Predicting the effects of cam manufacturing errors on the follower motion

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Abstract

In this study, a modern method for investigating the influence of manufacturing errors on the follower motion was evaluated and implemented. A 5th degree non-uniform periodic B-spline was used to approximate the cam contour in order to solve the three nonlinear equations of motion. The accuracy of the technique was confirmed by an analytical example. Finally, this technique was applied to predict the follower motion for a manufactured cam.

1. Introduction

Cam-follower systems are extensively used in motion control and production automation. Cam design and manufacturing have been dramatically improved in recent years. In mass production, manufacturing errors and assembly tolerances can not be avoided. Therefore, in design and manufacturing stages, it is very important to measure manufacturing errors and evaluating their impact on final product performance. In the past years, various attempts were made to investigate the influence of manufacturing errors of cam contours on the follower motion([1],[2],[3]) .

In this research, the effects of cam manufacturing errors on the follower motion were investigated by approximating the cam profile with B-splines of degree 5. For this purpose, first, translational follower motion equations were derived and then using these equations and digitized cam contour follower motion is computed. The effects of manufacturing errors on the follower motion were obtained by comparing computed motion with theoretical motion.

2. Deriving analytical follower motion equations for a general cam system

Based on the proposed method by Tsay in [6], three equations governing the motion of the follower for the system shown In Fig.1 were obtained as follow.

$$X(t)\cos\theta_c - Y(t)\sin\theta_c - r_f\cos\alpha_f = 0 \quad (1)$$

$$X(t)\sin\theta_c + Y(t)\cos\theta_c - r_f\sin\alpha_f - L_f = 0 \quad (2)$$

$$\left[X'(t)\cos(\theta_c) - Y'(t)\sin(\theta_c)\right]\cos(\alpha_f) + \left[X'(t)\sin(\theta_c) + Y'(t)Cos(\theta_c)\right]Sin(\alpha_f) = 0 \quad (3)$$

Where θ_c is the cam rotational angle, L_f is the follower rise and α_f is the cam-follower contact-point angle relative to the $S_f(x_f, y_f)$ moving coordinate system, *(X(t),Y(t))* and *(X'(t),Y'(t))* are cam profile and it's tangent vector relative to the coordinate system attached to the cam. Considering θ_c as input, t, l_f, α_f remain unknown which can be found by solving the above three nonlinear equations.

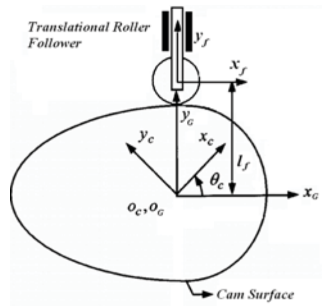


Fig.1- Coordinate system in translational cam-follower system

3. Non-uniform B-spline approach for cam profile approximation

In order to determine the follower displacement of the cam-follower mechanism shown in Fig.1 by solving Eqs. (1) to (3) first, second and third derivatives of cam surface are required. Thus, the degree of B-splines which approximate the cam contour must be greater than or equal to 4. In this research, Bezier curves of degree 5 have been used for constructing non-uniform periodic B-splines. The continuity conditions between two successive segments (i) and (i+1) of a composite curve are as follow.[4]

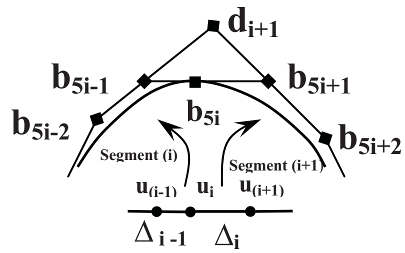


Fig.2- Two Bezier curves and their control polygon, b_{5i} is a junction point [4]

$$C^1 \rightarrow \Delta_i(b_{5i} - b_{5i-1}) = \Delta_{(i-1)}(b_{5i+1} - b_{5i}) \Rightarrow b_{5i} = \frac{\Delta_{(i-1)}}{\Delta_{(i-1)} + \Delta_i} \times b_{5i+1} + \frac{\Delta_i}{\Delta_{(i-1)} + \Delta_i} \times b_{5i-1}$$

$$C^2 \rightarrow \left(\frac{1}{\Delta_{(i-1)}}\right)^2 (b_{5i} - 2b_{5i-1} + b_{5i-2}) = \left(\frac{1}{\Delta_i}\right)^2 (b_{5i+2} - 2b_{5i+1} + b_{5i}) \quad (4,5)$$

Therefore C^2 continuity condition for a C^1 curve at u_i is the existence of a point like d_{i+1} such that

$$\begin{cases} b_{5i-1} = (1-t_i)b_{5i-2} + t_i d_{i+1} \\ b_{5i+1} = (1-t_i)d_{i+1} + t_i b_{5i+2} \end{cases} \quad (6)$$

$$C^3 \rightarrow f_1(b_{5i-1}, b_{5i-2}, b_{5i-3}) = g_1(b_{5i+1}, b_{5i+2}, b_{5i+3}) \quad (7,8)$$

$$C^4 \rightarrow f_2(b_{5i-1}, b_{5i-2}, b_{5i-3}, b_{5i-4}) = g_2(b_{5i+1}, b_{5i+2}, b_{5i+3}, b_{5i+4})$$

Due to lack of space, Eqs. (6),(7),(8) are summarized. Points b_{5i+5}, b_{5i} can be determined using Eq.(4) and knowing points $b_{5i+4}, b_{5i+6}, b_{5i+1}, b_{5i-1}$. For L Bezier curves there are total of 4L unknowns. Moreover, Eqs.(6)-(8) must be satisfied between every two successive segments including the first and last pieces. After establishing mentioned equations for all segments the following matrix equation will be derived.

$$A_{4L\times 4L}\,B_{4L\times 1} = C_{4LxL}\,D_{Lx1} \qquad \textbf{(9)}$$

Matrices A and C are the coefficient matrices. Matrix B includes unknowns $b_{5i+1}, b_{5i+2}, b_{5i+3}, b_{5i+4}$, $i = 0,\ldots.., L-1$

Solving above relation for B lead to:

$$B = A^{-1}C\,D \text{ , by letting } F = A^{-1}C = \left[f_{ij}\right] \text{ then } B = FD$$

Knowing matrix D, matrix B which includes control points will be found, therefore, each segment of Bezier curves will be determined completely.

Since the data set of the digitized cam contour is large and might be coupled with noises, the least square technique is employed for approximation. In the least square method, the following function must be minimized.[5]

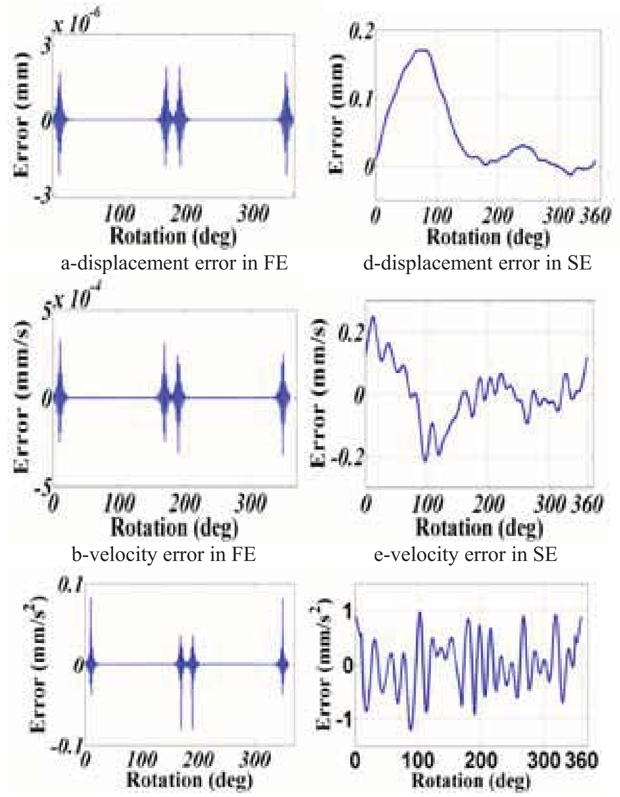
$$f = \sum_{k=1}^{m} \left|Q_k - C(\bar{u}_k)\right|^2 \quad , \; Q_k = data\ points \; , \quad k = 1,\ldots,m$$

Where $C(u_k)$ is the periodic B-spline developed by the method. In this study approximation with least square periodic non-uniform B-spline of degree 5 has been taken into consideration.

4. System implementation

To clarify the reliability and applicability of the developed procedure for computing follower motion, two examples were investigated. In the first example (FE) ,which demonstrates the accuracy of method, 720 data points on the cam profile were obtained by the analytical equations. In order to calculate follower motion by the mentioned technique a non-uniform periodic B-spline of degree 5 was fitted through these data points. The maximum errors for displacement, velocity and acceleration (Fig.3-a,b,c) are 0.002µm, 0.32µm/s, 80µm/s^2, respectively, which show the accuracy of the method. In the second example effects of manufacturing error on the follower motion is investigated. A 3-4-5 polynomial cam was manufactured and its contour was digitized by a Renishaw digitizer (Cyclone series II, the precision of the machine is 0.01 mm). 4887 data points are recorded for computing a translational roller follower motion.

The difference between computed curves and designed curves (Fig.3-d,e,f) are due to the manufacturing errors and the accuracy of measuring system. Regarding this fact that the cam was manufactured by CNC milling process, manufacturing errors are the consequences of tool deflection and vibration, tool wear, backlash in drive system and G-code accuracy for approximating spline tool path.



a-displacement error in FE | d-displacement error in SE

b-velocity error in FE | e-velocity error in SE

c -acceleration error in FE | f-acceleration error in SE

Fig.3- The results of First example(FE) and second example(SE)

5. Conclusions

In this research, a modern method for evaluating the influences of manufacturing errors on follower motion has been investigated. Based on the condition of contact between the cam and the follower, explained in Section 2, and fitting a non-uniform periodic B-spline of degree 5 to cam profile, the follower motion was predicted. Manufactured cam profile was digitized and approximated by non-uniform periodic B-spline. The follower motion was predicted using the developed technique. The accuracy of the method was verified by an analytical example. In addition, The effects of manufacturing errors of a manufactured cam on its follower motion was predicted by this method.

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