Feed and acceleration region optimization during milling along free-form tool path

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1. Introduction

CNC machine tools can simultaneously drive two or more axes of motion along tool path computed by CAD/CAM softwares. Several algorithms have been developed to compute reference points for each axis in real time; due to inertia loads, torque and power limitations of each axis, it is necessary to improve the current algorithms.

In order to machine along a free-form tool path, most of machine controllers require that the tool path to be divided into linear and circular segments which approximate the real path. Path segmentation impose a severe computational burden on achieving optimum feed and acceleration rates.

Several curve interpolation algorithms have been investigated which are based on approximation methods. Farouki [1] introduced the idea of using Phythagorean-Hodograph (PH) curve in CNC interpolators which offers exact computation of reference points. In his recent research continuos variation of feed rate, and along with physical constraints have been investigated [2].

In this paper, physical constraints on each CNC axis are reviewed. Then, an improved quadratic feed rate algorithm based on arc-length variations is proposed in order to obtain the minimum acceleration length. Lastly, feed dependant cutting forces encountered in real machining are included in the developed algorithm which finds the maximum constant feed rate and the minimum acceleration length.

2. Torque and power constraints on axis drivers

The requirements for CNC axis drivers are: 1) To overcome inertia forces, 2) To act against cutting forces, and 3) To overcome friction forces. Among the above motion constraints, inertia forces play an important role in High-Speed Machining (HSM), therefor they must taken into consideration.

DC motor operation is governed by two fundamental equations:

$$T = K_T I \quad , \quad E = K_E \omega \tag{1}$$

where T and E are motor torque and back emf, respectively. The maximum magnitude of axis acceleration is determined by the motor current limit. In addition, the power demand for each axis can be considered proportional to the product of linear speed v and acceleration a. Thus, the following constraints must be satisfied which ensure trouble-free machining:

$$|a| \le a_{\max} \quad , |va| \le p_{\max} \tag{2}$$

3. Minimum feed acceleration arc- length

In CNC machining, one data block can be divided into three regions, that is acceleration, constant feed and deceleration regions. Constant feed algorithm along with physical constraints are thoroughly discussed in [2]. Feed rate in the acceleration region can be implemented as a linear or quadratic function of arc-length. Due to shortcomings of linear arc-length feed rates such as large jerk variations [3], quadratic arc-length profile is chosen for implementation. In the curvilinear motion, acceleration is given by $a = \ddot{r} = \dot{V}\hat{t} + \kappa V^2 \hat{n}$. Thus, in the acceleration region inequalities (2) can be written along x-and y-axis as follows [2]:

$$\begin{aligned}
&a_{x,\max} \pm \left(\kappa V^2 \sin\theta - \dot{V}\cos\theta\right) \ge 0 \\
&a_{y,\max} \pm \left(\kappa V^2 \cos\theta + \dot{V}\sin\theta\right) \ge 0 \\
&p_{x,\max} \pm V\cos\theta\left(\kappa V^2 \sin\theta - \dot{V}\cos\theta\right) \ge 0 \\
&p_{y,\max} \pm V\sin\theta\left(\kappa V^2 \cos\theta + \dot{V}\sin\theta\right) \ge 0
\end{aligned} \tag{3}$$

Quadratic feed rate relation of arc-length expressed as [1]:

$$V(s) = V_0\left(1-\frac{s}{S}\right)^2 + 2V_1\left(1-\frac{s}{S}\right)\left(\frac{s}{S}\right) + V_m\left(\frac{s}{S}\right)^2 \tag{4}$$

where s, S , V0 and V1 are arc-length, total arc-length, initial feed and a weight, respectively. The maximum allowable constant feed rate, V_m is computed in [2] by considering torque and power constraints. Feed acceleration can be defined as follows:

$$\dot{V} = V\frac{dV}{ds} = \frac{2}{S}\left[V_0\left(1-\frac{s}{S}\right)^2 + 2V_1\left(1-\frac{s}{S}\right)\left(\frac{s}{S}\right) + V_m\left(\frac{s}{S}\right)^2\right] \times \left[\left(V_1 - V_0\right)\left(1-\frac{s}{S}\right) + \left(V_m - V_1\right)\left(\frac{s}{S}\right)\right] \tag{5}$$

where dot denote derivative with respect to time. Substituting $\dot{V}$ in the relations (3) and considering $A = V_0 - 2V_1 + V_m$, $B = 2(V_1 - V_0)$, $C = As + BS$ eight constraints can be derived as:

$$\begin{aligned}
&F_{a,x}^{\pm}(\xi, S) = a_{x,\max} S^4 \pm \\
&\left(Cs + V_0 S^2\right)\left[k\sin\theta\left(Cs + V_0 S^2\right) - \left(As + C\right)\cos\theta\right] \ge 0 \\
&F_{a,y}^{\pm}(\xi, S) = a_{y,\max} S^4 \pm \\
&\left(Cs + V_0 S^2\right)\left[k\cos\theta\left(Cs + V_0 S^2\right) + \left(As + C\right)\sin\theta\right] \ge 0 \\
&F_{p,x}^{\pm}(\xi, S) = p_{x,\max} S^6 \pm \\
&\left(Cs + V_0 S^2\right)^2 \cos\theta\left[k\sin\theta\left(Cs + V_0 S^2\right) - \left(As + C\right)\cos\theta\right] \ge 0 \\
&F_{p,y}^{\pm}(\xi, S) = p_{y,\max} S^6 \pm \\
&\left(Cs + V_0 S^2\right)^2 \sin\theta\left[k\cos\theta\left(Cs + V_0 S^2\right) + \left(As + C\right)\sin\theta\right] \ge 0
\end{aligned} \tag{6}$$

Where θ, κ, s are functions of curve parameter ξ. The above constraints are applied to a quintic PH curve shown in Fig.1. Using the graphical method discussed in [2], the minimum acceleration region for $V_m = 7.3$ is obtained and depicted in Fig. 2. It is clear that the safe region is the area above $F_i(\xi, S)=0$ and to left of the curve $Q(\xi, S)=s(\xi)-S=0$. Using quadratic feed rate , minimum acceleration length is equal to in $S_{min}=4.25$ while linear feed rate result in $S_{min}=13.76$.

4. Cutting force constraints

In applications that cutting forces are considerably high, it is important to include cutting force constraints in the algorithm. The average cutting forces in milling operation is estimated by[4]:

$$F_T = \frac{C_p}{\pi} \cdot \frac{w s_0^{(1-m)} h^{\left(1-\frac{m}{2}\right)} Z}{D^{\left(1-\frac{m}{2}\right)}} \qquad kg \qquad (7)$$

where C_p , m are material constraints and s_0, h, w ,D are feed per tooth, depth of cut, width of cut and tool diameter, respectively. Feed rate dependant cutting power can be expressed by: $P_{cut}= F_T.V$. Thus, x- and y-axis power constraints including cutting forces can be expressed as:

$$I_x V_m^3 \left|K \sin\theta \cos\theta\right| + \frac{C_p}{\pi} \cdot \frac{w h^{\left(1-\frac{m}{2}\right)} Z^m}{N^{(1-m)} D^{\left(1-\frac{m}{2}\right)}} V_m^{(2-m)} \cos^2\theta \le P_{x,\max}$$

$$I_y V_m^3 \left|K \sin\theta \cos\theta\right| + \frac{C_p}{\pi} \cdot \frac{w h^{\left(1-\frac{m}{2}\right)} Z^m}{N^{(1-m)} D^{\left(1-\frac{m}{2}\right)}} V_m^{(2-m)} \sin^2\theta \le P_{y,\max} \qquad (8)$$

for constant feed rate region and,

$$I_x V \left|\dot{V} \cos^2\theta - V^2 \kappa \sin\theta \cos\theta\right| + \frac{c_p}{\pi} \cdot \frac{w h^{\left(1-\frac{m}{2}\right)} Z^m}{N^{(1-m)} D^{\left(1-\frac{m}{2}\right)}} V^{(2-m)} \cos^2\theta \le P_{x,\max}$$

$$I_y V \left|\dot{V} \sin^2\theta + V^2 \kappa \sin\theta \cos\theta\right| + \frac{c_p}{\pi} \cdot \frac{w h^{\left(1-\frac{m}{2}\right)} Z^m}{N^{(1-m)} D^{\left(1-\frac{m}{2}\right)}} V^{(2-m)} \sin^2\theta \le P_{y,\max} \qquad (9)$$

for acceleration feed rate region. Considering maximum motor acceleration $a_x=a_y=18$, and maximum motor power $p_x=p_y=30$, unit mass and material constant as $c_p=210$, m=0.28, the proposed algorithm result in Vm=6.54 for cutting along curve shown in Fig.1. The graphical optimization method is shown in Fig.3. Also, S_{min} considering linear feed rate variation is 11.35, refer to Fig.4. It is obvious that cutting force constraints decrease feed rate and increase acceleration region.

5. Result and discussion

Inertial loads, torque and power limitations, along with cutting force encountered in real machining are included in the optimization algorithm. The minimum acceleration arc-length is obtained using quadratic feed rate variations. Also, the maximum constant feed rate and minimum acceleration arc-length subjecting to feed dependant cutting force constraints are computed by the proposed algorithm.

6. References

[1] R.T. Farouki , S. Shah ,“ Real-Time CNC Interpolators for Pythagorean-Hodograph Curves ”, Computer Aided Geometrictric Design 13, 1996

[2] R.T. Farouki, Y.F. Tsai ,C.S. Wilson , “Physical Constraints on Feed rates and Feed Accelerations along Curved Tool Paths “ , Computer Aided Geometric Design , 17 , 2000

[3] Y.Altintas , “ Manufacturing Automation “ , Cambridge University Press , 2000

[4] A. Bhattacharyya, “Metal Cutting Theory and Application “ , New Center Book Agency , 1998

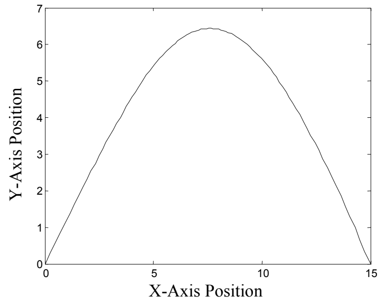

Fig.1 Quintic PH curve

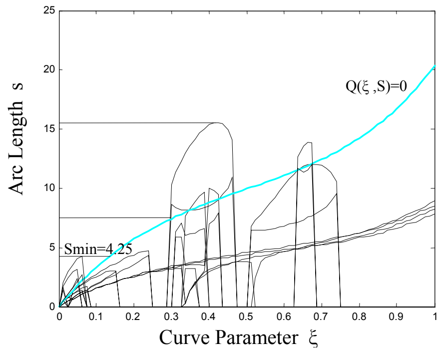

Fig.2 Constraint Curves Defined by Relations (6)

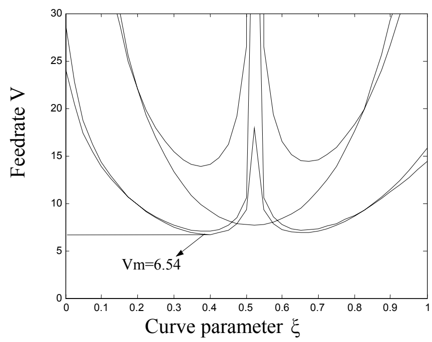

Fig.3 Constraint Curves Defined by Relations (8)

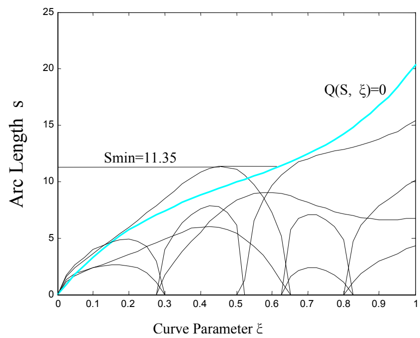

Fig. 4 Constraint Curves Defined by Relations (9)