Recursive Maximum SINR Blind Beamforming Algorithm for CDMA systems

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***Abstract*— Maximizing signal-to-interference plus noise ratio (SINR) at the output of beamformer significantly improves the performance of the receiver using an antenna array. In this paper, we propose a recursive maximum SINR (RMSINR) blind beamforming algorithm for coded-division multiple access (CDMA) systems. First, a blind beamformer algorithm called direct maximum SINR (DMSINR) that maximizes the SINR directly is introduced. Then, to reduce the computational complexity of the DMSINR algorithm, the RMSINR algorithm is developed, which achieves the maximum of SINR under a linear constraint in a recursive manner. In addition to a significant complexity reduction, as simulation results show, the RMSINR algorithm attains a very fast convergence rate with the high ability of tracking direction of arrival in time varying environments.**

***Index Terms* — Blind beamforming, CDMA systems, maximum signal-to-interference plus noise ratio (MSINR), recursive maximum SINR.**

I. Introduction

As the development of wireless communication systems evolves, advanced techniques are employed to achieve the demand for services with higher speed and quality. Many advantages of beamforming in array systems make it a desirable approach for increasing the capacity of wireless communications [1]. Two approaches are generally used in digital beamforming, training-based and blind methods. The training-based is suffering from the loss of bandwidth efficiency especially in time varying environments due to occupying a part of bandwidth by the training data. Blind beamforming, on the other hand, is an efficient approach to preserve the spectrum. However, blind beamforming algorithms are complex or have slow convergence rates that limit the applications of this approach.

A group of beamforming methods is based on the constant modulus algorithm (CMA). Stochastic gradient descent CMA (SGD-CMA), least squares CMA (LS-CMA), orthogonal CMA (O-CMA) and recursive least squares CMA (RLS-CMA) are adaptive algorithms that have been proposed for blind beamforming under the framework of the CMA [2]-[4]. Although the proposed blind beamforming algorithms based on the CMA are low complex and simple, their convergence rates are slow such that the RLS-CMA, whish has the fastest convergence rate, requires more than one hundred symbols to converge [4]. Moreover, the CMA criterion is not related to the SINR maximization that is a receiver performance improvement criterion.

Beamforming based on the maximum SINR significantly improves the performance of the CDMA systems [5]. It has been shown in [5] that maximizing SINR is equivalent to maximize the ratio of the despread signal power to the spread signal power at the output of beamformer. Based on this result an adaptive blind beamforming method has been proposed in [5] by using SGD algorithm. However, the convergence rate of the proposed algorithm in [5] is slow due to the nature of the SGD algorithm.

In this paper, first we develop a blind beamforming algorithm called direct maximum SINR (DMSINR) that maximizes the SINR in a direct manner. Then, to reduce the computational complexity of the DMSINR algorithm, another algorithm called recursive maximum SINR (RMSINR) is proposed based on the linear constrained optimization approach that achieves maximum of the SINR recursively. The RMSINR algorithm, which employs the recursive least squares method, has a very fast convergence rate,

The rest of the paper is organized as follows. A CDMA system description with a general SINR definition is given in Section II. The DMSINR algorithm is introduced in Section III and in Section IV the RMSINR is developed. The performance of the DMSINR and the RMSINR blind beamforming algorithms are evaluated and compared by computer simulations in Section V. Finally, conclusion remarks are given in Section VI.

II. System discription

We consider a CDMA system in which an antenna array with M-element is employed. When G is the processing gain and $\mathbf{a}_{j,k}(n)$ is the vector channel for jth user at the kth path, $\mathbf{x}_{j,k}(l)$, the received signal vector through the kth path

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at time $l = Gn - l'$ (for $0 \le l' \le G-1$ and $-\infty < n < \infty$) is given as

$$\mathbf{x}_{j,k}(l) = A_j\, b_j(l)\, \mathbf{a}_{j,k}(n) + \mathbf{u}_{j,k}(l) \tag{1}$$

where $\mathbf{u}_{j,k}(l)$ is the interference signal from other users plus noise and $b_j(l)$ is defined as

$$b_j(l) = d_j(n) c_j(Gn + l') \tag{2}$$

where $d_j(n)$ is the nth symbol transmitted by the jth user and $c_j(Gn + l')$ is the l'th chip of the spreading sequence for the nth symbol of the jth user. The despread signal vector for the kth path of the jth user is given as

$$\begin{aligned} \mathbf{y}_{j,k}(n) &= \frac{1}{\sqrt{G}} \sum_{l'=0}^{G-1} c_j(Gn + l')\, \mathbf{x}_{j,k}(Gn + l') \\ &= \sqrt{G}\, \mathbf{s}_{j,k}(n) + \boldsymbol{\zeta}(n) \end{aligned} \tag{3}$$

where

$$\mathbf{s}_{j,k}(n) = A_j\, d_j(n)\, \mathbf{a}_{j,k}(n) \tag{4}$$

$$\boldsymbol{\zeta}(n) = \frac{1}{\sqrt{G}} \sum_{l'=0}^{G-1} c_j(Gn + l')\, \mathbf{u}_{j,k}(Gn + l') \tag{5}$$

Defining $\mathbf{w}_{j,k} = [w_{j,k}(1), \cdots, w_{j,k}(M)]^T$ as the weight vector of the beamformer, the output of the beamformer for the kth path of the jth user becomes

$$\mathbf{z}_{j,k}(n) = \mathbf{w}_{j,k}^H \mathbf{y}_{j,k}(n) \tag{6}$$

where $(\cdot)^T$ and $(\cdot)^H$ denote transposed and transposed complex conjugate, respectively.
Due to simplicity and without loss of generality, we drop the subscripts j and k. By substituting (3) in (6), $\sqrt{G}\mathbf{w}^H \mathbf{s}(n)$ and $\mathbf{w}^H \boldsymbol{\zeta}(n)$ are the desired signal and the interference plus noise, respectively. We employ the signal-to-interference plus noise (SINR) maximization as a criterion for estimating the weight vector of the beamformer, $\mathbf{w}$.

$$\hat{\mathbf{w}} = \arg\max_{\mathbf{w}} \frac{E\left[\left|\mathbf{w}^H \mathbf{s}(n)\right|^2\right]}{E\left[\left|\mathbf{w}^H \boldsymbol{\zeta}(n)\right|^2\right]} \tag{7}$$

From (5), it is straightforward to show that

$$E\left[\left|\mathbf{w}^H \boldsymbol{\zeta}(n)\right|^2\right] = \mathbf{w}^H \mathbf{R}_{uu} \mathbf{w} \tag{8}$$

where $\mathbf{R}_{uu}$ is autocorrelation function of $\mathbf{u}(l)$. Defining $\mathbf{R}_{ss}$ as the autocorrelation of the $\mathbf{s}(n)$, the estimation criterion of $\mathbf{w}$ becomes

$$\hat{\mathbf{w}} = \arg\max_{\mathbf{w}} \frac{\mathbf{w}^H \mathbf{R}_{ss} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{uu} \mathbf{w}} \tag{9}$$

Since the autocorrelation matrices of the $\mathbf{s}(n)$ and $\mathbf{u}(l)$ are not available at the receiver, the $\mathbf{w}$ cannot be estimated directly from (9). It has been shown in [5] that maximization of $\mathbf{w}^H \mathbf{R}_{ss} \mathbf{w} / \mathbf{w}^H \mathbf{R}_{uu} \mathbf{w}$ in (9) is equivalent to maximization of $\mathbf{w}^H \mathbf{R}_{yy} \mathbf{w} / \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w}$, where $\mathbf{R}_{yy}$ and $\mathbf{R}_{xx}$ are the autocorrelation functions of $\mathbf{y}(n)$ and $\mathbf{x}(l)$, respectively. Thus the criterion of $\mathbf{w}$ estimation is equivalent to

$$\hat{\mathbf{w}} = \arg\max_{\mathbf{w}} \frac{\mathbf{w}^H \mathbf{R}_{yy} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{xx} \mathbf{w}} \tag{10}$$

III. DIRECT MAXIMUM SINR BLIND BEAMFORMING ALGORITHM

To develop the blind beamforming in a time varying environment which is a general case in mobile communication systems, we replace the E operation in (10) by the weighted sum as follows

$$\hat{\mathbf{w}}(n) = \arg\max_{\mathbf{w}} \frac{\sum_{k=0}^{n} \lambda^{n-k} \mathbf{w}^H \mathbf{y}(k) \mathbf{y}(k)^H \mathbf{w}}{\sum_{k=0}^{n} \sum_{l'=0}^{G-1} \lambda^{n-k} \mathbf{w}^H \mathbf{x}(Gk + l') \mathbf{x}(Gk + l')^H \mathbf{w}} = \frac{N(n)}{D(n)} \tag{11}$$

where $0 < \lambda \le 1$ is a forgetting factor. Defining $\mathbf{x}^{(m)}(k) = [x^{(m)}(Gk), x^{(m)}(Gk-1), \cdots, x^{(m)}(Gk-G+1)]$ as the received vector by the mth antenna and $X(k) = [\mathbf{x}^{(1)}(k)^T, \mathbf{x}^{(2)}(k)^T, \cdots, \mathbf{x}^{(M)}(k)^T]^T$, the denominator of (11) can be written as

$$\begin{aligned} D(n) &= \lambda^n \mathbf{w}^H \left(\sum_{k=0}^{n} \lambda^{-k} X(k) X(k)^H \right) \mathbf{w} \\ &= \lambda^n \mathbf{w}^H \mathbf{X}(n) \Lambda(n) \mathbf{X}(n)^H \mathbf{w} \end{aligned} \tag{12}$$

where $\mathbf{X}(n) = [X(0), X(1), \cdots, X(n)]$ is a $M \times G(n+1)$ matrix, $\Lambda(n) = \mathrm{diag}(I_G, \lambda^{-1} I_G, \cdots, \lambda^{-n} I_G)$ is a $G(n+1) \times G(n+1)$ diagonal matrix and I_G is the $G \times G$ identity matrix. The despread received signal at the mth antenna is given as

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$$y^{(m)}(k) = \mathbf{x}^{(m)}(k)\,\mathbf{c}(k) \tag{13}$$

where $\mathbf{c}(k) = \frac{1}{\sqrt{G}}[c(Gk;0), c(Gk;1), \cdots, c(Gk;G-1)]^T$.

From (3) and using (13), we have

$$\mathbf{y}(k) = X(k)\mathbf{c}(k) \tag{14}$$

By using (14), the numerator of (11) can be performed as

$$N(n) = \lambda^n \mathbf{w}^H \left(\sum_{k=0}^{n} \lambda^{-k} X(k)\mathbf{c}(k)\mathbf{c}(k)^H X(k)^H \right)\mathbf{w}$$
$$= \lambda^n \mathbf{w}^H \mathbf{X}(n)\mathbf{C}(n)\Delta(n)\mathbf{C}(n)^H \mathbf{X}(n)^H \mathbf{w} \tag{15}$$

where $\Delta(n) = \mathrm{diag}(1, \lambda^{-1}, \cdots, \lambda^{-n})$ is a $(n+1)\times(n+1)$ diagonal matrix and $\mathbf{C}(n)$ is a $G(n+1)\times(n+1)$ matrix that is defined as

$$\mathbf{C}(n) = \begin{bmatrix} \mathbf{c}(0) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{c}(1) & \cdots & \vdots \\ \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{c}(n) \end{bmatrix} \tag{16}$$

Defining $\mathbf{Q}_n = \mathbf{X}(n)\mathbf{C}(n)\Delta(n)\mathbf{C}(n)^H \mathbf{X}(n)^H$, since $\mathbf{Q}_n$ is a Hermittan positive definite matrix, it can be performed as $\mathbf{Q}_n = \mathbf{\Gamma}_n^H \mathbf{\Gamma}_n$, where $\mathbf{\Gamma}_n$ is a $M \times M$ square matrix. When define

$$\mathbf{v}(n) = \mathbf{\Gamma}_n \mathbf{w} \tag{17}$$

The nominator of (11) becomes

$$N(n) = \lambda^n \mathbf{v}(n)^H \mathbf{v}(n) \tag{18}$$

From (17), we have

$$\mathbf{w} = \mathbf{\Gamma}_n^{-1}\mathbf{v}(n) \tag{19}$$

By substituting $\mathbf{w}$ from (19) in (12) and define $\mathbf{P}_n = \mathbf{\Gamma}_n^{-H}\mathbf{X}(n)\mathbf{\Lambda}(n)\mathbf{X}(n)^H\mathbf{\Gamma}_n^{-1}$, the denominator of (11) becomes

$$D(n) = \lambda^n \mathbf{v}(n)^H \mathbf{P}_n \mathbf{v}(n) \tag{20}$$

By substituting (18) and (20) in (11), the estimation criterion of $\mathbf{w}$ is given as

$$\hat{\mathbf{w}}(n) = \arg\max_{\mathbf{w}} \frac{\mathbf{v}(n)^H \mathbf{v}(n)}{\mathbf{v}(n)^H \mathbf{P}_n \mathbf{v}(n)} \tag{21}$$

From (21), the estimation of beamformer weight vector, $\mathbf{w}$, at time n based on the defined criterion in (11) becomes

$$\hat{\mathbf{w}}(n) = \mathbf{\Gamma}_n^{-1}\mathbf{u}_{\min} \tag{22}$$

where $\mathbf{u}_{\min}$ is eigenvector of $\mathbf{P}_n$ corresponding to the minimum eigenvalue of $\mathbf{P}_n$.

IV. RECURSIVE BLIND BEAMFORMING BASED ON SINR MAXIMIZATION

Although the developed algorithm in Section III is able to estimate the weight vector of the beamformer at each time based on maximizing SINR in a blind manner, it is a complex algorithm due to involving matrix decomposition. In this Section we would like to drive an algorithm that estimate the weight vector in a recursive manner with less complexity.

We define following cost function, which is based on SINR maximization, by minimizing the denominator of (11) subject to keeping constant the numerator of (11).

$$\tilde{\mathbf{w}}(n) = \arg\min_{\mathbf{w}} \left\{ \mathbf{w}^H \mathbf{X}(n)\mathbf{\Lambda}(n)\mathbf{X}(n)\mathbf{w} + \beta_n \left(1 - \mathbf{w}^H \mathbf{Q}_n \mathbf{w}\right) \right\} \tag{23}$$

where β_n is the Lagrange multiplier for constraint $\mathbf{w}^H \mathbf{Q}_n \mathbf{w} = 1$. Due to the defined constraint in (23) that is a nonlinear function of $\mathbf{w}$, estimating procedure becomes complicate for computing β_n [5]. To estimate $\mathbf{w}$ with less complex adaptive procedure, we modify the constrained optimization, (23), as follows

$$\tilde{\mathbf{w}}(n) = \arg\min_{\mathbf{w}} \left\{ \mathbf{w}^H \mathbf{X}(n)\Lambda(n)\mathbf{X}(n)^H \mathbf{w} + \beta_n \left(1 - \mathbf{w}^H \mathbf{Q}_n \tilde{\mathbf{w}}(n-1)\right) \right\} \tag{24}$$

Based on criterion defined in (24), one can show that [6]

$$\tilde{\mathbf{w}}(n) = \left(\tilde{\mathbf{w}}(n-1)^H \mathbf{Q}_n \left(\mathbf{X}(n)\Lambda(n)\mathbf{X}(n)^H\right)^{-1} \mathbf{Q}_n \tilde{\mathbf{w}}(n-1) \right)^{-1} \times \left(\mathbf{X}(n)\Lambda(n)\mathbf{X}(n)^H\right)^{-1} \mathbf{Q}_n \tilde{\mathbf{w}}(n-1) \tag{25}$$

Defining $\mathbf{\Phi}_n = \mathbf{X}(n)\mathbf{\Lambda}(n)\mathbf{X}(n)^H$, and

$$\mathbf{\Phi}_n^{(g)} = \lambda\,\mathbf{\Phi}_{n-1} + \sum_{l'=0}^{g-1} \mathbf{x}(Gn + l')\mathbf{x}(Gn + l')^H, \tag{26}$$

by using the matrix inversion lemma[1] and selecting initial values of $\tilde{\mathbf{w}}(-1)$, $\mathbf{\Sigma}_{-1}$, $\mathbf{Q}_{-1} = \mathbf{0}$ and λ, the recursive estimating algorithm of $\mathbf{w}$ based on (25) becomes

[1] $(A + BC)^{-1} = A^{-1} - A^{-1}B(I + CA^{-1}B)CA^{-1}$

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For $n = 0,1,\cdots$ {

$$\Phi_n^{(0)^{-1}} = \lambda^{-1}\Sigma_{n-1}$$

For $g = 0,\cdots,G-1$ {

$$\Phi_n^{(g+1)^{-1}} = \Phi_n^{(g)^{-1}} - \Phi_n^{(g)^{-1}}\mathbf{x}(Gn+g) \times\left(1+\mathbf{x}(Gn+g)^H\Phi_n^{(g)^{-1}}\mathbf{x}(Gn+g)\right)^{-1} \times\mathbf{x}(Gn+g)^H\Phi_n^{(g)^{-1}}$$

}

$$\Sigma_n = \Phi_n^{(G)^{-1}}$$

$$\mathbf{Q}_n = \lambda\mathbf{Q}_{n-1} + X(n)\mathbf{c}(n)\mathbf{c}(n)^H X(n)^H$$

$$\tilde{\mathbf{w}}(n) = \left(\tilde{\mathbf{w}}(n-1)^H\mathbf{Q}_n\Sigma_n\mathbf{Q}_n\tilde{\mathbf{w}}(n-1)\right)^{-1}\Sigma_n\mathbf{Q}_n\tilde{\mathbf{w}}(n-1)$$

}.

V. Computer simulations

A CDMA system with the processing gain $G = 64$ and a M-element uniform linear array is considered in simulations for different values of M. The average SINR (before despreading) defined as follows is employed to evaluate the performance of the proposed DMSINR and RMSINR blind beamforming algorithms.

$$SINR = \frac{1}{N}\sum_{j=1}^{N} SINR_j \tag{27}$$

where N is the number of users and $SINR_j$ is the SINR of the jth user before despreading that is defined as

$$SINR_j = \frac{\sum_{k=1}^{J}\hat{\mathbf{w}}_{j,k}^H R_{ss_{jk}}\hat{\mathbf{w}}_{j,k}}{\sum_{k=1}^{J}\hat{\mathbf{w}}_{j,k}^H R_{uu_{jk}}\hat{\mathbf{w}}_{j,k}} \tag{28}$$

where J is the number of paths for each user, $R_{ss_{jk}}$ is the autocorrelation matrix of the received signal vector (before despreading) of the kth path for jth user and $R_{uu_{jk}}$ is the autocorrelation matrix of the interference plus noise signal for the kth path of the jth user and $\hat{\mathbf{w}}_{j,k}$ is the estimated beamformer weight vector of the kth path for jth user. One hundred independent realizations are used for each simulation with $\lambda = 0.99$. For each user, the angle of arrival is uniformly selected $0^0 \le \theta_{ik} \le 180^0$ in a random manner at each realization.

The performance of the DMSINR algorithm is shown in Fig. 1 for different values of M and N based on signal-to-noise ratio (SNR). The SNR is defined as the ratio of each user power (for all paths) to the noise power. It is assumed a perfect power control system in which the received powers of all users are equal. Under different situations, the convergence rate of the RMSINR has been evaluated in Fig.2, Fig.3 and Fig.4. As seen in these figures, the RMSINR approximately achieves the performance of the DMSINR algorithm after the first iteration. As indicated in the RMSINR algorithm procedure, It should be noted that each iteration is equal to one symbol duration time that is equal to G-chip duration.
To evaluate the convergence rate of the RMSINR algorithm within a symbol duration, in Fig. 5, the estimation of the beamformer weight vector has been updated at each chip duration. When the number of users is 20 and M=6, 8 and 10, the performance of the RMSINR algorithm is shown in Fig. 5 before despreading for SNR =20dB. As seen in Fig. 5 the RMSINR algorithm converges close to the final SINR after twenty chips. This result shows the RMSINR algorithm attains a very fast convergence rate.

VI. Conclusions

Blind beamforming is a technique that in addition to preserve bandwidth efficiency can provide high performance in wireless communications. In this paper the DMSINR and the RMSINR blind beamforming algorithms have been developed based on maximizing the SINR for CDMA systems. The DMSINR algorithm achieves the maximum of SINR in a direct manner, but it is complex due to involving matrix decomposition. To reduce the complexity of the DMSINR algorithm, the RMSINR algorithm has been derived based on linear constrained optimization that maximizes the SINR recursively. Computer simulations have shown that the RMSINR algorithm has a very fast convergence rate. The RMSINR algorithm achieves a performance close to the steady state situation after one symbol duration.

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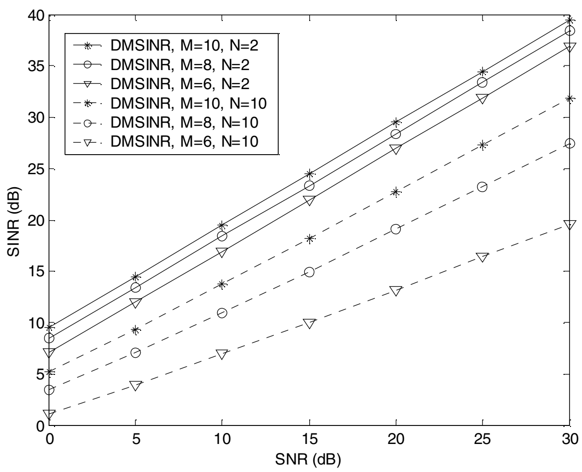

Fig. 1. The performance of the DMSINR algorithm versus SNR for different values of M and N.

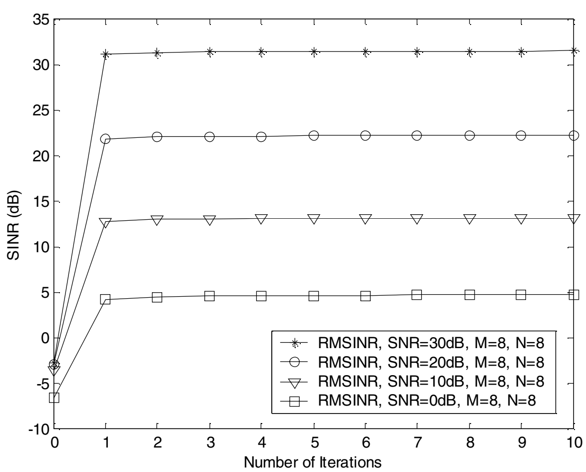

Fig. 2. Convergence rate of the RMSINR algorithm for SNR=0dB, 10dB, 20dB and 30dB, when M=8 and N=8.

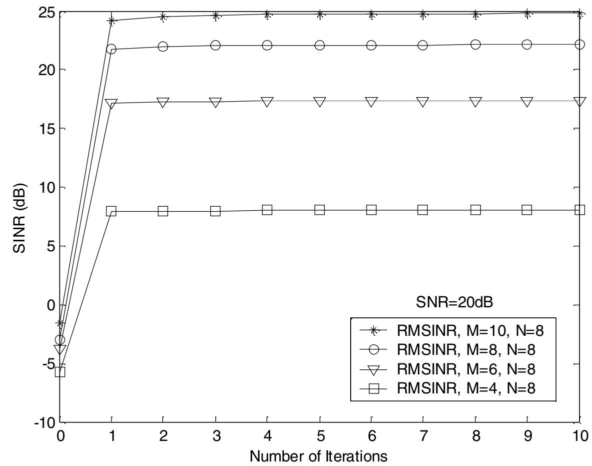

Fig. 3. Convergence rate of the RMSINR algorithm for SNR=20dB and N=8 when M=4,6,8 and 10.

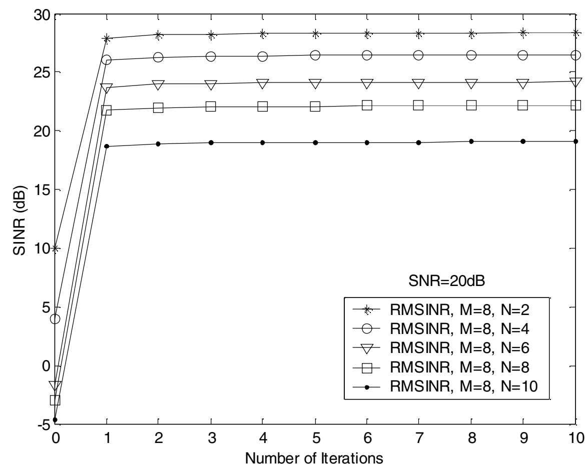

Fig. 4. Convergence rate of the RMSINR algorithm for SNR=20dB and M=8 when N=2,4,6,8 and 10.

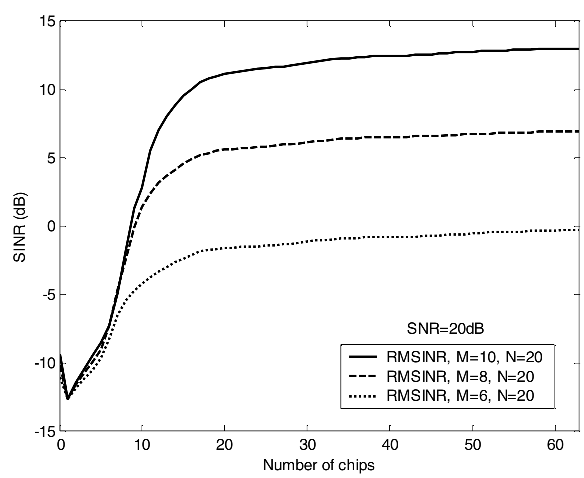

Fig. 5. Convergence rates of the RMSINR algorithms before despreading for SNR=20dB when the beamformer weight vector is updated at each chip duration.

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