Adaptive Channel SVD Estimation for MIMO-OFDM Systems

H. Zamiri-Jafarian[1, 2] and G. Gulak[1]

[1]Department of Electrical and Computer Engineering, University of Toronto, Toronto, Ontario, Canada

[2]Electrical Engineering Department, Ferdowsi University, Mashhad, Iran

zamiri@comm.utoronto.ca

***Abstract*—In this paper we propose an adaptive estimation algorithm for channel matrix singular value decomposition (SVD) in multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems. The SVD method is an efficient approach to design space-time coding/decoding and detection algorithms in MIMO-OFDM systems. However, the SVD estimation may involve complex nonlinear optimization methods. The proposed algorithm in this paper is developed based on a two-step recursive method by utilizing the linear constrained least mean square (LMS) technique.**

***Index Terms*— MIMO-OFDM systems, MIMO channel estimation, SVD estimation, constrained LMS.**

I. Introductiion

Channel matrix singular value decomposition (SVD) method is employed in MIMO-OFDM systems in order to over come subchannel interference, to allocate transmitted power through subchannels in an optimum manner and also to design the space-time coding algorithm efficiently [1]. Thus SVD estimation is an important technique to exploit the full capability of MIMO-OFDM systems. Different methods have been proposed to estimate MIMO channels [2]-[4] and the SVD can be obtained from the estimated channel matrix. However, the SVD is a nonlinear function and its estimation is very sensitive to the error of estimated channel matrix because of the nonlinear procedures involved [5]. In this paper we propose an adaptive SVD estimation algorithm that uses a two-step recursive method to estimate the SVD of the channel matrix directly from the received signal. The estimation algorithm is developed based on the linear constrained LMS method. The performance of the proposed algorithm is evaluated by computer simulations under different scenarios.

The organization of the paper is as follows. After the introduction, in Section II the MIMO-OFDM communication system model is described. The adaptive SVD estimation algorithm is derived in Section III. Section IV contains computer simulation results and conclusions are presented in Section V.

II. System Models

A discrete model of a MIMO-OFDM system with N transmitting antennas, M receiving antennas and L subcarriers is shown in Fig. 1. when $S(k)=[\boldsymbol{s}^{(1)}(k),\ldots,\boldsymbol{s}^{(L)}(k)]$ is a transmitted OFDM symbol matrix and $\boldsymbol{s}^{(l)}(k)=[s_1^{(l)}(k),\ldots,s_N^{(l)}(k)]^T$ is a symbol vector transmitted from the lth subcarrier. Note that $(.)^T$ represents the transpose operation. Defining $\mathbf{H}=[\boldsymbol{H}^{(1)},\ldots,\boldsymbol{H}^{(L)}]^T$ as an L-point FFT of the $M\times N$ channel impulse response matrix, $H(k)$, after removing the cyclic prefix, the $M\times 1$ received vector of the lth subcarrier, $\boldsymbol{x}^{(l)}(k)$, becomes

$$\boldsymbol{x}^{(l)}(k)=\boldsymbol{H}^{(l)}\boldsymbol{s}^{(l)}(k)+\boldsymbol{n}^{(l)}(k) \qquad \text{for } l=1,\cdots,L \tag{1}$$

where $\boldsymbol{H}^{(l)}$ is an $M\times N$ channel matrix of the lth subcarrier and $\boldsymbol{n}^{(l)}(k)$ is the $M\times 1$ vector of the complex additive white Gaussian noise (AWGN) with zero-mean and autocorrelation matrix $R_{\mathbf{n}_l}(k)=\sigma_n^2\boldsymbol{I}_M\delta(k)$ for $l=1,\cdots,L$, while $\boldsymbol{I}_M$ is the $M\times M$ identity matrix. The SVD of the lth subcarrier channel matrix, $\boldsymbol{H}^{(l)}$, can be given as

$$\boldsymbol{H}^{(l)}=\boldsymbol{U}^{(l)}\Sigma^{(l)}\boldsymbol{V}^{(l)H} \tag{2}$$

where $\boldsymbol{U}^{(l)}$ and $\boldsymbol{V}^{(l)}$ are $M\times P$ and $N\times P$ unitary matrices, respectively. Note that P is the rank of $\boldsymbol{H}^{(l)}$ where $P\le\min(M,N)$. $\Sigma^{(l)}=\mathrm{diag}(\sigma_1^{(l)},\sigma_2^{(l)},\cdots,\sigma_P^{(l)})$ is a diagonal matrix containing the singular values of the lth subcarrier channel matrix and $(.)^H$ denotes transposed complex conjugate. Knowing the $\boldsymbol{U}^{(l)}$, $\boldsymbol{V}^{(l)}$ and $\Sigma^{(l)}$ matrices for all subcarriers $l=1,\cdots,L$ are vital information to design space-time coding, decoding and detection schemes efficiently. Due to the unitary property of $\boldsymbol{U}^{(l)}$ and $\boldsymbol{V}^{(l)}$

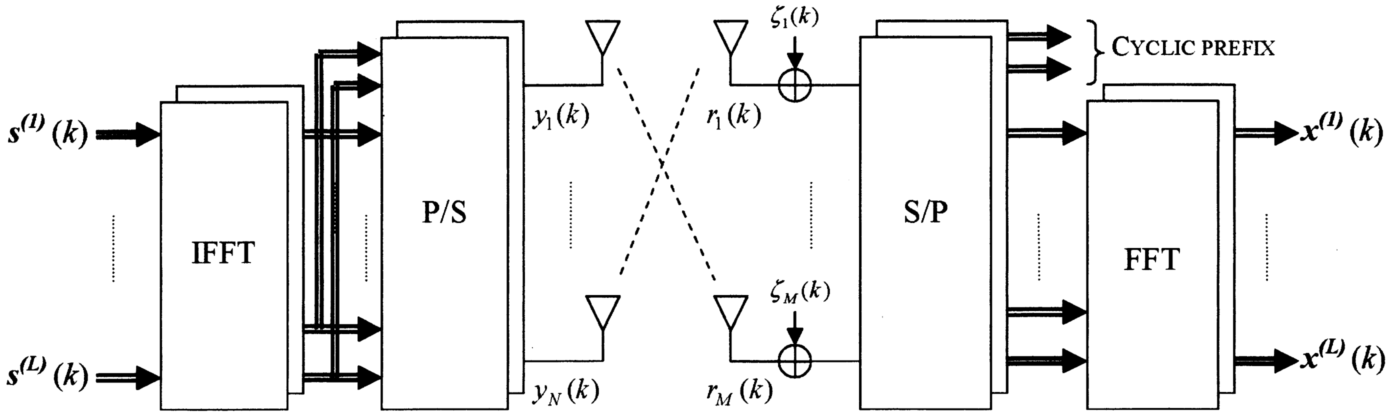

Fig. 1. A discrete MIMO-OFDM system with N transmitting antennas and M receiving antennas.

matrices, estimation of the subcarrier channel SVD at the receiver side based on minimizing the mean square error criterion is a complicated procedure because of nonlinear constraints $U^{(l)H}U^{(l)} = I_P$ and $V^{(l)H}V^{(l)} = I_P$. A two-step adaptive method is proposed in the Section III based on the constrained least mean square (LMS) algorithm that employs linear constraints to estimate the SVD of subcarrier channel matrices.

III. Adaptive SVD estimation

We consider one subcarrier channel matrix, $H^{(l)}$, and develop the adaptive algorithm to estimate its SVD. Also, due to simplicity, we drop the superscript of *(l)* in deriving the algorithm.

By defining the following matrices

$$W_1 = U\Sigma \tag{3}$$

$$W_2 = V\Sigma \tag{4}$$

It is straightforward to show

$$\mathbf{w}_{1i} = H\mathbf{v}_i = \sigma_i \mathbf{u}_i \tag{5}$$

$$\mathbf{w}_{2i}^H = \mathbf{u}_i^H H = \sigma_i \mathbf{v}_i^H \tag{6}$$

where $\mathbf{u}_i$, $\mathbf{v}_i$, $\mathbf{w}_{1i}$ and $\mathbf{w}_{2i}$ are the ith columns of U, V, W_1 and W_2, respectively, and σ_i is the ith diagonal element of Σ.

We assume that the training sequence is an independent and identically distributed (iid) signal such that $E[s(k)s(k)^H] = \sigma_s^2 I_N$ for all subcarriers (note that we drop the superscript l) and also $E[s(k)n(j)^H] = 0$ for all k and j. By assuming $\sigma_s^2 = 1$, from (5) one can show that

$$\mathbf{w}_{1i} = E[x(k)s(k)^H \mathbf{v}_i] \qquad \text{for } i = 1, \cdots, P \tag{7}$$

Based on (7), in the first step, by receiving $x(k)$ and assuming $\hat{\mathbf{v}}_i(k-1)$ is the estimated $\mathbf{v}_i$ at time $k-1$, the improved estimation of $\mathbf{w}_{1i}$ at time $k-1$, $\hat{\mathbf{w}}_{1i}^+(k-1)$, based on the estimation of $\mathbf{w}_{1i}$ at time $k-1$ becomes

$$\begin{aligned}\hat{\mathbf{w}}_{1i}^+(k-1) = \hat{\mathbf{w}}_{1i}(k-1) \\ + \mu_1\left(x(k)s(k)^H \hat{\mathbf{v}}_{1i}(k-1) - \hat{\mathbf{w}}_{1i}(k-1)\right) \\ \text{for } i = 1, \cdots, P\end{aligned} \tag{8}$$

where μ_1 is a positive scalar step-size of the LMS algorithm. Due to the orthogonality property of the column vectors of W_1, the estimation of $\mathbf{w}_{1i}$ at time k, $\hat{\mathbf{w}}_{1i}(k)$, should be obtained under the following constraint

$$\hat{\mathbf{w}}_{1j}(k)^H \hat{\mathbf{w}}_{1i}(k) = 0 \qquad \text{for } j < i \tag{9}$$

Defining

$$\hat{\mathbf{w}}_{1i}(k) = \hat{\mathbf{w}}_{1i}^+(k-1) + \mathbf{v}_{1i}(k) \qquad \text{for } i = 1, \cdots, P \tag{10}$$

$$\hat{W}_{1i}(k) = [\hat{\mathbf{w}}_{11}(k), \cdots, \hat{\mathbf{w}}_{1i-1}(k)] \qquad \text{for } i = 1, \cdots, P \tag{11}$$

where $\mathbf{v}_{1i}(k)$ is a $M \times 1$ vector. $\hat{\mathbf{w}}_{1i}(k)$ is obtained under the defined constraint (9) so that $\mathbf{v}_{1i}(k)^H \mathbf{v}_{1i}(k)$ is minimized [6]. For $j < i$, the constraint (9) can be given as

$$\hat{W}_{1i}(k)^H \hat{\mathbf{w}}_{1i}(k) = 0 \qquad \text{for } i = 1,\cdots,P \quad (12)$$

Using the Lagrange multiplier method, the criterion of obtaining $\mathbf{v}_{1i}(k)$ becomes

$$\mathbf{v}_{1i}(\mathrm{k}) = \arg\min_{\mathbf{v}_{1i}(k)} \{\mathbf{v}_{1i}(k)^H \mathbf{v}_{1i}(k) + \boldsymbol{\lambda}_{1i}(k)^H \hat{W}_{1i}(k)^H \left(\hat{\mathbf{w}}_{1i}(k-1) + \mathbf{v}_{1i}(k)\right)\} \qquad \text{for } i = 1,\cdots,P \quad (13)$$

where $\boldsymbol{\lambda}_{1i}(k) = [\lambda_{11}(k),\ldots,\lambda_{1i-1}(k)]^T$ is the Lagrange multiplier vector. By doing some manipulations, one can show that

$$\mathbf{v}_{1i}(k) = \hat{W}_{1i}(k) \times \left(\hat{W}_{1i}(k)^H \hat{W}_{1i}(k)\right)^{-1} \hat{W}_{1i}(k)^H \hat{\mathbf{w}}_{1i}^{+}(k-1) \qquad \text{for } i = 1,\cdots,P \quad (14)$$

Substituting $\mathbf{v}_{1i}(k)$ and $\hat{\mathbf{w}}_{1i}^{+}(k-1)$ in (10), $\mathbf{w}_{1i}$ at time k can be estimated by

$$\hat{\mathbf{w}}_{1i}(k) = \left(I_M - \hat{W}_{1i}(k)(\hat{W}_{1i}(k)^H \hat{W}_{1i}(k))^{-1} \hat{W}_{1i}(k)^H\right) \times \left(\hat{\mathbf{w}}_{1i}(k-1) + \mu_1 \left(x(k)s(k)^H \hat{\mathbf{v}}_i(k-1) - \hat{\mathbf{w}}_{1i}(k-1)\right)\right) \qquad \text{for } i = 1,\cdots,P \quad (15)$$

Based on (5), the estimation of $\mathbf{u}_i$ at time k is given by

$$\hat{\mathbf{u}}_i(k) = (\hat{\mathbf{w}}_{1i}(k)^H \hat{\mathbf{w}}_{1i}(k))^{-\frac{1}{2}} \hat{\mathbf{w}}_{1i}(k) \quad \text{for } i = 1,\cdots,P \quad (16)$$

By having $\hat{\mathbf{u}}_i(k)$ from (16), in the second step, $\mathbf{w}_{2i}$ at time k can be estimated with similar procedure of estimating $\mathbf{w}_{1i}$. From (2) and (6), one can show that

$$\mathbf{w}_{2i}^H = E[\mathbf{u}_i^H x(k)s(k)^H] \qquad \text{for } i = 1,\cdots,P \quad (17)$$

The improved estimation of $\mathbf{w}_{2i}$ at time $k-1$, $\hat{\mathbf{w}}_{2i}^{+}(k-1)$, can be given by

$$\hat{\mathbf{w}}_{2i}^{+}(k-1) = \hat{\mathbf{w}}_{2i}(k-1) + \mu_2\left(s(k)x(k)^H \hat{\mathbf{u}}_i(k) - \hat{\mathbf{w}}_{2i}(k-1)\right) \qquad \text{for } i = 1,\cdots,P \quad (18)$$

where $\hat{\mathbf{u}}_i(k)$ is obtained from the first step, *x(k)* is the received signal at time k and μ_2 is a positive scalar step-size. Due to the orthogonality property of the column vectors of W_2 defined in (4), the $\mathbf{w}_{2i}$ at time k should be estimated so that the following constraint is hold

$$\hat{W}_{2i}(k)^H \hat{\mathbf{w}}_{21i}(k) = 0 \qquad \text{for } i = 1,\cdots,P \quad (19)$$

where $\hat{W}_{2i}(k) = [\hat{\mathbf{w}}_{21}(k),\cdots,\hat{\mathbf{w}}_{2i-1}(k)]$. By defining

$$\hat{\mathbf{w}}_{2i}(k) = \hat{\mathbf{w}}_{2i}^{+}(k-1) + \mathbf{v}_{2i}(k) \quad \text{for } i = 1,\cdots,P \quad (20)$$

$\mathbf{w}_{2i}$ is estimated by minimizing $\mathbf{v}_{2i}(k)^H \mathbf{v}_{2i}(k)$ under the constraint (19). By using the Lagrange multiplier method and following the similar procedure of estimating $\mathbf{w}_{1i}(k)$, one can show that

$$\hat{\mathbf{w}}_{2i}(k) = \left(I_N - \hat{W}_{2i}(k)(\hat{W}_{2i}(k)^H \hat{W}_{2i}(k))^{-1} \hat{W}_{2i}(k)^H\right) \times \left(\hat{\mathbf{w}}_{2i}(k-1) + \mu_2\left(s(k)x(k)^H \hat{\mathbf{u}}_i(k) - \hat{\mathbf{w}}_{2i}(k-1)\right)\right) \qquad \text{for } i = 1,\cdots,P \quad (21)$$

By using $\hat{\mathbf{w}}_{2i}(k)$ from (21), σ_i and $\mathbf{v}_i$ for $i = 1,\cdots,P$ at time k are estimated by

$$\hat{\sigma}_i(k) = (\hat{\mathbf{w}}_{2i}(k)^H \hat{\mathbf{w}}_{2i}(k))^{\frac{1}{2}} \qquad \text{for } i = 1,\cdots,P \quad (22)$$

$$\hat{\mathbf{v}}_i(k) = (\hat{\mathbf{w}}_{2i}(k)^H \hat{\mathbf{w}}_{2i}(k))^{-\frac{1}{2}} \hat{\mathbf{w}}_{2i}(k) \quad \text{for } i = 1,\cdots,P \quad (23)$$

After choosing the initial value of $\mathbf{v}_i^{(l)}(0)$ for all subcarriers $l = 1,\cdots,L$ and $i = 1,\cdots,P$, the adaptive SVD estimation algorithm can be summarized as follows

For $l = 1,\cdots,L$ {
For $i = 1,\cdots,P$ {
For $k = 1,2,\cdots$ {

$$\hat{\mathbf{w}}_{1i}^{(l)}(k) = \left(I_M - \hat{W}_{1i}^{(l)}(k)(\hat{W}_{1i}^{(l)}(k)^H \hat{W}_{1i}^{(l)}(k))^{-1} \hat{W}_{1i}^{(l)}(k)^H\right) \times \left(\hat{\mathbf{w}}_{1i}^{(l)}(k-1) + \mu_1^{(l)}\left(x^{(l)}(k)s^{(l)}(k)^H \hat{\mathbf{v}}_i^{(l)}(k-1) - \hat{\mathbf{w}}_{1i}^{(l)}(k-1)\right)\right)$$

$$\hat{\mathbf{u}}_i^{(l)}(k) = (\hat{\mathbf{w}}_{1i}^{(l)}(k)^H \hat{\mathbf{w}}_{1i}^{(l)}(k))^{-\frac{1}{2}} \hat{\mathbf{w}}_{1i}^{(l)}(k)$$

$$\hat{\mathbf{w}}_{2i}^{(l)}(k) = \left(I_N - \hat{W}_{2i}^{(l)}(k)(\hat{W}_{2i}^{(l)}(k)^H \hat{W}_{2i}^{(l)}(k))^{-1} \hat{W}_{2i}^{(l)}(k)^H\right) \times \left(\hat{\mathbf{w}}_{2i}^{(l)}(k-1) + \mu_2^{(l)}\left(s^{(l)}(k)x^{(l)}(k)^H \hat{\mathbf{u}}_i^{(l)}(k) - \hat{\mathbf{w}}_{2i}^{(l)}(k-1)\right)\right)$$

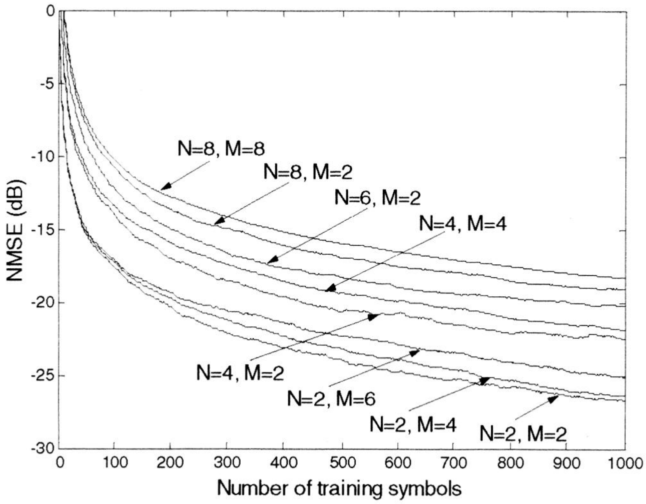

Fig. 2. The normalized MSE of the adaptive estimation algorithm versus the number of training symbols.

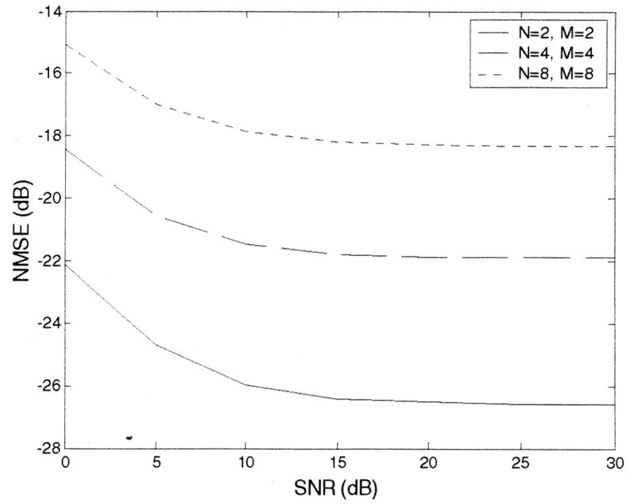

Fig. 3. The normalized MSE of the adaptive estimation algorithm versus SNR

$$\hat{\sigma}_i^{(l)}(k) = (\hat{\mathbf{w}}_{2i}^{(l)}(k)^H \hat{\mathbf{w}}_{2i}^{(l)}(k))^{\frac{1}{2}}$$

$$\hat{\mathbf{v}}_i^{(l)}(k) = (\hat{\mathbf{w}}_{2i}^{(l)}(k)^H \hat{\mathbf{w}}_{2i}^{(l)}(k))^{-\frac{1}{2}} \hat{\mathbf{w}}_{2i}^{(l)}(k)$$

}

}

}.

IV. COMPUTER SIMULATIONS

A MIMO-OFDM system with L=64 for different N and M is considered in the simulations when the channel impulse response, $H(k)$, has an exponential delay spread profile with duration of $L_C = 16$ that is equal to the cyclic prefix interval. The elements of the $H(k)$ are mutually independent complex Gaussian random variables with zero-mean. To evaluated the performance of the estimation algorithm, the normalized mean-square error (NMSE) criterion is employed.

$$\text{NMSE }(\hat{\boldsymbol{H}}) = \frac{1}{L}\sum_{l=1}^{L} \frac{E\left[\left\|\hat{\boldsymbol{H}}^{(l)} - \boldsymbol{H}^{(l)}\right\|_F^2\right]}{E\left[\left\|\boldsymbol{H}^{(l)}\right\|_F^2\right]} \tag{24}$$

where $\hat{\boldsymbol{H}}^{(l)}$ is the estimation of $\boldsymbol{H}^{(l)}$ and $\|\cdot\|_F$ denotes Frobenius norm. Note that the MIMO channel $\boldsymbol{H}^{(l)}$ is estimated based on its SVD estimation from the following relation.

$$\hat{\boldsymbol{H}}^{(l)} = \hat{\boldsymbol{U}}^{(l)}\hat{\Sigma}^{(l)}\hat{\boldsymbol{V}}^{(l)H} \tag{25}$$

One hundred independent channel impulse response matrices are generated for evaluating the performance of the algorithm.

For each subcarrier, a 16QAM training sequence of independent and identically distributed signal vector, $\boldsymbol{s}^{(l)}(k)$, is sent such that $R_s = E[\boldsymbol{s}^{(l)}(k)\boldsymbol{s}^{(l)}(k)^H] = I_N$ for all $l = 1,\cdots,L$. Also each subchannel autocorrelation matrix of zero-mean additive white Gaussian noise vector, $R_{n_l} = E[\boldsymbol{n}^{(l)}(k)\boldsymbol{n}^{(l)}(k)^H] = \sigma_{n_l}^2 I_M$, is chosen in order to achieve the following SNR when $\sigma_n^2 = \sigma_{n_l}^2$ for $l = 1,\cdots,L$.

$$\text{SNR} = \frac{\sum_{l=1}^{L} E[\|\boldsymbol{H}^{(l)}\boldsymbol{s}^{(l)}(k)|^2]}{\sum_{l=1}^{L} E[\|\boldsymbol{n}^{(l)}(k)|^2]} \tag{26}$$

Fig. 2 shows the performance of the proposed adaptive method for different values of N and M when SNR=30dB. As can be seen, the NMSE is decreased rapidly when the size of the training sequence is less than one hundred symbols. Also, by comparing "*N=2, M=4*" with "*N=4, M=2*" and "*N=6, M=2*" with "*N=6, M=2*" cases, we see that the performance of the algorithm is degraded more by increasing N.

The NMSE of the estimated subcarrier channel matrices versus SNR for N=M=2,4 and 8 has been shown in Fig. 3. As can be seen the performance improvement of the estimated algorithm is insignificant for SNR>15dB. To evaluate the unitary property of the estimated SVD, we perform $\Delta^{(l)}$ matrix as follows.

$$\Delta^{(l)} = \hat{\mathbf{u}}^{(l)}H^{(l)}\hat{\mathbf{v}}^{(l)} \qquad \text{for } l = 1,\cdots,L \tag{27}$$

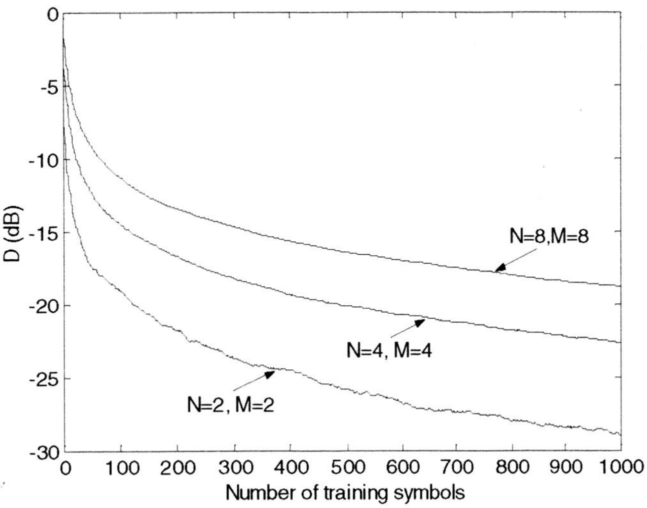

Fig. 4. The D performance criterion e of the adaptive estimation algorithm versus number of training symbols at SNR=30dB.

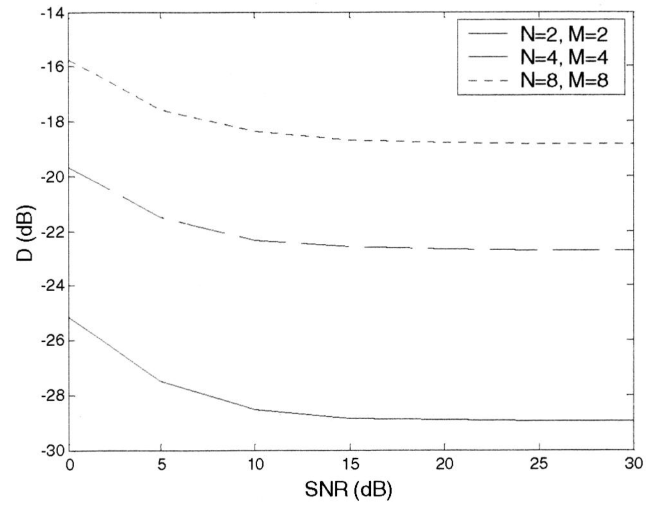

Fig. 5. The D performance criterion of the adaptive estimation algorithm versus SNR.

When there is no error in the SVD estimation of $\boldsymbol{H}^{(l)}$, the $\Delta^{(l)}$ becomes a diagonal matrix. To measure the performance of the unitary property we use the following criterion

$$D = \frac{1}{L}\sum_{l=1}^{L} \frac{E\left[\left\|\Delta^{(l)}\right\|_F^2 - |\delta^{(l)}|^2\right]}{E[\left\|\Delta^{(l)}\right\|_F^2]} \tag{28}$$

where $\delta^{(l)}$ is a vector that contains diagonal elements of $\Delta^{(l)}$ matrix, $\delta^{(l)} = [\Delta_{1,1}^{(l)}, \Delta_{2,2}^{(l)}, \ldots, \Delta_{P,P}^{(l)}]$. Fig. 4 shows the D performance criterion for N=M=2,4 and 8 versus the number of training symbols at SNR=30dB. The performance improvement is significant before one hundred training symbols. The D performance criterion versus SNR for N=M=2,4 and 8 shows in Fig. 5 when one thousand symbols are used for training. We can see that increasing the number of transmitting and receiving antennas degrades the performance significantly. Also, the increase in performance is not substantial for SNR>15dB. In the computer simulations, the training sequence vectors are selected in a random manner such that $\boldsymbol{E}[\boldsymbol{s}^{(l)}(k)\boldsymbol{s}^{(l)}(j)^H] = 0$ for all $k \neq j$. It should be noted that the convergence of the estimation algorithm can be improved by selecting an orthogonal training sequence.

V. CONCLUSIONS

The SVD is an efficient method to exploit the capability of MIMO systems. However, obtaining the SVD from the estimated channel matrix may produce enormous error due to nonlinearity issues. An adaptive SVD estimation algorithm has been derived in this paper for MIMO-OFDM systems. The SVD is estimated directly from the received signal for each subcarrier MIMO channel in the OFDM system. The adaptive algorithm estimates the SVD based on the linear constraint LMS method such that the constraint satisfies the unitary property of the SVD matrices. The performance of the proposed adaptive algorithm has been evaluated by computer simulations for different transmitting and receiving antennas.

REFERENCES

[1] H. Sempath, P. Stoica and A. Paularj, "Generalized linear precoder and coder design for MIMO channels using the weighed MMSE criterion," *IEEE Trans. Commun.*, vol. 49, no. 12, pp. 2198-2206, Dec. 2001.

[2] G. Q. Sun, D. C. Cox, H. C. Huanq and A. Lozano, "Estimation of continuous flat fading MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 1, pp. 449-553, Oct. 2002.

[3] Y. (G.) Li, "Simplified channel estimation for OFDM systems with multiple transmit antennas," *IEEE Trans. Wireless Commun.*, vol. 1, Jan. 2002.

[4] M. Shin , H. Lee and C. Lee, " Enhanced channel-estimation technique for MIMO-OFDM systems," *IEEE Trans. Vehicular Technology*, vol. 53, pp. 261-265. Jan. 2004.

[5] G. Lebrun, S. Spiteri and M. Faulkner, "Channel estimation for an SVD-MIMO system," *IEEE International Conference on Communications* 2004, vol. 5, pp. 3025-3029, June 2004.

[6] B. Farhang-Boroujeny, *Adaptive Filters: Theory and Applications.* New York: John Wiley, 1998.