Elasto-Plastic Analysis of Material with Uniaxial Symmetric Tresca Yield Condition and Cyclic Loading

by

Morteza A. M. Torkamani[1] M. ASCE. and Mohammad R. Pajand[2]

1. INTRODUCTION

In this study the Uniaxial Symmetric Tresca yield surface has been selected for the yield condition. The kinematic hardening rule, which is suitable for loading and unloading and considers Bauschinger's effect, is chosen for establishing the conditions for subsequent yield from a plastic state. The associated flow rule for the Uniaxial Symmetric Tresca yield condition, and the kinematic hardening for linear hardening material, have been employed to relate the plastic strain increments to the stresses and stress increments.

2. YIELD CONDITION

The equation of the uniaxial symmetric Tresca yield condition, in terms of principal stresses, can be obtained by writing equations for all six sides of the hexagon as:

$$
\begin{aligned}
F^1 &= \sigma_1 - \sigma_o = 0 \\
F^2 &= -\sigma_2 - a\sigma_o = 0 \\
F^3 &= a\sigma_1 - \sigma_2 - a\sigma_o = 0 \\
F^4 &= \sigma_2 - \sigma_o = 0 \\
F^5 &= -\sigma_1 - a\sigma_o = 0 \\
F^6 &= a\sigma_2 - \sigma_1 - a\sigma_o = 0
\end{aligned}
\tag{1}
$$

[1]Asst. Prof. of Civ. Engrg., Univ. of Pittsburgh, Pittsburgh, Pa. 15261.

[2]Grad. Student of Civ. Engrg., Univ. of Pittsburgh, Pittsburgh.

By invoking the relationship between principal stresses and cartesian stresses these yield functions may be expressed in the cartition coordinate system.

3. ELASTO-PLASTIC MATRIX FOR SIDES

The yield function for side i of the Uniaxial Symmetric Tresca yield surface is expressed by:

$$F^i[\{\sigma - \alpha\}] = 0 \tag{2}$$

In this equation $\{\sigma\}$ is the state of stress, and $\{\alpha\}$ represents the position of the yield surface origin with respect to the origin of the initial stress space. Reference [2] presents the derivation of the elasto-plastic matrix for sides. The final result is:

$$\{\delta\sigma\} = [[D]_e - [D]_e\{\frac{\delta F^i}{\delta\sigma}\}[M]_i^{-1}\{\frac{\delta F^i}{\delta\sigma}\}^T[D]_e]\{\delta\epsilon\} \tag{3}$$

where

$$[M]_i = H\{\frac{\delta F^i}{\delta\sigma}\}^T\{\frac{\delta F^i}{\delta\sigma}\} + \{\frac{\delta F^i}{\delta\sigma}\}^T[D]_e\{\frac{\delta F^i}{\delta\sigma}\} \tag{4}$$

The Prager-Ziegler rule is applied, with the consideration of the yield function for each side. The incremental form of the plastic strain-stress is expressed by:

$$\{\delta\epsilon^P\} = \frac{\{\frac{\delta F^i}{\delta\sigma}\}\{\frac{\delta F^i}{\delta\sigma}\}^T\delta\sigma\}}{H\ \{\frac{\delta F^i}{\delta\sigma}\}^T\{\frac{\delta F^i}{\delta\sigma}\}} \tag{5}$$

The above equation can be utilized to evaluate the hardening coefficient, H. This may be done by imposing the uniaxial stress-strain relationship upon equation 5.

The constitutive laws for sides 1-3 and derivation of the constitutive laws for the corners, are presented in the reference [2].

The constant strain triangular element (CST) is used in this study. Finite element formulation of the incremental equation of equilibrium, relating the nodal incremental displacements to incremental forces through elasto-plastic stiffness matrix, is described in [1, 2]. The incremental force-displacement relationship of whole structure in global coordinate system is in the form

$$\{\delta Q\} = [K]_{ep}\{\delta q\} \tag{6}$$

4. EXAMPLE

A fixed-fixed end beam with rectangular cross-section under a gradually and uniform distributed cyclic load is considered for this example. The beam is assumed to be made of material with linear elastic and perfect plastic behavior. The initial yield strength in tension is assumed to be 350.00 psi (2.413 MPa) and the corresponding value in compression is 10 times higher, 3500.00 psi (24.133 MPa). The modulus of elasticity is assumed to be 3,500,000.00 psi (24132.500 MPa) and Poisson's ratio is taken to be 0.15.

A graphical representation of the beam and finite element mesh is given in Figure (4-1). In this figure, L=72.0 in. (1828.800 mm) and H=12.0 in. (304.800 mm), the thickness of the beam is assumed to be 3.0 in. (76.200 mm).

The applied load is sinosoidal in the form of $F(t)=A.\mathrm{Sin}(\omega t)$, where A=750.0 lb/in (131.339 N/mm) and ω=1.0 rad/sec. Here t is a parameter to control the cyclic behavior of the load.

The load is applied incrementally, with starting parameter increment of Δt=0.02 second. Since the applied load should be small in the plastic analysis, this parameter increment was reduced, once the plastic region started and progressed. A total, 950 increments, equivalent to a parameter value of 13.024 seconds, are considered in this case. This is more than two cycles of loading.

Due to the low tensile strength of the material, the first elements which yielded are those located at the top surface of the beam at the fixed supports. Progression of plastic enclaves continue at the supports toward the center of the beam, until the number of yielded elements at each support reaches two. At this point another plastic region started at the bottom and center of the beam. By increasing the applied load, the progression of plastic enclaves continued vertically at the supports and both horizontally and vertically at the center of the beam. Another region of plasticity started at the bottom of the beam at the fixed supports, when the direction of applied load changed, and loading started in the opposite direction.

The load-deflection relationship for point A, located at the center and bottom of the beam (see Figure 4-1), is shown in the Figure (4-2). The horizontal shift of the hysteresis loops, with no vertical shifting, is an indication of elastic-perfect plastic material behavior.

For the sake of comparison, a load-deflection curve of the same beam, considering elastic behavior and the same applied load for a duration of 20.0 seconds, and the equivalent of 1000 load increments, is shown in the Figure (4-2).

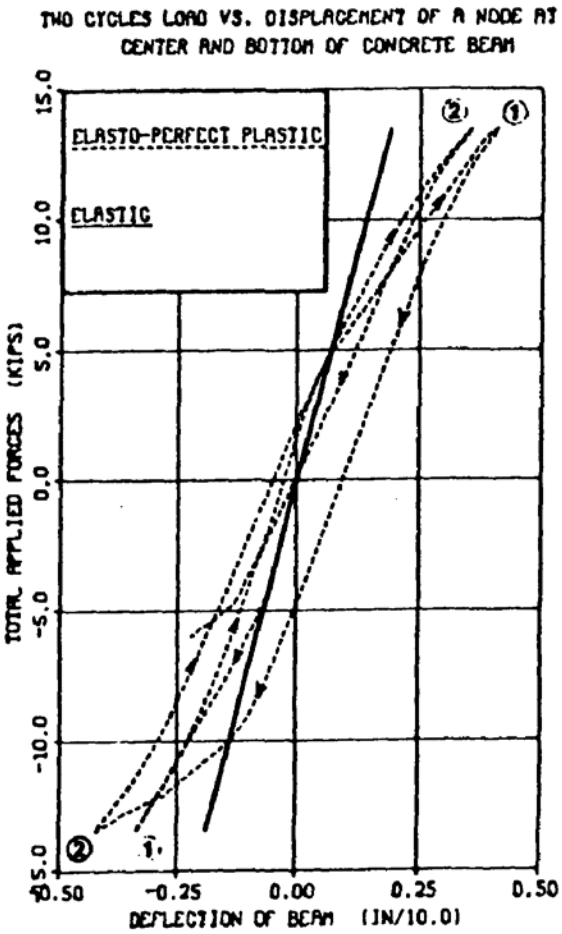
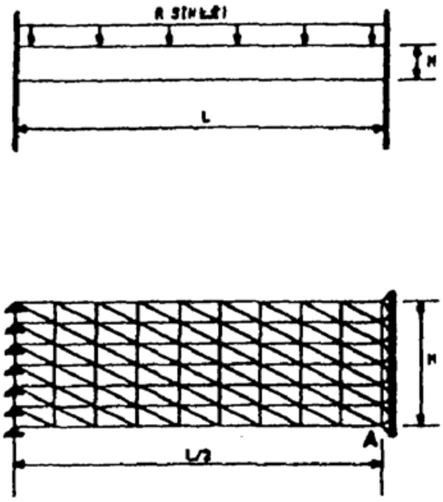


Figure 4-2: Two Cycle Load vs. Displacement of a Node at Center and Bottom of Concrete Beam



Figue 4-1: Concrete Beam

5. REFERENCES

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