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Research Note

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An Investigation on the Performance of Approximate Methods in the Representation of Stressed Power Systems

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Abstract. *Heavily loaded stressed power systems exhibit complex nonlinear dynamic behavior, which cannot be analyzed and described accurately by conventional linear methods, such as eigen-analysis. A normal form of the vector fields theory, a well established mathematical method and the Modal Series technique (a relatively newly established approach) have been used as tools to analyze, characterize and quantify some of the stressed power system's sophisticated nonlinear behavior such as low frequency inter-area oscillations. The normal form method has been used extensively in recent years for the analysis of nonlinear modal interaction and the role of this interaction in causing inter-area oscillations after the occurrence of large disturbances. However, the normal form has some shortcomings, which must be further highlighted. In this paper, some of these shortcomings are addressed by the use of simple examples. Linear modal, normal form and Modal Series methods are used to simulate a two-area, 4-machine power system test case and the results are compared with its accurate nonlinear simulation to asses the performance and accuracy of these three methods. It is shown that: 1) Normal form techniques cannot simulate stressed power systems well in some regions of its operating space, 2) In some regions of state space, even a linear modal method provides more accurate results than a normal form, and 3) Modal Series' results are consistently the most accurate of the three.*

Keywords: *Power system dynamics; Normal forms method; Modal Series technique; Modal analysis; Nonlinear interaction.*

INTRODUCTION

These days, more and more large interconnected power systems are operating in stressed conditions, especially with the advent of restructuring and in deregulated environments. Heavily loaded and stressed power systems exhibit complex dynamic behavior when subjected to large or small disturbances. When a stressed power system is subjected to a large disturbance, it exhibits complex behavior, not detectable from linear systems analysis. For example, the inter-area mode phenomenon in stressed power systems and auto- and hetero-parametric resonances can be addressed as some of this complex behavior [1-3]. In recent years, the Normal Form (NF) analysis has been used to investigate and quantify nonlinear interactions between power system modes [4-8]. Applications include control system design [4,9-12], approximation of stability boundary [13-15] and the predicting of inter-area separation [6,7,16,17]. In [18], a relatively complete survey on applications of the normal form method in the small signal stability of power systems has been presented. The method of normal forms was introduced by Poincare and is a well established mathematical procedure for simplifying nonlinear differential equations [19-23]. Using this method, provided that certain conditions are met, a set of nonlinear differential equations can be transformed, up to a specified order, into a set of linear differential equations by performing a sequence of nonlinear coordinate transformations. The transformed equations are in their simplest form, i.e. in their normal form, and allow for the study of essential modal characteristics. An important characteristic of

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this approach is that it provides a closed form solution for the system state variables. These last two features are key elements in the analysis of power system stability. Normal forms are used to identify the nonlinear interaction among the power system's natural modes of oscillation. These interactions are then quantified in terms of the modal solutions in the original system states. The interaction coefficients obtained via the NF analysis clearly identify the interacting modes. However, the normal form technique is an approximate method and its responses have some differences with actual responses of the system. In [24], this feature of the work has been considered and indices for the better utilization of the NF method are represented; the drawbacks to this approach have not been yet discussed. The method of a Modal Series (MS) is another approximate technique for the analysis and study of nonlinear dynamical systems. It is looked up as an efficient complement or alternative to NF [25,26].

This paper addresses some of the issues relating to the accuracy of the normal form method. It will show that in some regions around the stable equilibrium point, even near it, the NF technique fails to simulate the nonlinear dynamic behavior of the power system. It will also show that the region of validity of the NF technique becomes small near the resonance condition and, at some regions of state space, even linear analysis works better than NF. Furthermore, it will show that the Modal Series works better than both linear analysis and NF.

This paper has been organized as follows: First a summary of Linear Modal Analysis and Normal Form methods are presented. Then, the Modal Series technique is represented and the normal form technique's accuracy issues are discussed. Following that, the results are confirmed in applying the methods to a power system. Finally, the conclusion of study is presented.

TAYLOR SERIES EXPANSION, JORDAN FORM TRANSFORMATION AND LINEAR MODAL ANALYSIS

A wide class of nonlinear dynamical systems, including power systems, can be modeled by differential equations of the form:

$$\dot{X} = F(X), \tag{1}$$

where X is the N dimensional state vector and $F : R^N \to R^N$ is a smooth vector field (when only sinusoidal nonlinearity is considered, it would be analytic as well). Often, the behavior of the system in the neighborhood of an equilibrium point is desired and studied. Expanding Equation 1 in a Taylor series about a stable equilibrium point, X_{SEP}, and using again X and x_i as the new state vector and state variables to refer to $X - X_{SEP}$ and $x_i - x_i^{SEP}$, yields the following representation:

$$\dot{x}_i = A_i X = \frac{1}{2}\sum_{k=1}^{N}\sum_{l=1}^{N} H_{kl}^i x_k x_l + \frac{1}{6}\sum_{P=1}^{N}\sum_{Q=1}^{N}\sum_{R=1}^{N} P_{PQR}^i x_P x_Q x_R + \cdots, \tag{2}$$

where X belongs to the convergence domain of the Taylor series, $\upsilon \subseteq R^N$, and $i = 1, 2, \cdots, N$. A_i is the ith row of Jacobian matrix $A = (\partial F/\partial X)|_{X_{SEP}}$. $H^i = (\partial^2 F_i/\partial X^2)|_{X_{SEP}}$ is the Hessian matrix, $P_{pqr}^i = (\partial^3 F_i/\partial x_p \partial x_q \partial x_r)|_{X_{SEP}}$, and so on. Assuming the system has N distinct eigenvalues, $\lambda_j, j = 1, 2, \cdots, N$ and denoting by U and V the matrices of the right and left eigenvectors of A, respectively, the transformation, $X = UY$, yields the following equivalent set of differential equations for Equation 2:

$$\dot{y}_i = \lambda_j y_j + \sum_{k=1}^{N}\sum_{l=1}^{N} C_{kl}^j y_k y_l + \sum_{p=1}^{N}\sum_{q=1}^{N}\sum_{r=1}^{N} D_{pqr}^j y_p y_q y_r + \cdots, \tag{3}$$

where, Y belongs to the linear mapping of υ, denoted by $\nu \subseteq C^N$, under defined linear transformation;

$$[C_{kl}^j] = \frac{1}{2}\sum_{p=1}^{N} V_{jp}^T [U^T H^p U], \tag{4}$$

$$[D_{pqr}^j] = \frac{1}{6}\sum_{P=1}^{N}\sum_{Q=1}^{N}\sum_{R=1}^{N} P_{PQR}^j V_p^P V_q^Q V_r^R, \tag{5}$$

V_p^P is the pth element of the Pth left eigenvector and so forth. Also, here and from now on $j = 1, 2, \cdots, N$.

Linear Modal Analysis

Starting with the set of Equations 3, in the linear modal method of analysis, only the linear first term is retained to obtain:

$$\dot{y} = \lambda_j y_j, \tag{6}$$

$$y_j(t) = y_{j0} e^{\lambda_j t}. \tag{7}$$

Inverse transformation yields:

$$x_i(t) = \sum_{j=1}^{N} u_{ij} y_{j0} e^{\lambda_j t} = \sum_{j=1}^{N} L_{i,j}^{\text{linear}} e^{\lambda_j t}, \quad i = 1, 2, ..., N, \tag{8}$$

where y_{j0} is the jth element of $Y_0 = U^{-1}X_0$, X_0 is the initial condition in physical state-space, and $L_{i,j}^{\text{linear}} = u_{ij}y_{j0}$. In linear systems, terms $e^{\lambda_j t}$ are called modes of system. Although a linear modal analysis gives a physical insight into system behavior, it is only valid in a small region around the equilibrium point.

NORMAL FORM ANALYSIS

The normal form of a vector field is the simplest member of an equivalence class of vector fields, all exhibiting the same qualitative behavior [19]. To get this normal form, successive polynomial transformations are used. By these transformations, in the absence of a resonance condition, which is defined later, the smallest order of nonlinear terms in the new coordinates is increased. To have the closed form approximate solution, higher order terms in the new coordinate are ignored and a linear system is obtained [21]. This linear system has the form $\dot{Z} = \Lambda Z$, where Λ =diag$(\lambda_1, \lambda_2, \cdots, \lambda_n)$ and λ_i is a linear mode of the original system for each i. The response of the linear system with initial condition z_0 is calculated and, by applying successive transformation, an approximate closed form solution in the original coordinates is obtained. In the normal form theory, a set of system modes is said to be resonant of order r, if $\lambda_i = \sum_{j=1}^{N} m_j\lambda_j$ and $r = \sum_{j=1}^{N} m_j$ for $i \in \{1, \cdots, N\}$. Here, r and m_j are integer and λ_j's are linear modes of the system. By neglecting the third and higher order terms in Equation 3, and considering the case without second order resonance, the normal form technique offers the transformation:

$$Y = Z + h2(Z), \tag{9}$$

where:

$$h2^j(Z) = \sum_{k=1}^{N}\sum_{l=1}^{N} h2_{kl}^j z_k z_l, \quad j = 1, \cdots, N, \tag{10}$$

$$h2_{kl}^j = \frac{C_{kl}^j}{\lambda_k + \lambda_l - \lambda_j}. \tag{11}$$

In Z-coordinates, the system (Equation 3) takes the form:

$$\dot{z} = \lambda_j z_j + \text{Order}(|z|^3). \tag{12}$$

By neglecting higher order terms in Equation 12, an explicit second order approximate solution can be found as [4-6]:

$$z_j(t) = z_{j0}e^{\lambda_j t}. \tag{13}$$

By replacing Equation 13 in Equations 9 and 10, the solutions for Equation 3 are then given by:

$$y_j(t) = z_{j0}e^{\lambda_j t} + \sum_{k=1}^{N}\sum_{l=1}^{N} h2_{kl}^j z_{k0} z_{l0} e^{(\lambda_k+\lambda_l)t}, \tag{14}$$

where z_{j0} is the initial condition of the normal form variable, z_j. Then the second-order approximate solution to the original system (Equation 1) can be found by applying the transformation $X = UY$ to Equation 14. The resulting solution is:

$$x_i(t) = \sum_{j=1}^{N} u_{ij} z_{j0} e^{\lambda_j t} + \sum_{j=1}^{N} u_{ij}\left[\sum_{k=1}^{N}\sum_{l=1}^{N} h2_{kl}^j z_{k0} z_{l0} e^{(\lambda_k+\lambda_l)t}\right], \tag{15}$$

where:

$$Y_0 = VX_0 = U^{-1}X_0, \tag{16}$$

$$Y_0 = Z_0 + h2(Z_0). \tag{17}$$

Solving the ill-conditioned nonlinear algebraic Equation 17 to find initial condition Z_0 is a major source of inaccuracy and imposes heavy calculating burdens, even for moderately sized systems.

MODAL SERIES METHOD

By using the Modal Series, it is possible to represent nonlinear dynamic systems as well as stressed power systems in a manner which yields a good deal of physical insight into the problem under consideration [25-26]. This method of solution also has the great conceptual advantage of presenting a nonlinear system as a rather straightforward generalization of the linear case. Moreover, this method provides a solution to the differential equations, even in the case of a resonance condition. As with the normal form technique, this method is restricted to polynomial nonlinearity. Taylor series expansions of other nonlinearity types are needed for the application of this method.

It has been shown that the solution of Equation 3 for the initial condition, $Y_0 = [y_{10}, y_{20}, ..., y_{N0}]^T$, can be written as [25]:

$$\begin{aligned} y_j(t) &= y_{1j}(t) + y_{2j}(t) \\ &= \left(y_{j0} - \left\{\sum_{k=1}^{N}\sum_{l=1}^{N} h2_{kl}^j y_{k0} y_{l0}\right\}_{(k,l,j)\notin R_2'}\right) e^{\lambda_j t} \\ &\quad + \left\{\sum_{k=1}^{N}\sum_{l=1}^{N} h2_{kl}^j y_{k0} y_{l0} e^{(\lambda_k+\lambda_l)t}\right\}_{(k,l,j)\notin R_2'} \\ &\quad + \left\{\left(\sum_{k=1}^{N}\sum_{l=1}^{N} C_{kl}^j y_{k0} y_{l0}\right) t e^{\lambda_j t}\right\}_{(k,l,j)\in R_2'}. \end{aligned} \tag{18}$$

Similar to the normal form case, the second-order approximate solution to the physical system in the Modal Series frame can be found by applying the transformation, $X = UY$, to Equation 18. The resulting solution will be:

$$x_i(t) = \sum_{j=1}^{N} u_{ij}\left(y_{j0} - \left\{\sum_{k=1}^{N}\sum_{l=1}^{N} h2_{kl}^{j} y_{k0} y_{l0}\right\}_{(k,l,j)\notin R_2'}\right)e^{\lambda_j t} + \left\{\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} u_{ij} h2_{kl}^{j} y_{k0} y_{l0} e^{(\lambda_k+\lambda_l)t}\right\}_{(k,l,j)\notin R_2'} + \left\{\sum_{j=1}^{N}\left(\sum_{k=1}^{N}\sum_{l=1}^{N} u_{ij} C_{kl}^{j} y_{k0} y_{l0}\right) t e^{\lambda_j t}\right\}_{(k,l,j)\in R_2'}. \quad (19)$$

The coefficients, C_{kl}^{j} and $h2_{kl}^{j}$ s, are as in Equations 4 and 11, respectively, and set R_2 contains all three tuples, (k, l, j), which cause the second order resonance condition, i.e. satisfy $\lambda_k + \lambda_l = \lambda_j$. A similar procedure may be carried out to calculate higher order terms. The condition $|\lambda_k + \lambda_l - \lambda_j| \leq 0.001\,|\lambda_j|$ is a so-called second order quasi-resonance and denotes by R_2' the set of all three tuples, (k, l, j), which cause the second order quasi-resonance. Defining new constants, L_j^i, K_{kl}^i and M_j^i as:

$$L_j^i \triangleq u_{ij}\left(y_{j0} - \sum_{k=1}^{N}\sum_{l=1}^{N} h2_{kl}^{j} y_{k0} y_{l0}\right)_{(k,l,j)\notin R_2'}, \quad (20)$$

$$K_{kl}^i \triangleq y_{k0} y_{l0} \sum_{j\in J_{kl}} u_{ij} h2_{kl}^{j},$$

$$J_{kl} = \{j \,|\, (k,l,j) \notin R_2'\}, \quad (21)$$

$$M_j^i \triangleq \sum_{k=1}^{N}\sum_{l=1}^{N} u_{ij} C_{kl}^{j} y_{k0} y_{l0}, \quad (k,l,j) \in R_2', \quad (22)$$

and rearranging Equation 19, we obtain:

$$x_i(t) = \sum_{j=1}^{N} L_j^i e^{\lambda_j t} + \sum_{j=1}^{N} M_j^i t e^{\lambda_j t} + \sum_{k=1}^{N}\sum_{l=1}^{N} K_{kl}^i e^{(\lambda_k+\lambda_l)t} = \sum_{j=1}^{N}\left(L_j^i + M_j^i t\right) e^{\lambda_j t} + \sum_{k=1}^{N}\sum_{l=1}^{N} K_{kl}^i e^{(\lambda_k+\lambda_l)t}. \quad (23)$$

SHORTCOMINGS OF NORMAL FORM TECHNIQUE

There are a few issues that limit the use of the NF method in studying stressed power systems. These issues can be addressed as follows:

(i) The nonlinear transformation needed for the NF method is neither onto nor one to one. This may result in multiple solutions in some cases or non-convergence of the algorithm used for the calculation of initial conditions, z_0's.

(ii) The nonlinear transformation introduces variables that do not directly correspond to physical state or modal phenomena.

(iii) Solving nonlinear algebraic Equation 17 to obtain z_0 for practical large size power systems is not an easy task.

(iv) In general, the normal form technique fails to obtain a closed form solution when there is a second or higher order resonance condition. However, an approximation solution can be found when some conditions hold [8].

The shortcomings of the normal form technique to simulate a nonlinear system behavior are illustrated using two examples. In the first example, a one-dimensional system is used to show the problems that are caused by the nonlinear transformation in the NF method. The aim of the second example is to demonstrate the effect of the near resonance condition on the accuracy of its results.

Example 1

By using a normal form, we want to eliminate the second order term of the system:

$$\dot{y} = \lambda y + \frac{1}{2} H y^2. \quad (24)$$

From Equations 9 to 11, one can obtain the following transformation:

$$y = z + \frac{1}{2\lambda} H z^2. \quad (25)$$

Applying this transformation to Equation 24 yields:

$$\dot{z} = \lambda z + \left(\frac{1}{4\lambda} H^2 Z^3 \times \frac{2 + \frac{1}{2\lambda} H z}{1 + \frac{1}{\lambda} H z}\right). \quad (26)$$

Expanding the fractional term of Equation 26 by introducing limitative assumption, $\left|(\frac{1}{\lambda})Hz\right| < 1$ yields:

$$\dot{z} = \lambda z + \left(\frac{1}{2\lambda} H^2 z^3 - \frac{1}{8\lambda^2} H^3 z^4 + \frac{1}{8\lambda^3} H^4 z^5 + \cdots\right). \quad (27)$$

From now on, any effort to eliminate higher order terms is subject to that limitative assumption and introduces other limitations. These sequential limitations cause the region of validity of the closed form solution obtained from the normal form to shrink. Neglecting the nonlinear term in Equation 26 provides an approximate closed form solution for Equation 24, called a second order modal as:

$$y(t) = z_0 e^{\lambda t} + \frac{1}{2\lambda} H z_0^2 e^{2\lambda t}. \tag{28}$$

The linear approximate solution of Equation 24, called a linear modal, is given as:

$$y(t) = y_0 e^{\lambda t} = \left(z_0 + \frac{1}{2\lambda} H z_0^2 \right) e^{\lambda t}. \tag{29}$$

Is the accuracy of Equation 28 always better than that of Equation 29. To answer this question, let us define a nonlinearity measure as the ratios of the absolute value of the nonlinear term to the linear term in Equations 24 and 26 by R_y and R_z, respectively:

$$R_y = \left| \frac{H}{2\lambda} y \right| = \left| \frac{H}{2\lambda} (z + \frac{1}{2\lambda} H z^2) \right| = \left| Z^2 - Z \right|, \tag{30}$$

$$R_z = \left| \frac{Z^2 (2 - Z)}{(1 - 2Z)} \right|, \tag{31}$$

where $Z = -(H/2\lambda) z$. In Figure 1, R_y and R_z are plotted. As shown in the figure, $R_z > R_y$ for $Z > 0.2324$, i.e. the linear modal is more accurate than the second order normal form for $Z > 0.2324$. The simulation result in Figure 2 proves this conclusion for $Z = 0.5$, equivalent to $y_0 = 0.05$, $H = 1$ and $\lambda = -0.1$.

Another problem originates from the nonlinear transformation used by the normal form technique,

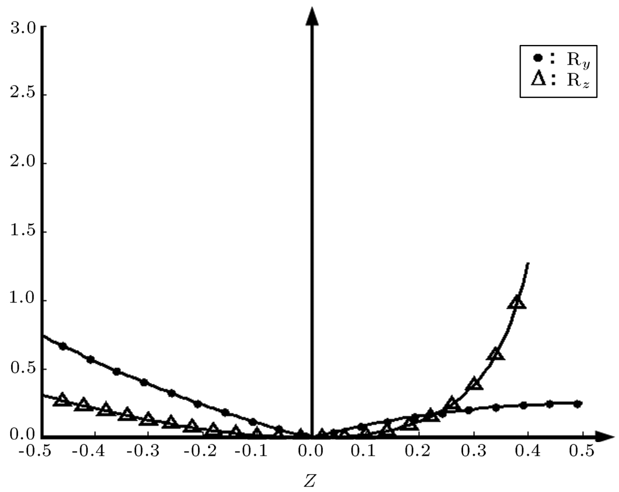

Figure 1. Plot of nonlinearity measures.

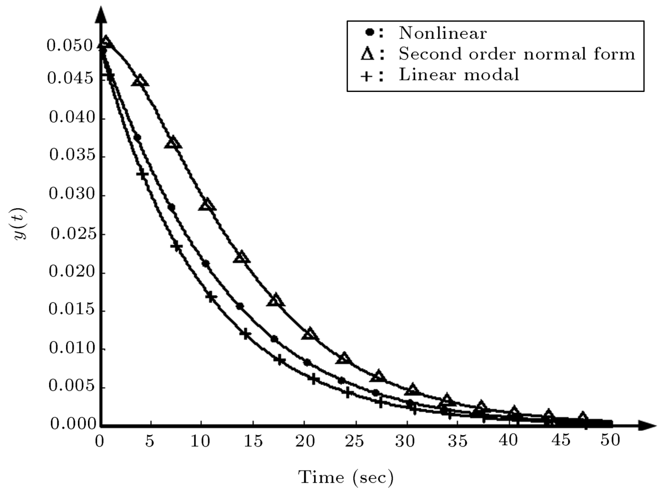

Figure 2. Time domain simulation results.

which is neither one to one nor onto and results in some parts of the state space not being covered by the transformation.

For example, assuming λ to be real and negative, Figure 3, plot of Equation 25, shows that for the region specified by $y > -\lambda/2H$, there is no solution for Z. Therefore, if the initial condition is such that y_0 belongs to that region, Equation 17 will not have any solution and the normal form fails.

Example 2

This second example shows that when resonance conditions are approached, the aforementioned problem is exacerbated. Let $x_{il}(t)$, $x_{in}(t)$, $x_{im}(t)$ and $x_{i\,NL}(t)$ denote linear modal, second order normal form, Modal Series and nonlinear time domain step by step simulation results for state i, respectively, and also $X_{il}(\omega)$, $X_{in}(\omega)$, $X_{im}(\omega)$ and $X_{i\,NL}(\omega)$ denote their Fourier transform magnitudes. A likeness or similarity index between nonlinear and other approximated simulations

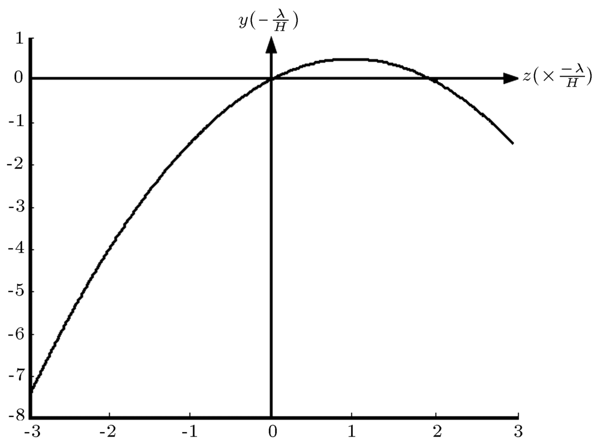

Figure 3. Plot of normal forms technique transformation.

is defined as:

$$L_m = \sum_{i=1}^{N} \int_{0}^{+\infty} |X_{i\mathrm{NL}}(\omega) - X_{i\,\mathrm{other}}(\omega)|\, d\omega. \tag{32}$$

For the following numerical system, the domain within $L_m \leq 2.5$ is obtained for each approximate solution and for two values of α.

$$\dot{X} = AX + \begin{bmatrix} X^T H^1 X & X^T H^2 X \end{bmatrix}^T, \tag{33}$$

where:

$$X = \begin{bmatrix} x_2 \\ x_2 \end{bmatrix}, \quad A = \begin{bmatrix} -0.2 & -0.5 \\ 0 & \alpha \end{bmatrix},$$

$$H^1 = 0.02 \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}, \quad H^2 = 0.02 \begin{bmatrix} -1 & -4 \\ -4 & -1 \end{bmatrix}.$$

Also, to be able to compare the Modal Series and normal form solutions under similar conditions, only two terms of the Taylor series expansion are used in both cases. This system has two eigenvalues, $\lambda_1 = -0.2$ and $\lambda_2 = \alpha$. If α is chosen, such that $2\lambda_1 = \lambda_2$, then, the normal form fails to give a closed form solution, because a resonance condition results. The region of stability of the system has been shown in Figures 4a and 4b. The filled areas in these figures are the regions where NF cannot span. These figures show that the domain of attraction is not changed considerably but, for $\alpha = -0.41$, i.e. near resonance condition (Figure 4a), the region around the origin where the normal form is applicable becomes smaller than that for $\alpha = -0.45$ (Figure 4b). For more details, we focus our attention on the region that is surrounded by the rectangle in each figure.

In Figure 5, circular, ellipsoidal and lunulate show the regions of validity of linear, Modal Series and normal form approximation for $L_m \leq 2.5$. It can be seen that the accuracy region of the Modal Series technique is wider than that of the normal form and linear modal methods. On the other hand, there are some regions around the stable equilibrium point where the accuracy of the linear approximation is better than that of the second order normal form.

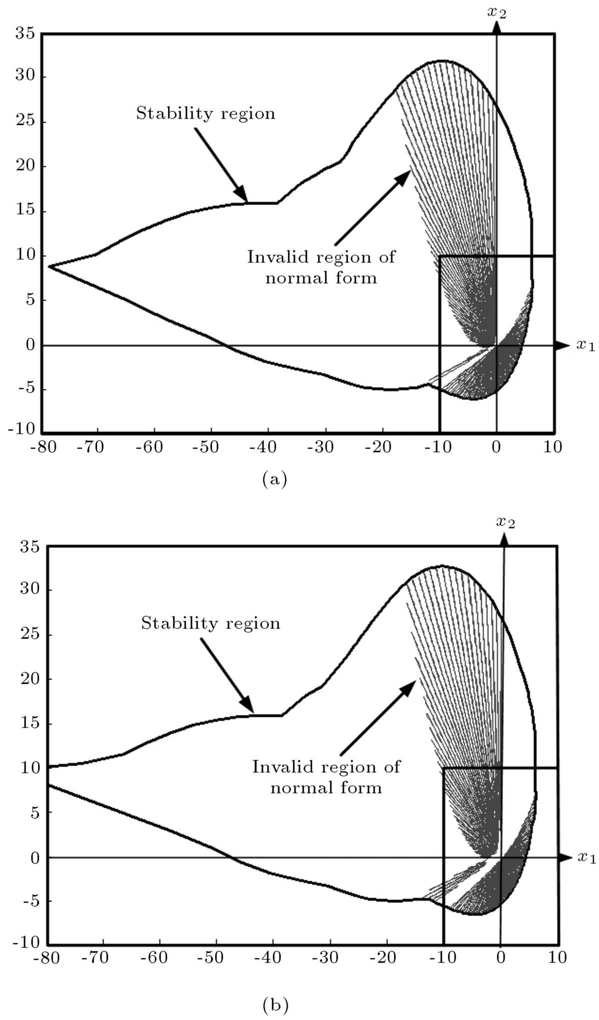

Figure 4. The region of stability and invalid region of normal form technique.

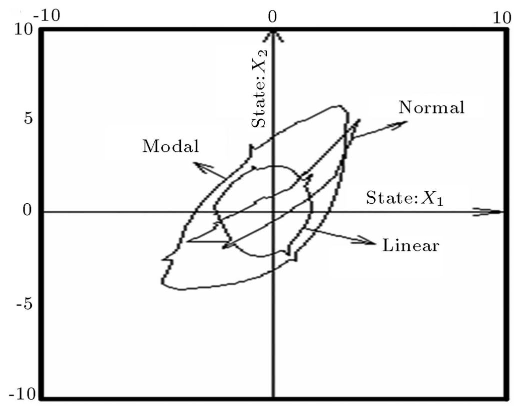

Figure 5. Validity regions in phase-plane for linear, normal form and modal series method.

STUDY ON THE STRESSED POWER SYSTEM

The test system selected for this work, shown in Figure 6, is a four-generator system, which was introduced in [28-29] and thus, has become a de facto benchmark for analyzing electromechanical oscillations, modal interactions, inter-area oscillations and small-signal stability analysis.

Generators are modeled using a two-axis model with one winding in each axis. Moreover, each is equipped with an AVR with a transient gain reduction of 10 and a fast-response exciter represented by a single time constant and a gain. The block diagram of the exciter model is shown in Figure 7. The loads, $L1$ and $L2$, are represented by constant impedances; the

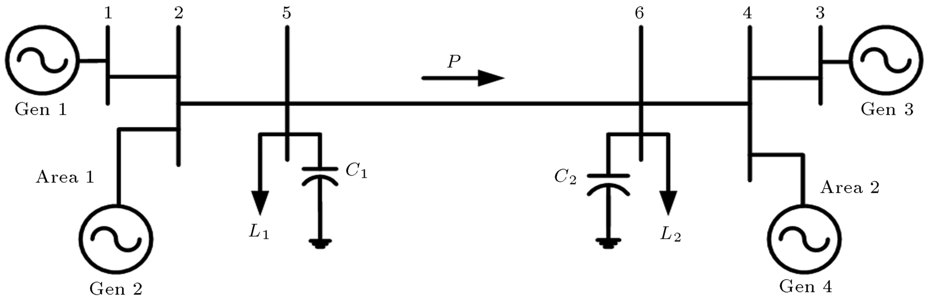

Figure 6. Two-area, four-machine power system.

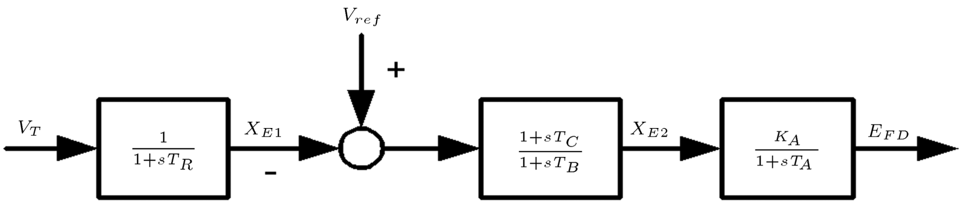

Figure 7. Excitation system model (IEEE type AC4A).

network is reduced to the generator internal nodes [29]. System data are adopted from [18].

The dynamics of this system can be described by Equation 1 with state vector X:

$$X = [E'_{q1}, E'_{d1}, \omega_1, \delta_1, E_{FD1}, X_{E1_1}, X_{E2_1},$$
$$\cdots, E'_{q4}, E'_{d4}, \omega_4, \delta_4, E_{FD4}, X_{E1_4}, X_{E2_4}]^T,$$

where:

E'_d, E'_q:	transient direct and quadrature axes EMFs, respectively;
ω:	rotor speed, with respect to a synchronous reference frame;
δ:	rotor angle;
E_{FD}:	stator EMF corresponding to the field voltage;
X_{E1}, X_{E2}:	exciter state variables as shown in Figure 1.

There is one inter-area mode associated with the oscillations of the two areas, and two local modes associated with the oscillations of the generators within each area. The system is operated in a highly stressed regime close to voltage collapse, characterized by a tie line flow of 410 MW from Area 1 to Area 2. This operating condition was selected to more readily expose the nonlinear characteristics of the system. The system is subjected to a three-phase stub fault at bus 5, which is cleared in 0.035 s with no line switching. X_{cl}, at the end of the disturbance, is determined using a conventional time domain simulation. The post-disturbance Stable Equilibrium Point (SEP), X_{SEP} , of the system is also determined using a load flow solution. The Taylor series expansion of the system around the SEP is obtained and then an eigen-analysis is performed. The initial conditions, $X_0 = X_{cl} - X_{SEP}$, are determined and transformed to Y_0 using Equation 16. Results of nonlinear simulation, linear modal analysis, the normal form technique and Modal Series method for various states are obtained and compared. Due to space limitation, only time evolutions of six states are illustrated in Figure 8.

Figure 8 shows considerable differences between the obtained results from different methods. The Modal Series simulation follows the full nonlinear simulation most accurately, whereas the normal form response has considerable deviations from it. This shows that the normal form technique cannot simulate stressed power systems well in some regions of the operating space. The NF response is, in some instances, even less accurate than that obtained by linear modal analysis.

To quantify the comparison of the performance and accuracy of different approximate methods, the following error index (or distance measure) is defined:

$$P_{m_{i,pu}} = \frac{\int_0^\infty |X_{i_{NL}}(\omega) - X_{i_{\text{other}}}(\omega)|\, d\omega}{\int_0^\infty X_{i_{NL}}(\omega) d\omega}, \tag{34}$$

where $X_{i_{NL}}(\omega)$ and $X_{i_{\text{other}}}(\omega)$ are the the Fourier transform of the nonlinear simulation and other methods, respectively.

Numerical values for distance measures of the three methods for a number of states have been calculated and shown in Table 1, which shows that the Modal Series method has more accurate performance than either the normal form or linear modal analysis.

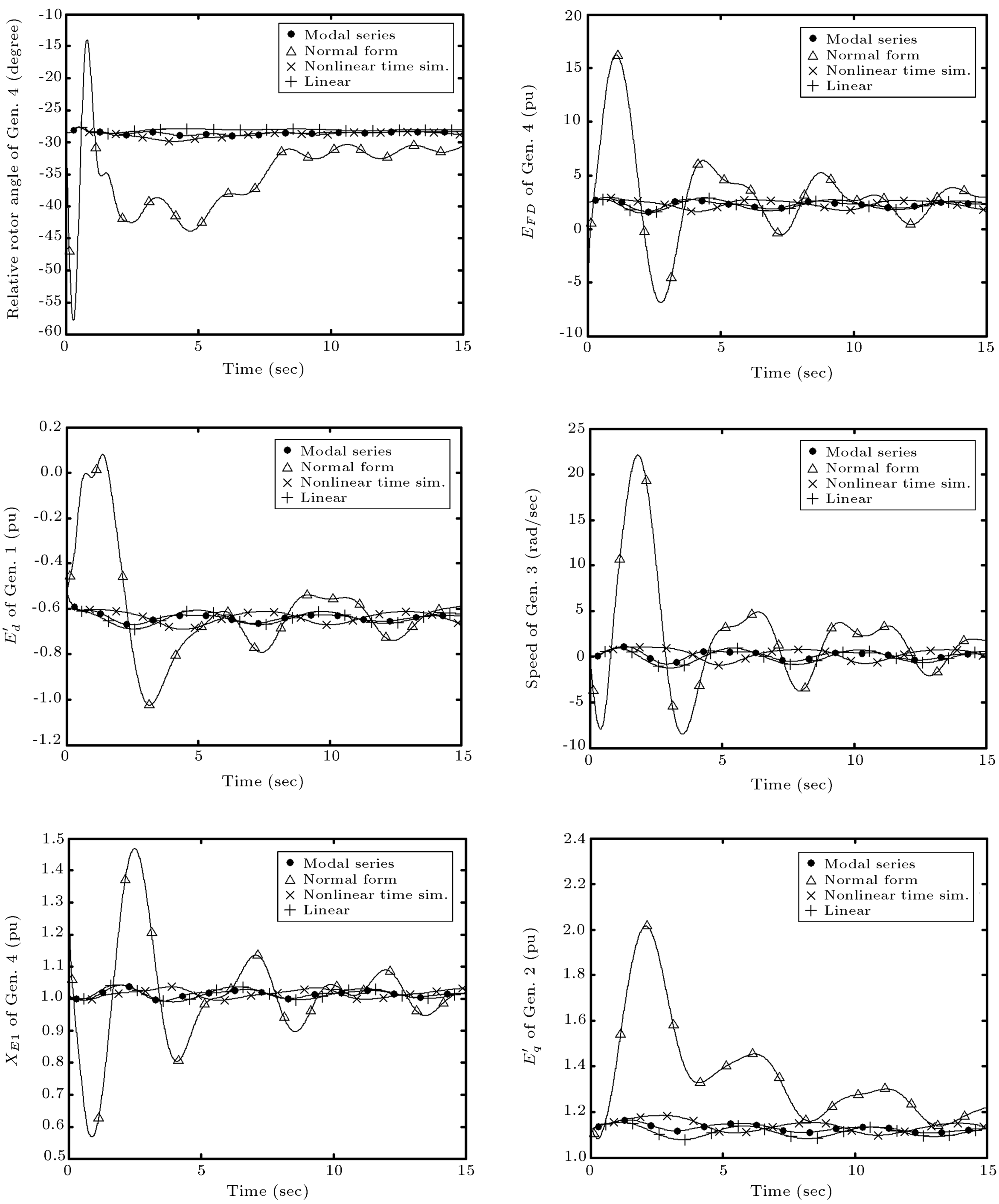

Figure 8. Comparison of time domain responses of various states obtained by approximate methods and full nonlinear simulation.

It should be emphasized that, in general, the normal form technique provides a more accurate and structurally improved approximation of the system dynamic behavior than the linear modal method. It is only in some regions of state space that NF fails to represent an accurate response. Our numerous simulations of different power system test cases, under varying operating conditions, have shown that the Modal Series method consistently provides an accurate presentation of the system response; better than either NF or linear modal methods. On the other hand, while the response obtained by the NF technique is, in general, better than the linear modal response, under some operating conditions like the one just reported, not only the Modal Series but also the linear modal analysis perform more accurately than the NF method.

Table 1. Error indices obtained from different approximate methods for sample states.

State #	Linear Modal	Normal Form	Modal Series
δ_4	14.65%	92.04%	9.39%
E'_{d1}	13.29%	81.77%	10.12%
E'_{q2}	11.76%	73.04%	8.80%
ω_3	7.83%	98.54%	3.75%
E_{Fd4}	10.53%	59.90%	5.08%
ω_2	18.24%	65.39%	12.65%
δ_1	8.23%	99.46%	7.54%
X_{E14}	3.54%	67.48%	2.91%
δ_2	28.16%	79.99%	15.83%

CONCLUSION

The stressed power system exhibits highly nonlinear dynamic behavior when subjected to disturbances, either small or large. The method of the normal form of vector fields has been used as an efficient tool for the study of these complicated dynamic phenomena in recent years. In spite of its widespread applications, the method suffers from some shortcomings. These shortcomings are: (a) In general, it fails under resonance conditions, (b) Its nonlinear transformation is neither onto nor one to one, resulting in multiple solutions in some and lack of solution in other regions, and (c) Determination of initial conditions in normal form coordinates, z_0's, specifically for highly stressed cases, may not be possible. Under these situations, the numerical solution algorithms have difficulty converging. The Modal Series method, on the other hand, does not require or use nonlinear transformation and, therefore, does not suffer from the above shortcomings. Using a simple example, it was shown that the normal form may return more than a single solution, even for a low dimension dynamical system. The same example showed that for a range of initial conditions the normal form failed to provide any solution. A second example showed that changing the operating point of the system and moving closer to resonance condition, shrinks the validity region of the normal form method. Validity regions of the linear modal, normal form and Modal Series methods were compared. The results demonstrated that the Modal Series has a much larger validity region. A two-area, 4-machine power system was used as a test case. The accurate nonlinear simulation of it was obtained and compared with the simulations provided by the linear modal, normal form and Modal Series methods. The results, yet again, confirmed the above conclusions. We should note that in spite of normal form shortcomings in most cases, it provides a relatively accurate approximate analytic solution, in terms of natural modes of the system and their interactions. The structural form of this solution is especially well suited for analysis and design. The Modal Series method provides, structurally, the same solution as the normal form, but without the shortcomings of that method and, in many cases, with a better accuracy. Further work will include: finding new applications for the Modal Series method and formulating improved numerical algorithms for the calculation of initial conditions required in the normal form method.

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