Channel Impulse Response Matrix Shortening Algorithm for MIMO Communication Systems

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***Abstract*- Efficient multiple access interference cancellation and users signal separation could be achieved by space-time equalization. However, receiver complexity like MLSE increases greatly with the number of transmit and receive antennas and channel memory. The problem of designing an equalizer to shorten the channel impulse response and reduce the complexity of MLSE equalization for a MIMO system is addressed in this paper. Impulse response shortening (IRS) equalizer is generalized for a MIMO system in two different methods: Decomposed-MIMO-IRS and MIMO-IRS. Finally, a channel impulse response matrix shortening (CIRMS) algorithm is introduced which decreases the effective channel's memory, which is the multiplication of the number of transmit antennas and channel's memory, instead of just decreasing the channel's memory.**

II. INTRODUCTION

MAXIMUM likelihood sequence detection and estimation (MLSD/MLSDE) is an optimal receiver technique designed for intersymbol interference-contaminated multipath fading channels, which minimizes the sequence detection error rate [1]. However, the major problem associated with such a nonlinear receiver is its computational complexity, which increases exponentially with the channel's memory and is too high; especially with multiple transmit antennas and large signal constellations (to achieve higher spectral efficiencies). More specifically, the number of MLSE states required in general is $q^{M\times L}$,where q is the signal constellation size, M is the number of transmit antennas, and L is the memory of the overall channel impulse response.

A considerable amount of research has been conducted in order to find less complex algorithms, which most of them emphasize on decreasing the overall effective memory length of the channel/equalizer, in order to reduce the number of states in the trellis diagram and, hence, the number of required computations for each branch metric [2]–[5].

In this paper, the impulse response shortening (IRS) equalizer in [4]-[5] for SISO systems is generalized for MIMO systems in two different methods: Decomposed-MIMO-IRS, MIMO-IRS. Finally, a channel impulse response matrix shortening (CIRMS) algorithm is introduced, which decreases the effective memory ($M\times L$), instead of only decreasing channel's memory(L).

II. GENERALIZED IRS ALGORITHM

Considering a $M\times N$ MIMO communication systems, channel output at time instant k in matrix form can be expressed as

$$\mathbf{y}(k)=\mathbf{H}\mathbf{x}(k)+\mathbf{w}(k) \tag{1}$$

where

$$\begin{aligned}
&\mathbf{x}(k)=\begin{bmatrix}\mathbf{x}_1(k) & \cdots & \mathbf{x}_M(k)\end{bmatrix}^T\\
&\mathbf{x}_i(k)=\begin{bmatrix}\mathbf{x}_i(k) & \cdots & \mathbf{x}_i(k-L-1) & \mathbf{x}_i(k-L)\end{bmatrix}^T, i=1,\cdots,M\\
&\mathbf{y}(k)=\begin{bmatrix}\mathbf{y}_1(k) & \cdots & \mathbf{y}_N(k)\end{bmatrix}^T\\
&\mathbf{H}=\begin{bmatrix}\mathbf{h}_1 & \cdots & \mathbf{h}_N\end{bmatrix}^T, \mathbf{h}_j=\begin{bmatrix}\mathbf{h}_{1j} & \cdots & \mathbf{h}_{Mj}\end{bmatrix},\ j=1,\cdots,N\\
&\mathbf{h}_{ij}=\begin{bmatrix}h_{ij}(0) & \cdots & h_{ij}(L)\end{bmatrix}
\end{aligned} \tag{2}$$

$\mathbf{h}_{ij}$ is channel impulse response (CIR) between the *i*th input and *j*th output with length of $L+1$ and $\mathbf{H}$ is the CIR matrix. $\mathbf{x}(k)$ and $\mathbf{y}(k)$ are respectively input and output vector and $\mathbf{w}(k)$ is noise vector with the same dimension as $\mathbf{y}(k)$. So the output vector at each receive antenna would be written as

$$\mathbf{y}_j(k)=\mathbf{h}_j\mathbf{x}(k)+\mathbf{w}_j(k)\ ,\ j=1,\cdots,N \tag{3}$$

In this section, the SISO channel-shortening algorithm in [4] and [5] is extended to MIMO systems in two different methods: Decomposed-MIMO-IRS, MIMO-IRS

II.A. DECOMPOSED-MIMO-IRS ALGORITHM

Decomposed-MIMO-IRS algorithm is the MISO extension of IRS algorithm while the same algorithm is repeated for multiple receive antennas. So for each receive antenna, a single equalizer is designed to shorten all MISO channels based on a SSNR maximization criterion (defined in [4,5]). Considering the block diagram of the proposed system shown in fig. 1.a, if f_i is the impulse response of IRS equalizer for *j*th receive antenna with length of t ,so *k*th tap of the overall channel impulse response (OIR) at *j*th output , g_i ($j=1,\cdots,N$) with length of $L_0\equiv L+t$ can be expressed as

$$g_j(k)=\sum_{i=1}^{M}\sum_{l=0}^{L}h_{ij}(k)f_j(l-k) \tag{4}$$

In matrix notation

$$\mathbf{g}_j=\sum_{i=1}^{M}\mathbf{H}_{ij}\mathbf{f}_j \tag{5}$$

where $\mathbf{g}_j=\begin{bmatrix}g_j(0) & \cdots & g_j(L_0-1)\end{bmatrix}^T$, $\mathbf{f}_j=\begin{bmatrix}f_j(0) & \cdots & f_j(t-1)\end{bmatrix}^T$ and $\mathbf{H}_{ij}$ is the Toeplitz convolution matrix whose first column is $\begin{bmatrix}\mathbf{h}_{ij} & \mathbf{0}_{1\times(t-1)}\end{bmatrix}^T$,where $\mathbf{0}_{1\times(t-1)}$ is a row vector of zeros of length of $t-1$. So $\mathbf{H}_{ij}$ is given by

$$\mathbf{H}_{ij}=\begin{bmatrix}
h_{ij}(0) & 0 & \cdots & 0\\
h_{ij}(1) & h_{ij}(0) & 0 & \vdots\\
\vdots & h_{ij}(1) & \ddots & 0\\
h_{ij}(L) & \vdots & \ddots & h_{ij}(0)\\
0 & h_{ij}(L) & \ddots & h_{ij}(1)\\
\vdots & 0 & \ddots & \vdots\\
0 & 0 & \cdots & h_{ij}(L)
\end{bmatrix} \tag{6}$$

Let $\mathbf{g}_{jwin}$ denote a time window of length v with a delay of d samples and $\mathbf{g}_{jwall}$ denote the remaining samples of $\mathbf{g}_j$.

$$\mathbf{g}_{jwin}=\begin{bmatrix}g_j(d) & g_j(d+1) & \cdots & g_j(d+v)\end{bmatrix}^T\equiv\sum_{i=1}^{M}\mathbf{H}_{ijwin}\mathbf{f}_j \tag{7}$$

$$\mathbf{g}_{jwall}=\begin{bmatrix}g_j(0) \cdots g_i(d-1)\ g_i(d+v+1) \cdots g_j(L_0-1)\end{bmatrix}^T\equiv\sum_{i=1}^{M}\mathbf{H}_{ijwall}\mathbf{f}_j \tag{8}$$

As a measure of the ISI for each equalizer output, we can measure the shortening SNR(SSNR) which is the ratio of the sum of the total channel energy in $\mathbf{g}_{jwin}$ to the sum of the total channel energy in $\mathbf{g}_{jwall}$ [4]. The IRS coefficients are optimized such that the SSNR is maximized. The expressions for the energy outside and inside the window can be written as

$$\mathbf{g}_{jwall}^H\mathbf{g}_{jwall}=\mathbf{f}_j^{\ H}\sum_{i=1}^{M}\mathbf{H}_{ijwall}^H\mathbf{H}_{ijwall}\mathbf{f}_j=\mathbf{f}_j^{\ H}\mathbf{A}_j\mathbf{f}_j \tag{9}$$

$$\mathbf{g}_{jwin}^H\mathbf{g}_{jwin}=\mathbf{f}_j^{\ H}\sum_{i=1}^{M}\mathbf{H}_{ijwin}^H\mathbf{H}_{ijwin}\mathbf{f}_j=\mathbf{f}_j^{\ H}\mathbf{B}_j\mathbf{f}_j \tag{10}$$

So the optimization problem based on SSNR maximizing could be written as

$$\begin{aligned} &\mathbf{f}_{opt\,j} : \arg\min_{\mathbf{f}_j}\{\mathbf{f}_j^{\;H}\mathbf{A}_j\mathbf{f}_j\} \text{ subject to } \mathbf{f}_j^{\;H}\mathbf{B}_j\mathbf{f}_j = 1, \; \textit{for } j = 1,2,\cdots,N \\ &\qquad\qquad\text{or} \\ &\mathbf{f}_{opt\,j} : \arg\max_{\mathbf{f}_j}\{\mathbf{f}_j^{\;H}\mathbf{B}_j\mathbf{f}_j\} \text{ subject to } \mathbf{f}_j^{\;H}\mathbf{A}_j\mathbf{f}_j = 1, \; \textit{for } j = 1,2,\cdots,N \end{aligned} \tag{11}$$

where $\mathbf{A}_j$ and $\mathbf{B}_j$ is defined as

$$\begin{aligned} \mathbf{A}_j &\overset{\text{def}}{=} \sum_{i=1}^{M} \mathbf{H}_{ij_{wall}}^{H}\mathbf{H}_{ij_{wall}} \\ \mathbf{B}_j &\overset{\text{def}}{=} \sum_{i=1}^{M} \mathbf{H}_{ij_{win}}^{H}\mathbf{H}_{ij_{win}} \end{aligned} \tag{12}$$

II.B. MIMO-IRS ALGORITHM

The proposed block diagram of this MIMO extension is shown in fig 1.b. If $\mathbf{H}$, $\mathbf{F}$ and $\mathbf{G}$ are respectively the matrixes of CIR, IRS and OIR and $\mathbf{h}_{ij}$, $\mathbf{f}_{ij}$ and $\mathbf{g}_{ij}$ are the impulse responses of channel, equalizer and overall channel between *i*th input node and *j*th output node, so MIMO matrix extension of proposed block diagram can be written as

$$\mathbf{G} = \mathbf{HF}$$

$$= \begin{bmatrix} \mathbf{g}_{11} & \cdots & \mathbf{g}_{1P} \\ \vdots & \ddots & \vdots \\ \mathbf{g}_{M1} & \cdots & \mathbf{g}_{MP} \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \cdots & \mathbf{h}_{1N} \\ \vdots & \ddots & \vdots \\ \mathbf{h}_{M1} & \cdots & \mathbf{h}_{MN} \end{bmatrix} \begin{bmatrix} \mathbf{f}_{11} & \cdots & \mathbf{f}_{1P} \\ \vdots & \ddots & \vdots \\ \mathbf{f}_{N1} & \cdots & \mathbf{f}_{NP} \end{bmatrix} \tag{13}$$

By constellating (11) to fit for IRS algorithm, an equation the same as (5) in extended for a MIMO system is achieved. Finally, the optimization problem would be solved by SSNR maximization criterion.

III. IRS ALGORITHM CONSIDERING NOISE

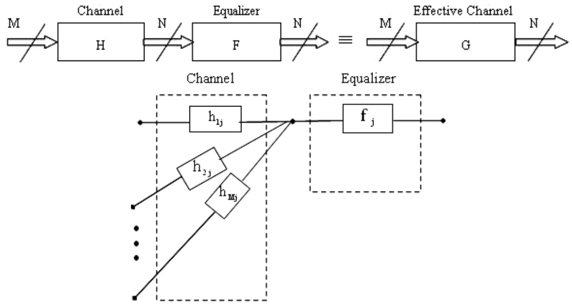
Considering noise, we hope any noise energy enhancement by IRS equalizer is minimized. However the noise sample at the equalizer output will be correlated, we choose not to include this noise correlation in the MLSE metric calculation to reduce receiver complexity at the expense of the some performance loss. So the equalizer coefficients are optimized by maximizing the ratio of the channel energy inside the window to the sum of channel energy outside the window plus the noise energy at the equalizer output.

IV. THE CIRMS ALGORITHM

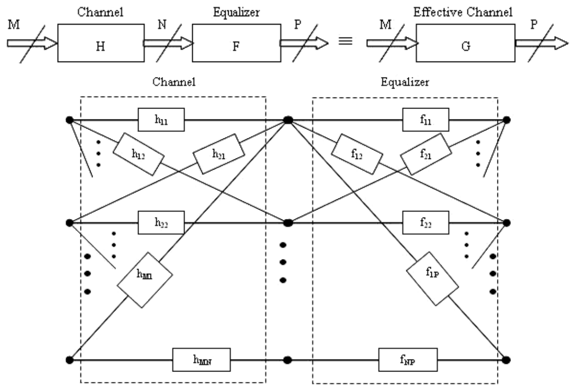
All method described in II-III focus on only decreasing the channel's memory (L). Here, we develop a channel impulse matrix shortening (CIRMS) algorithm, which not only decreases channel's memory but also the number of transmit antennas. In general, it decreases the effective memory's channel ($M \times L$). Given the desired truncated memory, the optimum shortened length for each channel is chosen, which result in maximum SSNR in all SSNRs associated with all permutations of channel lengths.

V. SIMULATION RESULTS

In the simulation, the worst case is considered, where channel is uniform. Considering fig. 1, channel/equalizer is shown by a $\mathrm{M} \times \mathrm{N} \times \mathrm{P}$ system. Figure 2 shows the power profile of OIR when the equalizer with t=16 taps is designed based on the $2 \times 2 \times 2$ Decomposed-MIMO-IRS, $2 \times 2 \times 2$ and $2 \times 2 \times 1$ MIMO-IRS algorithms for a uniform channel with $L = 4, v = 3$. These results predict better performance for MIMO-IRS algorithm than Decomposed-MIMO-IRS. Simulation results of CIRMS algorithm is shown in fig. 3 for a $2 \times 2 \times 2$ MIMO system, $L = 4, v = 3$ and $t = 16$. It's desired to reduce $M \times L = 8$ to 6 and less. The best permutation, which result in maximum SSNR in this realization is [4 , 2 ; 4 , 2]. However, the total complexity of MIMO receiver is reduced; the optimum shortened length according to the fading of channels is selected.



(a) Decomposed-MIMO block diagram



(b) MIMO block diagram

Fig. 1. Channel/Equalizer block diagram

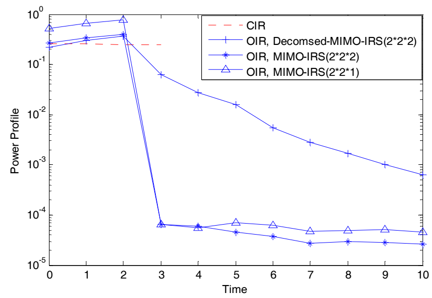


Fig. 2. Power profile of the OIR for Decomposed-MIMO-IRS and MIMO-IRS algorithms for different MIMO systems when *L*=4, *v*=3 and *t*=16

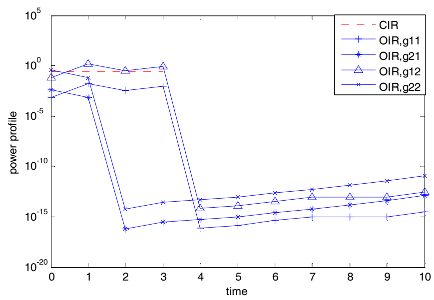


Fig. 3. Power profile of all OIRs for $2 \times 2 \times 2$ CIRMS algorithm when *L*=4, *t*=16

REFRENCES

[1] G. D. Forney, "Maximum-likelihood sequence estimation of digital sequences in presence of intersymbol interference," *IEEE Trans. Inform. Theory*, vol. IT-18, pp. 363–378, May 1972.

[2] D. D. Falconer and F. R. Magee, "Adaptive channel memory truncation for maximum-likelihood sequence estimation," *Bell Syst. Tech. J.*, vol. 9, pp. 1541–1562, Nov. 1973.

[3] N. Al-Dhahir, "FIR channel-shortening equalizers for MIMO ISI channels," *IEEE transactions on communication*, vol. 49, no. 2, Feb. 2001.

[4] P. J. W. Melsa, R. C. Younce, and C. E. Rohrs, "Impulse response shortening for discrete multitone transceivers," *IEEE Trans. Commun.*, vol. 44, pp. 1662-1672, Dec. 1996.

[5] C. Yin and G. Yue, "Optimal impulse response shortening for discrete multitone transceivers," *IEEE Electronics Letters*, vol. 34, pp. 35-36, Jan. 1998.