot{q} = v \qquad (6)$$

These equations are decoupled and linear.

Now we can use different linear methods for the design of the controller. For example: optimal control, pole placement, root locus, design in the frequency domain etc… . As an example we can use a PID controller. Suppose that $q_d(t)$ is the desired trajectory. Let

$$e(t) = q_d(t) - q(t) \qquad (7)$$

where $e(t)$ is the error vector.

If we use a PID controller we have:

$$v = K_P.e(t) + K_D.\frac{de}{dt} + K_I.\int_0^t e(\tau).d\tau) \qquad (8)$$

Where

K_P , K_D and K_I are the gain matrixes and are diagonal. By putting (8) in (5) we obtain the torque vector τ .

Robust control

The dynamical equations of the system are coupled and non linear. If we have a good model the system can be globally linearized. In this case the H infinity methods can be used. The design of the controller can be done in the frequency domain. In this case the controller is chosen so that the loop gain is high at low frequencies and the loop gain is low at high frequencies and we have a good stability margin near the Nyquist frequency.

In practice we never have a perfect model. The effect of the model-plant mismatch can be considered as a non linear perturbation. For measuring how big the perturbation is the L_2 and L_∞ norms can be used. The stability of the closed loop system can be studied by the input-output methods and the small gain theorem. A condition for the stability of the closed loop system and a bound on the norm of the error can be obtained [2], [3], [4] .

Real time identification

Suppose that we have a dynamic system with some inputs and outputs. Identification means to estimate the unknown parameters of the dynamic equations of the system by using the input-output data. In [15] many methods have been developed for the identification of dynamic systems in discrete time. Here we consider the identification of dynamic systems in continuous time. The differential equations of the robot are given by (1). These differential equations are nonlinear. These equations are linear in parameters. The parameters can be estimated by using the input-output data.

The differential equations (1) can be written as:

$$\tau = W(q,\dot{q},\ddot{q}).\varphi \qquad (9)$$

where φ is the vector of unknown parameters,
τ is the vector of the torques and
$W(q,\dot{q},\ddot{q})$ is a known nonlinear function.

Let:

$$\hat{\tau} = W.\hat{\varphi} \qquad (10)$$

Where $\hat{\tau}$ is the vector of estimated torques and
$\hat{\varphi}$ is the vector of estimated parameters.

Let:

$$\tilde{\tau} = \tau - \hat{\tau} \qquad (11)$$

The least squares real time identification is given by:

$$\dot{\hat{\varphi}} = p.W.\tilde{\tau} \qquad (12)$$

$$\frac{d(p^{-1})}{dt} = W^T W \qquad (13)$$

or

$$\dot{p} = -pW^T Wp \qquad (14)$$

Adaptive feedback linearization

When we use a feedback linearization we need a model for the system. The model can be estimated in real time as described in the previous section. The estimated model can be used in (5) . The block diagram of the adaptive feedback linearization is shown in the figure 3 .

Simulations

Here some simulations are given. As an example we consider a robot with two degrees of freedom.

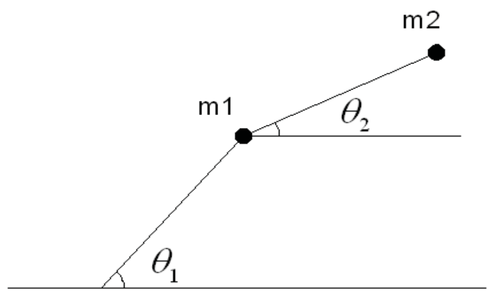

Figure 4 , A serial robot with two degrees of freedom

The dynamic equations of the robot are given by:

$$\underbrace{\begin{bmatrix} (m_1+m_2)a_1^2 + m_2 a_2^2 + 2m_2 a_1 a_2 \cos\theta_2 & m_2 a_2^2 + m_2 a_1 a_2 \cos\theta_2 \\ m_2 a_2^2 + m_2 a_1 a_2 \cos\theta_2 & m_2 a_2^2 \end{bmatrix}}_{\text{The inertia matrix}} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$

$$+ \underbrace{\begin{bmatrix} -m_2 a_1 a_2 (2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2)\sin\theta_2 \\ m_2 a_1 a_2 \dot{\theta}_1^2 \sin\theta_2 \end{bmatrix}}_{\text{Coriolis and centrifuge}} + \underbrace{\begin{bmatrix} (m_1+m_2) g a_1 \cos\theta_2 + m_2 g a_2 \cos(\theta_1+\theta_2) \\ m_2 g a_2 \cos(\theta_1+\theta_2) \end{bmatrix}}_{\text{gravitation}}$$

$$= \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{15}$$

where θ_1 , θ_2 are the joint angles τ_1, τ_2 are the torques and m1, m2 are the joint masses and a1, a2 are the length of the robot arms.

Robust and adaptive control

We use a robust and adaptive feedback linearization. At the beginning the estimations are not good and the errors are big. With the passage of time the estimations become good and the errors go to zero.

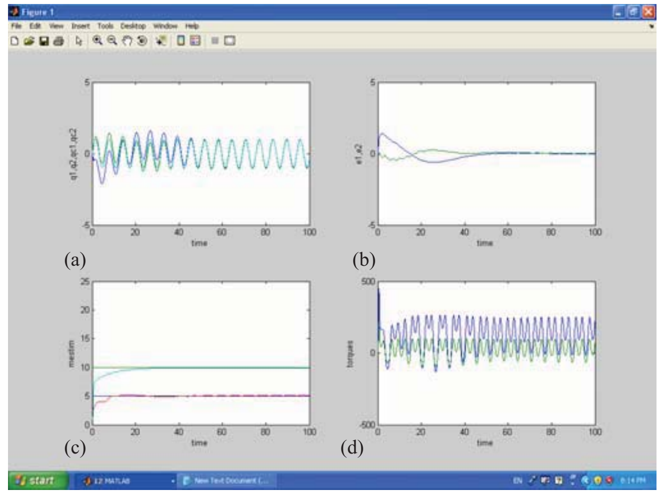

Figure 5, (a) joint angles and reference trajectory, (b) errors, (c) parameter estimates, (d) torques

Robust but not adaptive control

We use a feedback linearization with a PID controller. The simulation results are given in figure 6 . We suppose that the load of the robot changes. The mass m2 changes at t=100 and t=200 but the estimate of m2 doesn't change. We see that at the beginning the trajectory following is good. At t=100 and t=200 the model plant mismatch increases and the performance deteriorates.

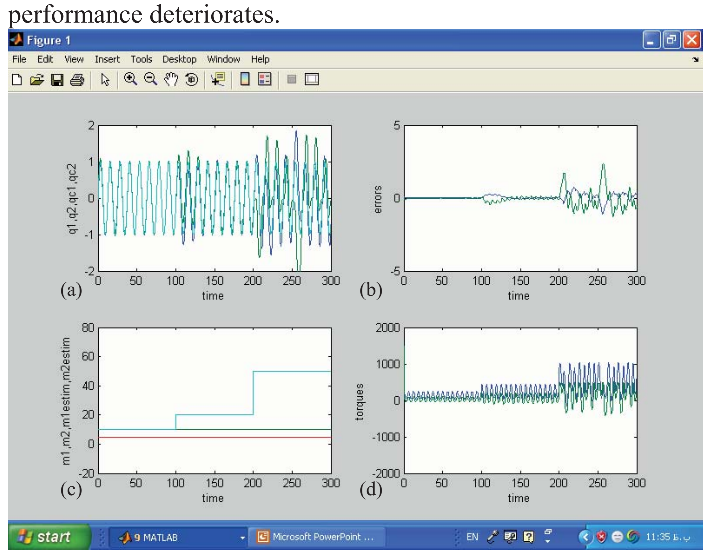

Figure 6, (a) joint angles and reference trajectory (b) errors (c) parameter estimates (d) torques

Robust and adaptive control

In this case the parameters are identified in real time and the controller is adapted by using the parameter estimates.

Simulation results are shown in figure 7 . As it is seen the model-plant mismatch is not big and the control performance is good.

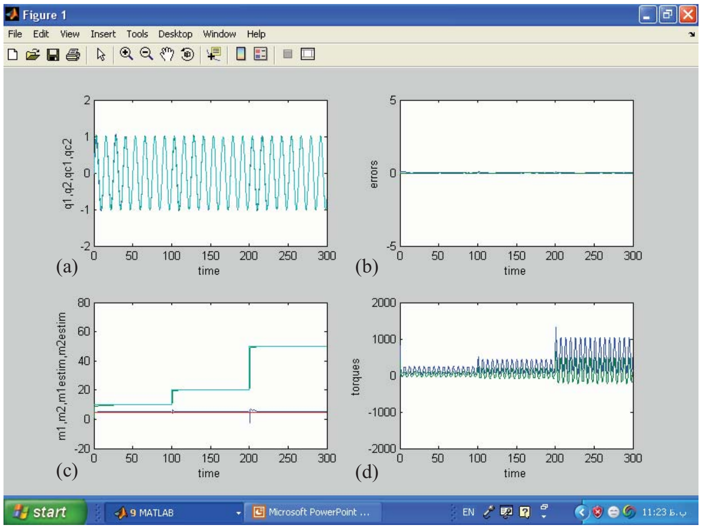

Figure 7, (a) joint angles and reference trajectory (b) errors (c) parameter estimates (d) torques

IV. VARIABLE STRUCTURE CONTROL

Variable structure control is a non linear control method. This method is a kind of high gain feedback and is very robust with respect to parameter uncertainty and non linearities [5], [6], [7], [8], [9], [10], [11] . The sliding mode is invariant with respect to parameter changes. One of the disadvantages of this method is the chattering witch can excite the neglected fast states. By using a boundary layer the problem of chattering can be improved.

Suppose that the dynamic equations of the robot is given by (1) . Let:

$$r = \Lambda . e + \dot{e} \qquad (16)$$

Where

$\Lambda = diag[\lambda_1 \quad \cdots \quad \lambda_n]$ and $\lambda_i > 0$

Consider the Lyapunov function candidate:

$$V(r) = \frac{1}{2} r^T M(q) r \qquad (17)$$

Then we have:

$$\dot{V}(r) = r^T (M\ddot{q}_r + C_m \dot{q}_r + G - \tau) \qquad (18)$$

Where:

$$\dot{q}_r = \Lambda . e + \dot{q}_d \qquad (19)$$

If the control is chosen as:

$$\tau = \hat{M}\ddot{q}_r + \hat{C}_m \dot{q}_r + \hat{G} + K.\text{sgn}(r) \qquad (20)$$

where

$K = diag[k_1 \quad \cdots \quad k_n]$, $k_i > 0$ and

$$\text{sgn}(r) = [\text{sgn}(r_1) \quad \cdots \quad \text{sgn}(r_n)]^T$$

If k_i is enough large the derivative of the Lyapunov function is negative. If we have

$$k_i \geq \left| [\tilde{M}\ddot{q}_r + \tilde{C}_m \dot{q}_r + \tilde{G}]_i \right| + \eta_i \qquad (21)$$

With $\eta_i > 0$,

$\tilde{M} = M - \hat{M}$, $\tilde{C}_m = C_m - \hat{C}_m$ and $\tilde{G} = G - \hat{G}$

then

$$\dot{V}(r) \leq -\sum_{i=1}^{n} \eta_i . |r_i| < 0 \qquad (22)$$

So r=0 is reached in finite time and the sliding phase begins. Once in the sliding mode the error e(t) converges exponentially to zero.

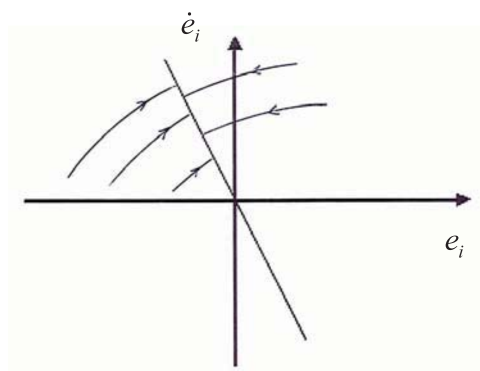

Figure 8 , Arrival phase

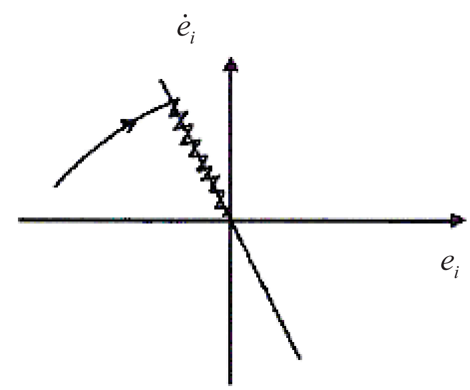

Figure 9 , Chattering during the sliding phase

Adaptive variable structure control

The model can be estimated in real time as described before. The estimated model can be used to evaluate the torques in (20).

Simulation, Robust and adaptive variable structure control

The robust and adaptive variable structure control described before is applied to the robot with two degrees of freedom. The simulation results are given in figure 10 .

The mass m2 changes at t=100 and t-200. We have a real time identification and the masses are identified and the controller is adapted in real time.

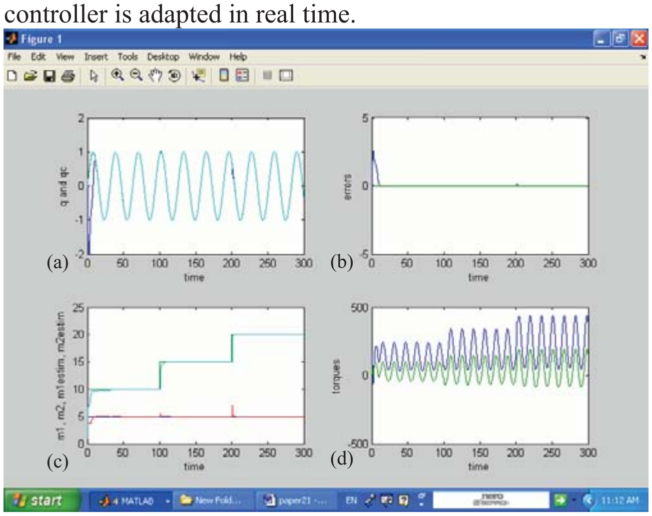

Figure 10 , (a) joint angles and reference trajectory (b) errors (c) parameter estimates (d) torques

V. CONCLUSION

The dynamical equations of a robot with n degrees of freedom can be obtained by using the Newton-Euler method or the Lagarange method. Different control methods exist in the literature. A robust and adaptive control has been proposed. This method gives a better performance than only robust and only adaptive methods. Two robust control methods have been considered. Variable structure control and H infinity control. A real time identification is used and the controller is adapted in real time. We have the Lyapunov stability or the input-output stability under some assumptions. The method is applied to robotics. The method can be applied to many other systems for example the control of machine tools and electrical motors. Simulation results are given. This method can be used in automatic production lines.

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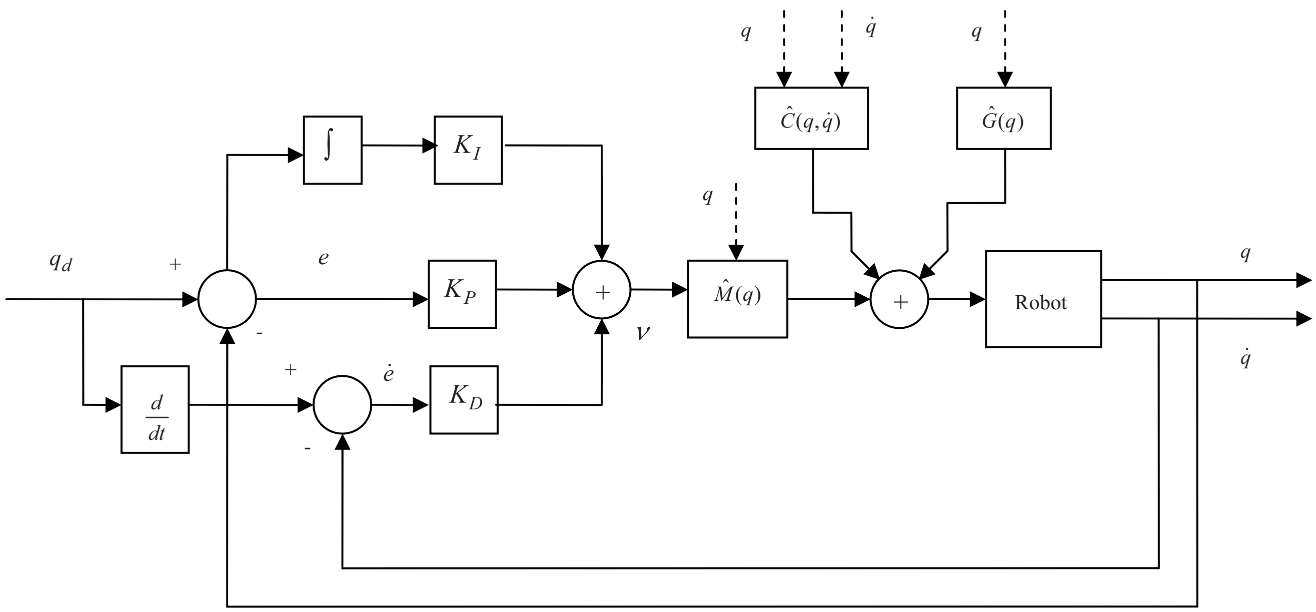

Figure 3 Adaptive feedback linearization

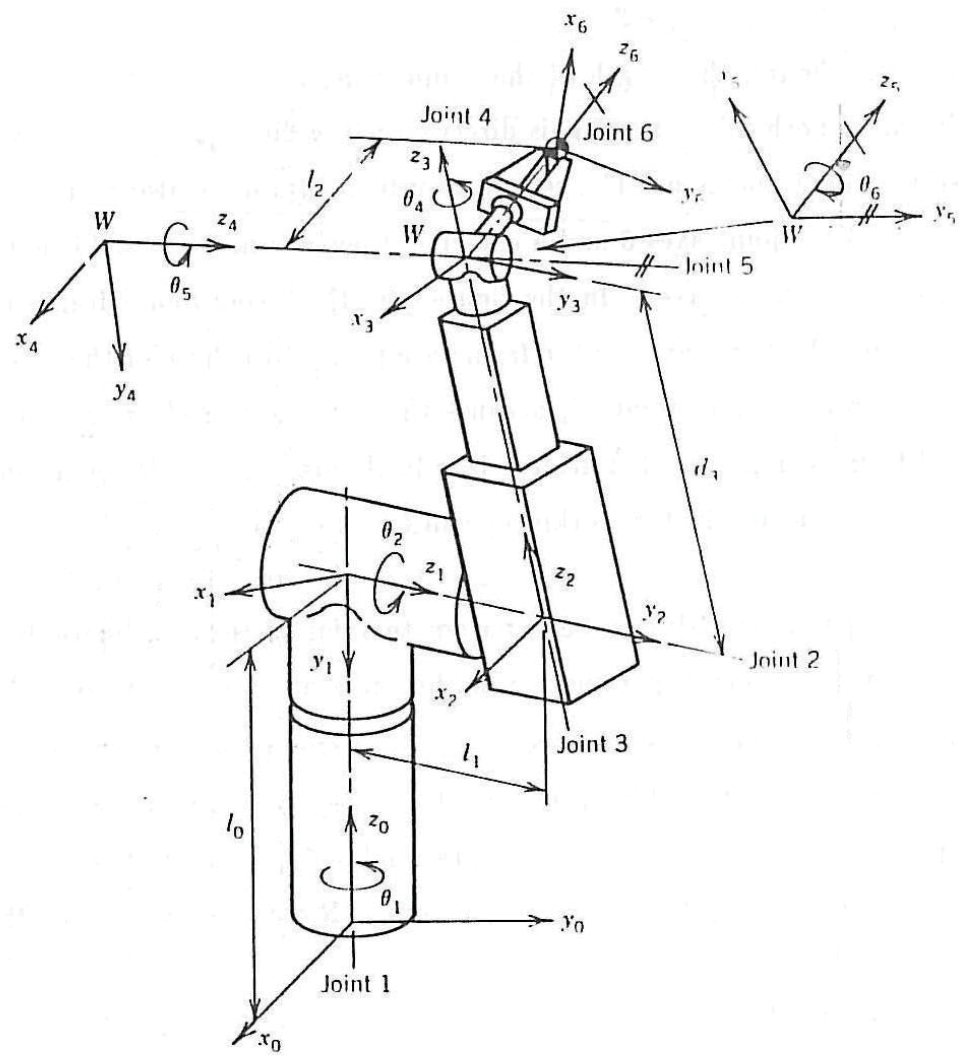

Figure 2 : A robot with six degrees of freedom