REDUCTION OF POSITION ERROR OF KINEMATIC MECHANISMS BY TOLERANCE ANALYSIS METHOD PART II: IMPLEMENTATION

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Abstract. A flexible crank-slider mechanism is considered to investigate the variations of Coupler Point (C.P.) position. The assembly specifications, i.e. C.P. position, have variations in x and y directions. The correlation between these variations also impresses the limits of variation. The bivariate distribution of the assembly specification is determined using the Direct Linearization Method (DLM). The Monte Carlo simulation is implemented to find the valid range of DLM solutions. Then, the percent contribution of each input variable to assembly specifications is computed by DLM. Critical variables, which have the highest contribution to the variations of the assembly specification, are recognized. By improving the tolerance limits of these critical variables and increasing area section r_2, the maximum error of mechanism can be decreased significantly. According to Figure 15, tolerance improvements result in 19% reduction of the maximum normal error at θ_2=3°. In addition, the curve which represents the minimum cost versus the maximum is derived based on the cost function and the regression equation. It has been observed that external loading will remarkably increase the amount of max error.

Keywords: **Tolerance Analysis, FEM, Multiple Regression**

1. INTRODUCTION

A common use of kinematic linkages is for accurate and precise positioning of an object, and various methods have been used to determine the effects of manufacturing tolerances on the position error. These methods can be categorized as either deterministic or probabilistic.
In contrast, probabilistic or statistical methods involve random variables that result in a probabilistic response [1]. Deterministic methods are used mostly where tolerances are given and the worst-case position error is to be determined. Statistical methods are used where dimensions have some type of random distribution, and the probability of being within a given tolerance is to be estimated.

When analyzing the position error in kinematic linkages, the goal is usually to find error bands around an ideal path. Error bands were first developed by Garrett and Hall [2], where they were applied to a function generating four-bar mechanism. When applied to coupler point position, there are numerous methods for determining the magnitudes of these error bands, involving both deterministic and probabilistic approaches [3].
In this paper, we use the DLM procedure applies for determine of variation limits assembly specification. In Section 2 of this paper, the kinematic model of a crank slider mechanism including tolerances of manufacturing variables is stated. In Section 3, the Direct Linearization Method is applied and the equations of vector loops, sensitivity matrix and position error are obtained [4]. In the following section, the DLM method is applied to find the bivariate distribution of the C.P. position error. The accuracy of DLM results is evaluated by means of Monte Carlo simulation in Section 5. The crank slider mechanism with external loading is modeled by FEM and the variations of the maximum normal-to-path error are computed during one cycle of motion. The maximum error functions with rigid and flexible component are obtained from multiple linear regression models and the percent contribution of each manufacturing variables are acquired in the following section. Finally, optimized product cost and max error curve is determined for the case of assembly with flexible parts.

2. IMPLEMENTING DLM FOR A CRANK SLIDER

According to Section 1, the C.P. path error of mechanism can be obtained by DLM. In order to implement DLM on crank slider mechanism, Equations 1 to 13 from first part of the research is used. Nominal dimensions and tolerances of the mechanism are according to Table 1 of part 1.

For example, matrices A, B, U are calculated at $\theta_2=3°$ and written as follows:

$$A=\begin{bmatrix}\dfrac{\partial h_X}{\partial X_i}\\[2mm]\dfrac{\partial h_Y}{\partial X_i}\end{bmatrix}=\begin{array}{c}\begin{matrix} r_2 & r_3 & r_4 & r_p & \beta & \theta_1 & \theta_2\end{matrix}\\ \begin{bmatrix} 0.99 & 0.99 & -1 & 0 & 0 & -25 & -8.72\\ 0.03 & 0.04 & 0 & 0 & 0 & -649.51 & 249.84\end{bmatrix}\end{array} \tag{1}$$

$$B=\begin{bmatrix}\dfrac{\partial h_X}{\partial U_i}\\[2mm]\dfrac{\partial h_Y}{\partial U_i}\end{bmatrix}=\begin{array}{c}\begin{matrix} r_1 & \theta_3\end{matrix}\\ \begin{bmatrix} -1 & -16.2751\\ 0 & 399.6688\end{bmatrix}\end{array} \tag{2}$$

$$U=\begin{bmatrix}\dfrac{\partial r_1}{\partial X_i}\\[2mm]\dfrac{\partial \theta_3}{\partial X_i}\end{bmatrix}=\begin{array}{c}\begin{matrix} r_2 & r_3 & r_4 & r_p & \beta & \theta_1 & \theta_2\end{matrix}\\ \begin{bmatrix} 1.0008 & 1.0008 & -1 & 0 & 0 & -51.44 & 1.4493\\ -0.0001 & -0.0001 & 0 & 0 & 0 & 1.6251 & -0.6251\end{bmatrix}\end{array} \tag{3}$$

And also, matrices C, D, S are calculated and written as below:

$$C=\begin{bmatrix}\dfrac{\partial C.P_X}{\partial X_i}\\[2mm]\dfrac{\partial C.P_Y}{\partial X_i}\end{bmatrix}=\begin{array}{c}\begin{matrix} r_2 & r_3 & r_4 & r_p & \beta & \theta_1 & \theta_2\end{matrix}\\ \begin{bmatrix} 0.9994 & 0 & 0 & 0.13 & -103.0700 & 0 & -8.7249\\ 0.0349 & 0 & 0 & 0.99 & 13.8772 & 0 & 249.8477\end{bmatrix}\end{array} \tag{4}$$

$$D=\begin{bmatrix}\dfrac{\partial C.P_X}{\partial U_i}\\[2mm]\dfrac{\partial C.P_{\cdot Y}}{\partial U_i}\end{bmatrix}=\begin{array}{c}\begin{matrix} r_1 & \theta_3\end{matrix}\\ \begin{bmatrix} 0 & -103.07\\ 0 & 13.87\end{bmatrix}\end{array} \tag{5}$$

$$S=\begin{bmatrix}\dfrac{\partial C.P_X}{\partial X_i}\\[2mm]\dfrac{\partial C.P_Y}{\partial X_i}\end{bmatrix}=\begin{array}{c}\begin{matrix} r_2 & r_3 & r_4 & r_p & \beta & \theta_1 & \theta_2\end{matrix}\\ \begin{bmatrix} 0.99 & -0.01 & 0 & 0.13 & -103.1 & 167.50 & -73.15\\ 0.03 & 0.001 & -0 & 0.99 & 13.9 & -22.55 & 258.59\end{bmatrix}\end{array} \tag{6}$$

After defining variance matrix of manufacturing variables (V), the bivariate distribution of C.P. of mechanism will be achieved. For example distribution contour of C.P. at $\theta_2=3°$ is computed and demonstrated in Figure 1. The maximum normal error is also illustrated in the Figure.

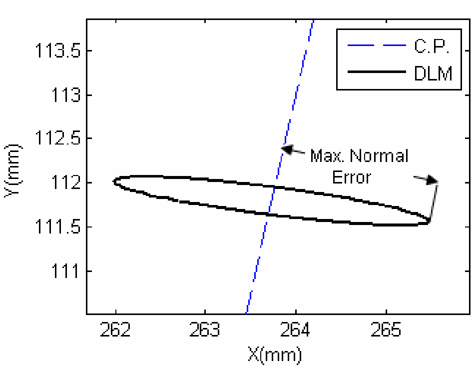

FIGURE 1. The bivariate distribution of C.P. at. $\theta_2=3°$

3. MONTE CARLO SIMULATION

In this section, the results obtained by DLM are compared to the Monte Carlo simulation. For this purpose, the Monte Carlo simulation, based on reliability of 95%, is performed with 400,000 samples of mechanism at each value of θ_2 [5]. Because of, we haven't data of product, distribution of all of manufacturing variables is suggested normal. Figure 2 demonstrates the comparison of DLM with Monte Carlo at $\theta_2=3°$. Mean values and covariance matrix of two approach are presented in Table 1.

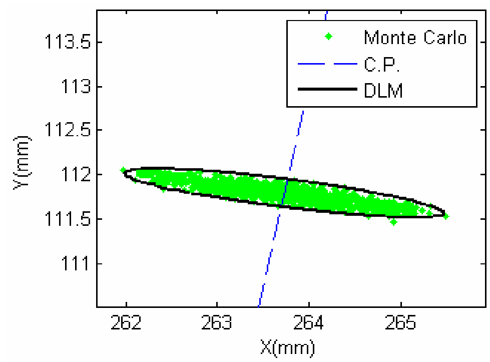

FIGURE 2. Comparison of DLM with Monte Carlo at $\theta_2=3°$.

TABLE 1. comparison of mean values and covariance matrix DLM with Monte Carlo.

Mechanism component	**mean**		**Variance Matrix**
	X	**Y**	
DLM	263.72	111.79	$\begin{bmatrix}0.3371 & -0.0433\\ -0.0433 & 0.0084\end{bmatrix}$
Monte Carlo	263.74	111.78	$\begin{bmatrix}0.3605 & -0.0470\\ -0.0470 & 0.0090\end{bmatrix}$

As noted in Figure 2 and Table1, there is a very small discrepancy between the results of DLM and Monte Carlo simulations. This is actually a very good result considering that the Monte Carlo simulation was based upon the nonlinear solution, whereas the bi-variate model uses direct linearization. This validation is important because DLM is much more efficient than Monte Carlo simulations.

After evaluation of the maximum normal-to-path error at each value of θ_2, the variations of the error for one cycle of motion is determined. In Figure 3, the maximum normal-to-path error using DLM method is compared to the Monte Carlo simulation.

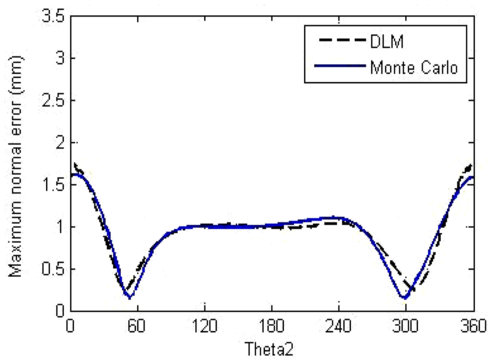

FIGURE 3. Comparison of maximum normal-to-path error using DLM with Monte Carlo.

4. DIMENSION VARIATIONS DUE TO LOADING

In order to determine the effect of loading on C.P. position error of a crank slider mechanism with flexible component, the FEM model of mechanism as shown in Figure 5 of Part 1, built using CALFEM. Beam elements are considered to have elastic modulus of 210 GPa, cross-sectional area of 100 mm^2 and area moment of inertia equals to 833.33 mm^4. Also, plane elements are considered to have elastic modulus of 210 GPa, Poisson ratio of 0.3 and thickness of 5 mm in the state of plane stress. The crank r_2 and the coupler are subdivided into 6 and 36 equal elements, respectively. Also the slider is divided into 32 identical triangular elements and 5 contact elements which are on contact surface the slider and the frame. Therefore, in total 79 elements are created for the FEM model.

A 10 KN load with angle of 45° respect to positive x axis is applied on the C.P. position. The magnitude and direction of this load is assumed to be constant during the cycle of motion. The internal load on each member of the mechanism varies with the rotation angle of the crank r2. Therefore, the input variables have different mean values and tolerances over the cycle of motion. The dimensional variation of each manufacturing variable is computed by FEM. The length variation of the bar can be evaluated by nodes displacements. The mean value and tolerance of r2 are estimated by Equations (23) and (24) of Part 1, at angular position of θ2. Figure 4 shows the variations of r2 for one cycle of motion.

After calculating the variations, new lengths and tolerances of each variables is used in DLM and the maximum normal error of mechanism is obtained. It is obvious that the variation of input variables influences on The bivariate distribution and the maximum error. For example, the distribution contour of C.P. for the flexible crank slider mechanism is computed at θ_2=3° and compared to that of the rigid mechanism in Figure 5. The illustrated contours are computed using DLM. Also, Table2 presents mean values and the covariance matrix of two cases (rigid and flexible) at θ_2=3°.

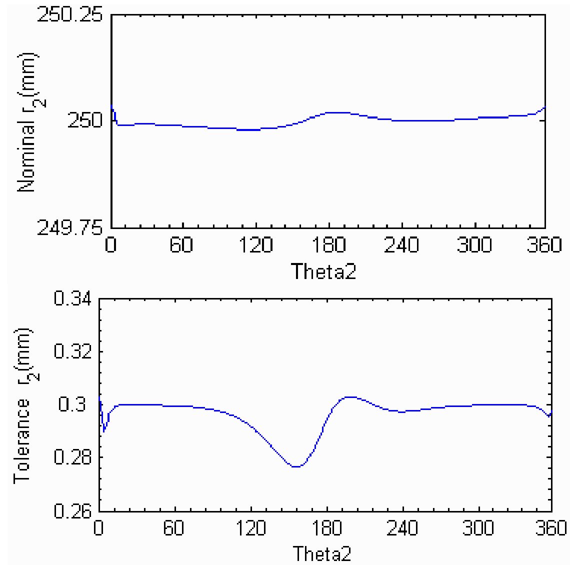

FIGURE 4. The variation of Mean and tolerance value r_2

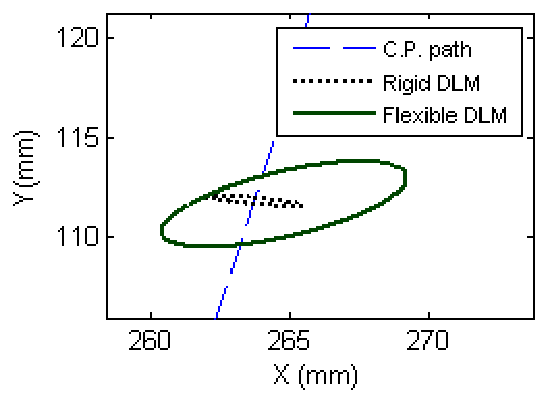

FIGURE 5. Comparison of the C.P. distribution contours at θ_2=3° with flexible and rigid parts.

TABLE 2. Comparison of mean values (X,Y) and Covariance of bivarite distribution C.P.

Mechanism component	mean X	mean Y	Variance Matrix
Rigid	263.72	111.79	$\begin{bmatrix} 0.3371 & -0.0433 \\ -0.0433 & 0.0084 \end{bmatrix}$
Flexible	264.79	111.61	$\begin{bmatrix} 0.5071 & -0.6541 \\ -0.6541 & 2.1673 \end{bmatrix}$

Table 2 and Figure 5 confirm that loading the flexible mechanism leads to shifts in the mean value and the variance of C.P. distribution. The shifts in mean values are 1.07 and -0.18 mm in X and Y direction, respectively. The distribution variances in X and Y directions have variations of 0.17 and +2.1589 mm^2, respectively. Therefore, the elliptic distribution contour is stretched in Y and X directions. The changes of the covariance, about 0.6108 mm^2, confirm that part flexibilities have a significant effect on covariance of the distribution.

Variations are calculated for one cycle of motion, i.e. θ_2 is changed from 0 to 360º with increment of 1º and obtained input variables are used in DLM. Therefore, the maximum normal error of mechanism with flexible component can be computed. The variation is shown in Figure 6.

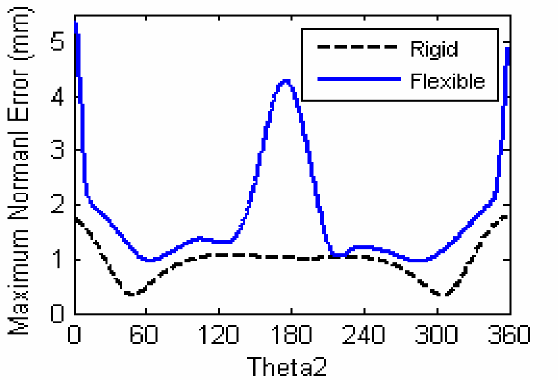

FIGURE 6. Comparison of maximum normal error.

5. MULTIPLE LINEAR REGRESSION MODEL

The maximum error of the mechanism is at θ_2=3°. In order to determine the maximum error function at this angle, the distribution of the error is obtained by changing the manufacturing tolerances within their specified limits. The error function is derived using multiple linear regression. For example, the error functions for the mechanism with rigid and flexible components computed at θ_2=3° are as follows, respectively:

$$Error_{rigid} = 0.05 + 0.35\mathrm{r}_2 + 3.34\mathrm{r}_p + .69\beta + 1.5068\theta_1 \quad (7)$$

$$Error_{flex} = 3.838 + 0.117r_2 + 0.049\beta + 0.2353\theta_1 + 0.263\theta_2 \quad .(8)$$

The valid-ness of the above relationship is verified by the normality test. According to before section, if P-value of test is less than 0.05 thus $H_0=\beta_i=0$ will rejected [6]. The P-value of above equations are 0.038 and 0.042, respectively. Therefore, these equations are valid.

In order to determine the percent contribution of each manufacturing variable on the error, β_i are divided by the summation of β_i and the sensitivity of each manufacturing variables is determined (Figures 7 and 8).

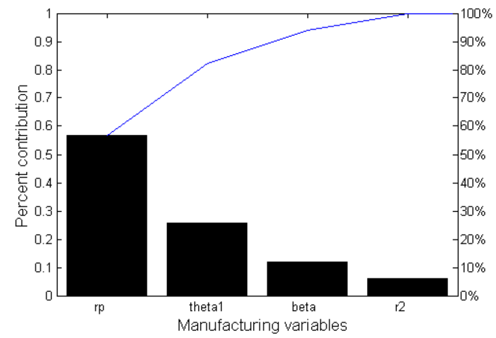

FIGURE 7. Present contribution of manufacturing variables at θ_2=3°(rigid component).

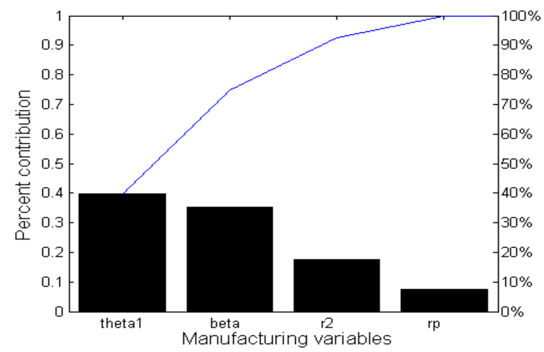

FIGURE 8. Present contribution of manufacturing variables at θ_2=3° (flexible component).

Figures 7 and 8 are shown which percent contributions of manufacturing variables are changed based on rigid or flexible component of mechanism. It can be observed that r_p and θ_1 have the major percent contribution of mechanism with rigid component whilst θ_1 and β on flexible mechanism is too. Thus, decreasing their tolerance limits can substantially decrease the error. Because of improving tolerance limits of angular dimension (i.e. θ_1, β) imposes manufacturing cost with higher rate, r_2 and r_p are chosen for decreasing the error and the modified tolerance limits are reported in Table 3.

TABLE 3. New tolerances of manufacturing variables (mm)

Manufacturing Variables	r_2	r_3	r_4	r_p	β	θ_1
Nominal Dimensions	250	400	25	104	80°	0°
Initial Tolerances	± 0.3	± 0.2	± 0.02	± 0.15	± 0.5°	± 0.5°
Modified Tolerances	± 0.15	± 0.2	± 0.02	± 0.07	± 0.5°	± 0.5°

Based on the improved parameters, the distributed of normal-to-path error at θ_2=3° along with the variation of maximum error for whole cycle are computed and depicted in Figures 9 and 10, respectively. Also, variation exerted on flexible mechanism and result of this variation is presented on Figures 11 and 12.

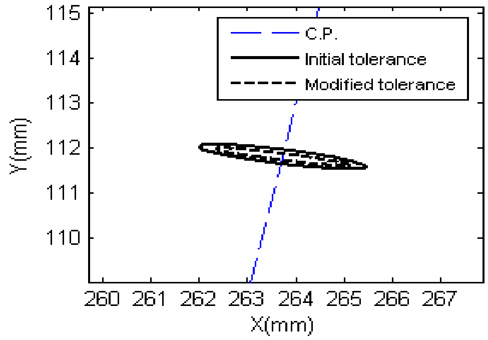

FIGURE 9. Comparison of bivariate distribution of C.P. at θ_2=3°,after and before modify (rigid component)

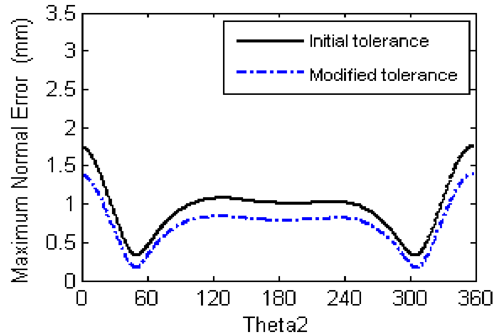

FIGURE 10. Variation of maximum error in cases, modified and initial tolerances (rigid component).

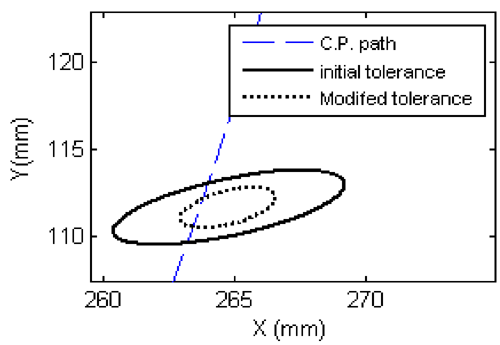

FIGURE 11. Bivariate distribution of C.P. position at θ_2=3°,after and before modify (flexible mechanism).

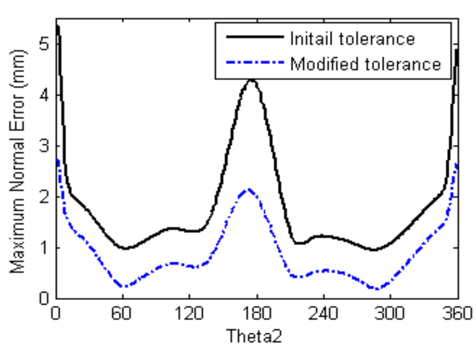

FIGURE 12. Variation of maximum error in cases, modified and initial tolerances (flexible component).

Variations of tolerance limits of input dimensions (r_2 and $r_{p)}$ result in variations of C.P. limit. For example at θ_2=3° shifts in mean values are 0.05 and 0 mm in X and Y directions, respectively. The variances in X and Y directions have the variations of -0.358 and -1.7986 mm^2, respectively. The results are summarized in Table 4.

TABLE 4. Comparison of mean values (X,Y) and Covariance of bivarite distribution C.P.

Mechanism component	mean X	mean Y	Variance Matrix
Initial Tolerances	264.79	111.61	$\begin{bmatrix} 0.5071 & -0.6541 \\ -0.6541 & 2.1673 \end{bmatrix}$
Modified Tolerances	264.75	111.61	$\begin{bmatrix} 0.1488 & -0.1199 \\ -0.1199 & 0.3687 \end{bmatrix}$

6. OPTIMIZATION OF ERROR AND MANUFACTURING COSTS

In this study, the tolerance limits of input variables are modified in order to reach optimized case where the total cost is minimized and the maximum error is kept within a certain limit. This can be accomplished by considering the error function equation (Eq. 8) and the cost equation (Eq. 9), simultaneously. Thus, it is possible to come up with minimum cost versus maximum error curve wich is depicted in Figure 13. Lt has been assumed that all of the components are produced by milling operations and fixed costs of product is suggested to be 2 dollars.

$$C = A + B/tol^k \qquad (9)$$

If data given in Table 3 is taken into consideration, Point A of Figure 13 will be the optimized solution.

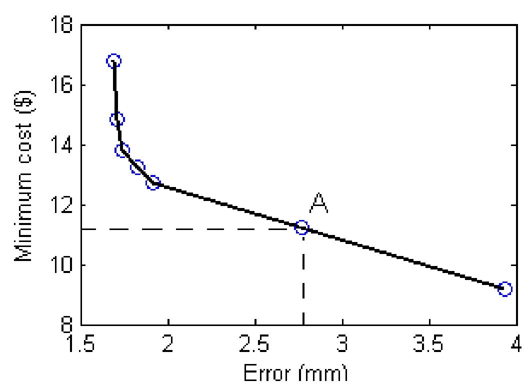

Figure 13. Optimized minimum cost versus max error

7. CONCLUSION

The Coupler Point (C.P.) position of a crank slider mechanism during one cycle of motion is considered as the assembly specification which has variations in two directions. The correlation between these variations also impresses the limit of variation. The bivariate distribution of the assembly specification is determined using the Direct Linearization Method (DLM). The valid range of DLM is firstly determined by means of the time-consuming Monte Carlo simulation and then the percent contribution of each input variable to assembly specification is computed by DLM. Thus, the most critical variables, which have the highest contribution to the variations of the assembly specification, can be recognized. By improving the tolerance limits of these critical variables and increasing area section r_2, the maximum error of mechanism can be decreased significantly. According to Figure 15, tolerance improvements result in 19% reduction of the maximum normal error at $\theta_2=3°$. In addition, the curve which represents optimize max error versus minimum cost is derived based on the regression equation and the cost function.

10. REFERENCES

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