# Dimensional and Geometrical Tolerance Analysis of Kinematic Assemblies

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## Abstract
Tolerance analysis of assemblies promotes concurrent engineering by bringing engineering requirements and manufacturing capabilities together in a common model. This analysis can be used for optimization of assembly performance before manufacturing. It can provide a quantitative design tool for predicting the effects of manufacturing variations on the performance and cost in a feature based CAD environment. Seat belt fastener is a kinematic assembly which is analyzed in this paper. Among methods of tolerance analysis Direct Linearization Method (DLM) is selected and perform step by step on this assembly. Sensitive dimensional and geometrical tolerances are determined. In addition the percent of rejected assembly are calculated and what-if studies performed to assign tolerances

**Keywords***:* Kinematic Assemblies, Quality Control, Tolerance Analysis

## 1 Introduction
Sensitivity analysis of mechanical assemblies is a new branch of tolerance analysis, which is mostly discussed in statistical quality control; however, this topic is in a closed relation with kinematics and dynamics of machinery. Integration of tolerance analysis with CAD systems was firstly proposed by Chase et.al.[1] which improves the kinematic performance of mechanical assemblies. They proposed Direct Linearization Method (DLM) for tolerance analysis. This method constructs vector loops of the assembly model. It applies matrix algebra and root sum squares error analysis to estimate tolerance stackup in assemblies. Since, the computational complexity of DLM is low; it is suitable for iterative design tasks. Tolerance sensitivity values and tolerance stack-up expressions may also be derived automatically.
Sources of variations can be categorized as follows: the dimensional variations, the geometrical variations and the kinematic adjustments. Dimensional variations account for small changes in size due to manufacturing processes. Geometric variations describe changes in shape, location and orientation of features. Kinematic variations describe the propagation of variation through an assembly by small adjustments between mating parts. [1]
DLM can be integrated easily with feature based CAD systems which have the ability of extracting the closed and open loops equations of these assemblies and solving them automatically. In such systems, part features are related together by parametric constraining equations. Thus, feature modifications can be easily applied through whole model.

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## 2 DLM algorithm :

DLM is a systematic method which can be implemented on CAD systems by the following procedure: [2]

**2.1** Critical assembly dimensions are represented by vector loops, which are described by a set of nonlinear algebraic equations.

$$
\begin{aligned}
h_x &= l_1 \cos(\theta_1) + l_2 \cos(\theta_2) + ........ + l_n \cos(\theta_n) = 0 \\
h_y &= l_1 \sin(\theta_1) + l_2 \sin(\theta_2) + ........ + l_n \sin(\theta_n) = 0 \\
h_\theta &= \theta_1 + \theta_2 + ........ + \theta_n = 0
\end{aligned} \tag{1}
$$

First two equations are derived by summing x and y components of the vector loop and the third equation represented 360° rotation constraint on the vector loop. In the above equation $L_i$ and $\theta_i$ represents length and orientation of vector i in the loop, respectively.

**2.2** The linearized loop equations may be written in matrix form as follows:

$$[A]\{\delta X\} + [B]\{\delta U\} = \{0\} \tag{2}$$

Where [A] Is the matrix of partial derivatives with respect to the component variables, [B] is the matrix of partial derivatives with respect to the assembly variables, {δX} is the vector of small variations in the component dimensions, and {δU} is the vector of corresponding closed loop assembly variations. After solving the above equation with respect to {δU}, tolerance sensitivity is obtained:

$$\{\delta U\} = -[B^{-1}A]\{\delta X\} \tag{3}$$

The matrix $[B^{-1}A]$ is the matrix of tolerance sensitivities for the closed loop assembly variables.

**2.3** The resulting expression is used to calculate tolerance sensitivities by partial differentiation of closed loop with respect to part and assembly variable.

$$
\begin{aligned}
\delta h_x &= \frac{\partial h_x}{\partial l_1}\delta l_1 + \frac{\partial h_x}{\partial l_2}\delta l_2 + ........ + \frac{\partial h_x}{\partial l_n}\delta l_n + \frac{\partial h_x}{\partial \theta_1}\delta\theta_1 + \frac{\partial h_x}{\partial \theta_2}\delta\theta_2 + ........ + \frac{\partial h_n}{\partial \theta_n}\delta\theta_n \\
\delta h_y &= \frac{\partial h_y}{\partial l_1}\delta l_1 + \frac{\partial h_y}{\partial l_2}\delta l_2 + ........ + \frac{\partial h_y}{\partial l_n}\delta l_n + \frac{\partial h_y}{\partial \theta_1}\delta\theta_1 + \frac{\partial h_y}{\partial \theta_2}\delta\theta_2 + ........ + \frac{\partial h_y}{\partial \theta_n}\delta\theta_n \\
\delta h_\theta &= \frac{\partial h_\theta}{\partial l_1}\delta l_1 + \frac{\partial h_\theta}{\partial l_2}\delta l_2 + ........ + \frac{\partial h_\theta}{\partial l_n}\delta l_n + \frac{\partial h_\theta}{\partial \theta_1}\delta\theta_1 + \frac{\partial h_\theta}{\partial \theta_2}\delta\theta_2 + ........ + \frac{\partial h_\theta}{\partial \theta_n}\delta\theta_n
\end{aligned} \tag{4}
$$

It is important to note that the assembly variations {δU} depend on part variables {δX}, which are DLM independent variables.

**2.4** The validness of obtained vector loops are evaluated by a set of rules which can be summarized as follows:

- The loops include only those control dimensions which contribute to assembly variation.
- All dimensions in vector loops are datum referenced.
- Joint DOF depends on the joint type. 'Figure1'

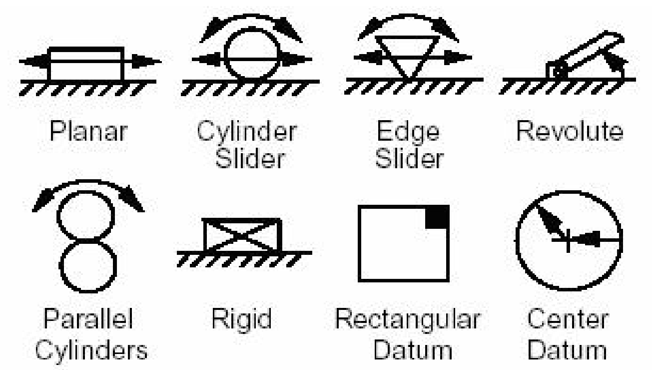



Figure 1: 2-D kinematic joint and datum type.[2]

**2.5** A comprehensive set of assembly tolerance requirements are introduces in terms of response functions (RF). In other words RF represent performance requirement of the assembly. Standard experimentations and the designer expertise determine the performance requirement or RF.

$$\delta RF = \frac{\partial RF}{\partial l_1}\delta l_1 + \frac{\partial RF}{\partial l_2}\delta l_2 + \ldots\ldots + \frac{\partial RF}{\partial l_n}\delta l_n + \frac{\partial RF}{\partial \theta_1}\delta\theta_1 + \frac{\partial RF}{\partial \theta_2}\delta\theta_2 + \ldots\ldots + \frac{\partial RF}{\partial \theta_n}\delta\theta_n \quad (5)$$

**2.6** Differentiation of complicated assembly expression is replaced by a single matrix operation*.*

**2.7** Geometrical tolerances like position tolerance, parallelism error or profile variation may be included in a vector loop assembly model.

## 3 Seat belt fastener components

Seat belt fastener components can be classified as bellow: 'Figure 2' [6]

**a)** **Frame:** this part is the most important part of the seat belt fastener assembly because all parts of the assembly connected to it directly or indirectly. Walls of the U form frame contain holes and slots which all of them perform special task.

**b)** **Plastic Frame:** This part is a revolute support for Plastic Tongue. It is placed on the bottom of the Frame.

**c)** **Plastic Tongue:** The task of this part is to guide the pin in L form slots of Frame. Plastic Tongue performs its task by means of its curved profile.

**d)** **Metal Tongue:** In the lock state of the fastener, this part is engaged with Frame and Main Tongue, thus it prevents the fastener from releasing.

**e)** **Pin:** This part is guided in the L-form slots of Frame and applies pressure on the top of Metal Tongue as a result prevents it from retracting.

**f)** **Plastic Plunger:** The task of this part is to release the assembly from the lock state.

**g)** **Main Tongue:** The Main Tongue has a secure joint with belt and when buckled up protects the passenger.

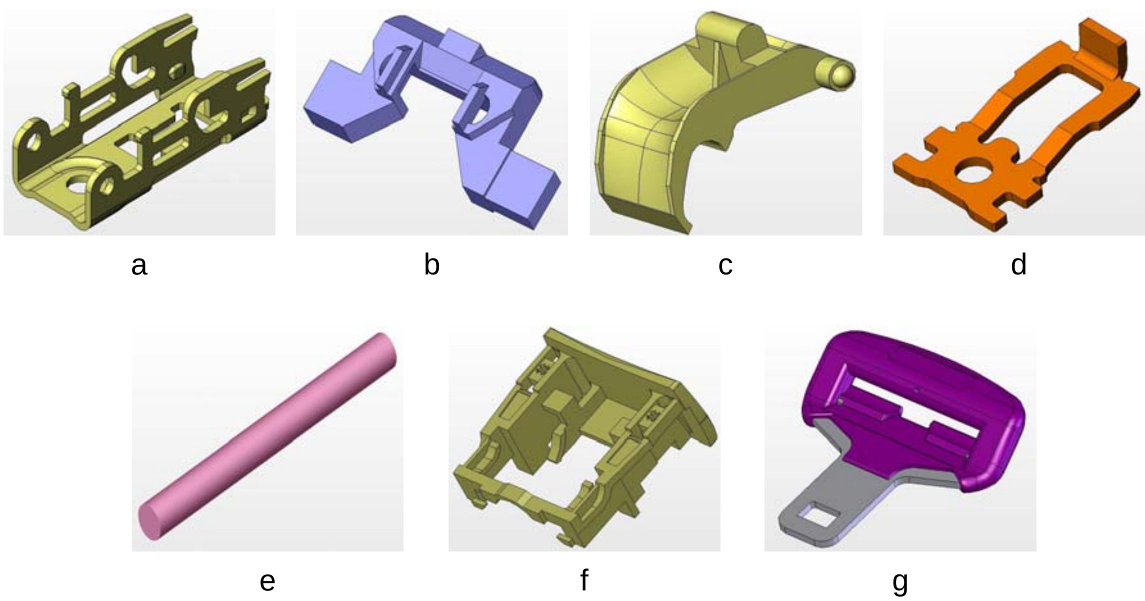



Figure 2: Seat Belt Fastener components

## 4 Applying DLM on Seat Belt Fastener

Having identified performance parameters of the assembly, DLM is applied on Seat Belt Fastener assembly as bellow:

**4.1** The first step of the DLM algorithm is to identify the number of effective parts on the assembly performance [2]: Number of parts = 7

The case which study in this paper is locked state of the assembly occurred when passenger is buckled up.

**4.2** The second step in DLM method is to determine the joint type and its degree of freedom (DOF). 'Table1'

**4.3** The next step is to calculate the required number of vector loops.

$$\text{Number of Joints} - \text{Number of Parts} + 1 = \text{Number of Loops} \tag{6}$$

Table 1

| DOF | Joint Type | Engaged Parts |
|---|---|---|
| DOF=2 | Edge Slider | Main Tongue with Frame |
| DOF=1 | Revolute | Plastic tongue with plastic Frame |
| DOF=1 | Planar | Main Tongue with Frame |
| DOF=2 | Edge Slider | Main Tongue with Metal Tongue |
| DOF=1 | Parallel Cylinder | Pin With Metal Tongue |
| DOF=2 | Cylinder Slider | Pin With Plastic Plunger |
| DOF=2 | Cylinder Slider | Pine with Frames L form profile |
| DOF=1 | Parallel Cylinder | Pin with Plastic tongue |
| DOF=0 | Fix | Plastic Plunger With Frame |

**4.4** The last step of the DLM algorithm is to derive the vector loop equations. 'Figure 3' [6]

$$\begin{aligned} h_x :\ & za_2 + u_3\cos(\beta_1+\beta_2) + sp_1\cos(180+\beta_1+\beta_2) + sp_0\cos(90+\beta_1+\beta_2) + \\ & zf_4\cos(90+\beta_1+\beta_2-180+\phi_1) + zf_2\cos(90+\beta_1+\beta_2+\phi_1) + \\ & u_2\cos(90+\beta+\beta_2+\phi_1+\theta) + za_1\cos(90+\beta_1+\beta_2+\phi_1+\theta+\phi_2) = 0 \end{aligned} \tag{7}$$

$$\begin{aligned} h_y :\ & u_3\sin(\beta_1+\beta_2) + sp_1\sin(180+\beta_1+\beta_2) + sp_0\sin(90+\beta_1+\beta_2) + \\ & zf_4\sin(90+\beta_1+\beta_2-180+\phi_1) + zf_2\sin(90+\beta_1+\beta_2+\phi_1) + \\ & u_2\sin(90+\beta+\beta_2+\phi_1+\theta) + za_1\sin(90+\beta_1+\beta_2+\phi_1+\theta+\phi_2) = 0 \end{aligned}$$

$$h_\theta : \phi_1 + \theta + \phi_2 + \beta_1 + \beta_2 = cte$$

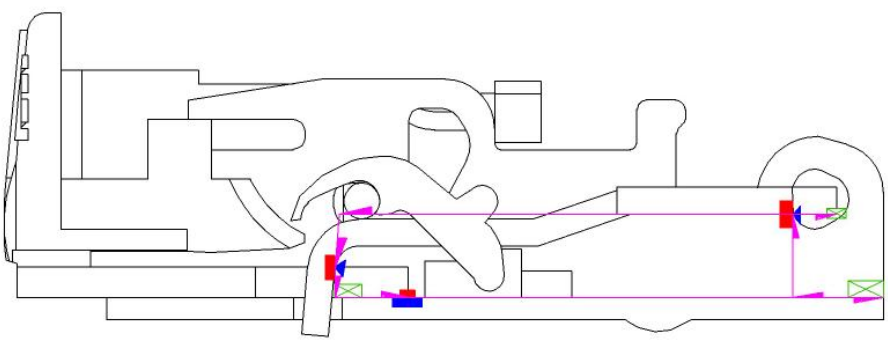


Figure 3: First closed loop diagram [6]

For applying DLM on Sear Belt Fastener assembly four closed loop equations are required. As sample calculation, the first closed loop equations are mentioned thoroughly.

In addition to dimensional tolerances these equations include geometrical tolerances [4]. The dimensional tolerances in kinematic assemblies are in form of length and angular variations which can be simply included in the closed loop equations as a vector. But the effects of geometrical variations on the dimensional assembly parameters must be analysed and considered in form of length or angular variation in the method.

First step in solving the assembly equations of variations $\{\delta U\}$ is to determine nominal values of $U_i$. These values are dimensional assembly parameters which can be calculated after the parts are assembled together. In simple assemblies these parameters can be obtained easily from algebraic equations; however in complex assemblies such as Seat Belt Fastener extracting and solving these equations are very difficult and time consuming. In such cases, DLM can be integrated with CAD systems in order to use the CAD ability to extract $U_i$ values. In this paper, firstly all parts are modelled using SolidWorks®2005 and then assembled together with proper mating constraints 'Figure 4'. The level of precision in modelling and mating the parts affects the accuracy of the results. At this stage, $U_i$ values can be extracted from the SolidWorks assembly model with default software accuracy. Matrices A and B are calculated for the assembly closed loop equations then $\{\delta U\}$ is obtained using equation (3):

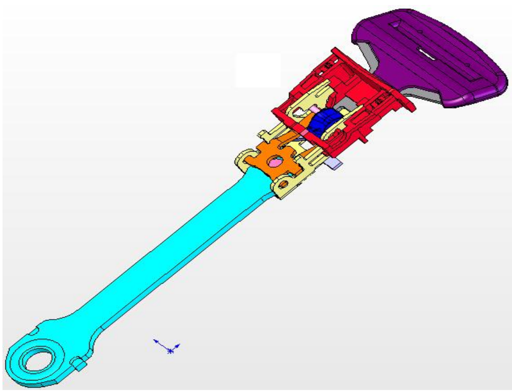


Figure 4: Seat Belt Assembly [6]

$$\delta u = \begin{bmatrix} \delta u_2 = 0.4561 \\ \delta u_3 = 0.2174 \\ \delta u_4 = 0.1568 \\ \delta u_5 = 0.4387 \\ \delta u_6 = 0.4169 \\ \delta\phi_1 = 0.0120 \\ \delta\phi_2 = 0.0287 \\ \delta\phi_4 = 0.0427 \\ \delta\phi_6 = 0.0514 \end{bmatrix} \quad (8)$$

One of the performance requirement or RF in the locked state of the Fastener is angle between the bent of the Metal Tongue and the line perpendicular to Main Tongue represented by $\phi_2$ 'Figure 5'. This RF is chosen according to standard releasing test of Fastener assembly. From calculated $\{\delta U\}$, the variation of $\phi_2$ is $\delta\phi_2$=0.0287 rad.

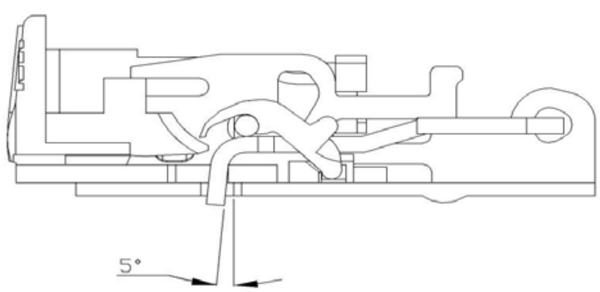


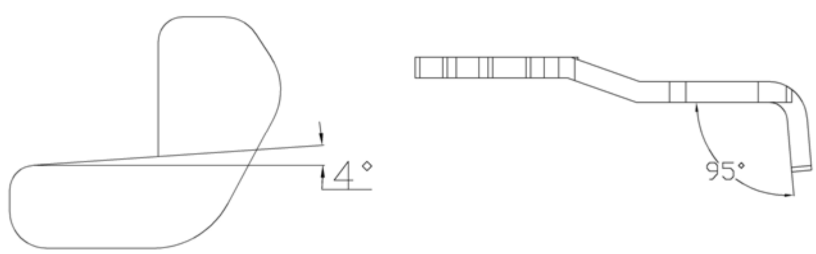

Figure 5: Location of selected RF

In addition to the assembly variations {δU}, the percent of contribution of each part dimension are calculated using the following relation [3,5]:

$$\%Contribution = \frac{(\frac{\partial RF}{\partial l_j}\delta l_j)^2}{\sum(\frac{\partial RF}{\partial l_i}\delta l_i)^2} \tag{9}$$

The contribution values tell the designer how each dimension contributes to the assembly variation (RF). The percent of contribution is calculated based on RSS variation estimation. The equation (9) reveals that the angles $\alpha$ and $\theta$ which are illustrated in 'Figure 5' have the maximum contribution to the RF variations:

Contribution of $\alpha$ = 14% (10)
Contribution of $\theta$ = 84%
Contribution of the rest = 2%




a: $\alpha$ b: $\theta$

Figure 5: $\alpha$ and $\theta$ locations

More details will be found in [6].
The above combination of contribution in which one or two part dimension variations are dominant is not suitable case, due to the fact that small changes in these dimensions will result in assembly failure during operation. Part tolerances must be optimized in order to decrease the rejected percent of assemblies. Tolerances of $\alpha$ and $\theta$ which have the maximum contributions are as follows:

$\delta\alpha = \pm 30'$ (11)
$\delta\theta = \pm 1.5°$

These values extracted from part detail drawings. The nominal value of RF and its tolerances are as bellow:

Nominal RF value = $5^\circ$ = 0.0872 rad (12)
RF upper limit = $7^\circ$ = 0.122 rad
RF lower limit = $4^\circ$ = 0.0698 rad

Using RSS method with above variations, the number of rejected assembly will be determined:

$$Z_{UL} = \frac{UL - \mu_{Gap}}{\sigma_{Gap}} = \frac{0.122 - 0.087}{.0096} = 3.65\sigma \Rightarrow R_{UL} = 100\,PPM \qquad (13)$$

$$Z_{LL} = \frac{LL - \mu_{Gap}}{\sigma_{Gap}} = \frac{0.0698 - 0.087}{.0096} = -1.8\sigma \Rightarrow R_{LL} = 35900\,PPM$$

## 5 Result review

Improving the quality along with decreasing the cost are challenging problems in manufacturing. Results of equation (13) reveal that there are 100 PPM rejects at the upper limit and 35900 PPM rejects at the lower limit. The amount of the rejects are not acceptable and a burden on the manufacturing expenses. The tolerance zone of the dimension with maximum contribution must be decreased. ‘Table 2’ illustrates that how changing the tolerance zone of the effective part dimensions will affect the number of rejects and decrease the assembly rejects.

Table 2

| | First case | Second case | Third case |
|---|---|---|---|
| Suggested value for $\delta\alpha$ | ±30’ | ±30’ | ±20’ |
| Suggested value for $\delta\theta$ | ±30’ | ±20’ | ±20’ |
| UL rejects | 0 PPM | 0 PPM | 0 PPM |
| LL rejects | 200 PPM | 100 PPM | 1 PPM |

**First case:** $\delta\alpha$ remain constant and $\delta\theta$ decrease to ±30’. It cause to UL rejects decrease from 100 to 0 and LL rejects decrease from 35900 to 200. These are reasonable rejects in manufacturing process.

**Second case:** $\delta\alpha$ remain constant and $\delta\theta$ decrease to ±20’. It cause to UL rejects decrease from 100 to 0 and LL rejects decrease from 35900 to 100. These tolerance zones are very tight and cause to cost increasing.

**Third case:** In the last case $\delta\alpha$ and $\delta\theta$ decrease to ±20’. It cause to UL rejects decrease from 100 to 0 and LL rejects decrease from 35900 to 1. In this case the rejects decrease but it will result in increasing the cost .This case is not recommended.

## Conclusion

In this research DLM is implemented on the seat belt fastener assembly and the following tasks are performed:

- Performing sensitivity studies to identify the critical sources of variation.
- Predicting percent of assemblies which will fail.
- Performing “what-if” studies and assigning tolerances throughout an assembly to minimize rejects.

Also DLM helps engineers and designers understand the effects and the importance of manufacturing tolerance early in the design process. It provides a quantitative tool for evaluating the consequences of manufacturing tolerances on assembled products. It can serve as a design tool by using estimated process variation to assign tolerances. It can also be used with actual process data to determine the affects of manufacturing variation on assemblies.

## Acknowledgment

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## References

[1] K. W. Chase, J. Gao, and S. P. Magleby, “General 2-D Assembly Tolerance Analysis of Mechanical Assemblies with Small Kinematic Adjustments,” J. of Design and Manufacturing, VOL 5, No.4. 1995

[2] K. W. Chase, J. Gao , and S. P. Magleby,”Tolerance Analysis of 2-D and 3-D Assemblies with Kinematic Adjustment , “chap.7 Basic Tools for Tolerance Analysis of Mechanical Assemblies, John Wiley , 1998, pp .103-137

[3] C.M. Creveling, ” Tolerance Design : a Handbook for Developing Optimal Specification “, Addison-Wesley ,1997

[4] Jeffrey G. Dabling, ” Incorporating Geometric Feature Variation with Kinematic Tolerance Analysis of 3D Assemblies”, M.Sc. Thesis, Department of Mechanical Engineering. Brigham Young University, December 2001

[5] M.F. Spotts , ”Dimensioning and Tolerancing for Quantity Production”, Prentice Hall , Englewood Cliffs ,New Jersey. 1983

[6] B.M. Imani, S.A. Hosseini, N. Irani and M. Gholami, ”3D Tolerance Analysis of Seat Belt Fastener Assembly”, Internship Project Report in Persian, Ferdowsi University of Mashhad, Engineering Faculty, Mechanical Engineering Department, 2004