Iterative MIMO Channel SVD Estimation

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***Abstract*— An iterative algorithm is proposed in this paper to estimate the singular value decomposition (SVD) of multiple-input multiple-output (MIMO) channel from the received signal. The proposed algorithm is based on the constrained minimum mean-square error (MMSE) criterion. For different numbers of transmitter and receiver antennas, simulation results show that the iterative algorithm achieves good performance with respect to the SVD estimation of the MIMO channel matrix.**

***Index Terms*— MIMO systems, MIMO channels estimation, SVD estimation, constrained MMSE.**

I. Introduction

Multiple-input multiple-output (MIMO) communication systems increase capacity and improve the bandwidth efficiency in rich scattering environments [1]. The performance of the MIMO system depends highly on the accuracy of channel state information. Although different techniques have been proposed to estimate the channel matrix [2]-[4], using the singular value decomposition (SVD) of the channel matrix is an efficient approach to obtain the channel state information and design coding, decoding and detection algorithms in MIMO communication systems [5]. Due to the nonlinearity issue, the procedure of obtaining the SVD from the estimated channel matrix may create more errors if the estimation of the channel matrix is not precise enough [6]. In this paper, an iterative algorithm is proposed to estimate the SVD of the MIMO channel matrix directly from the received signal. The algorithm is developed based on the constrained minimum mean-square error (MMSE) criterion.

The paper is organized as follows. The system model of a MIMO narrowband channel is described based on the SVD method in Section II. The iterative algorithm is developed in Section III. The performance of the proposed estimation algorithm is presented by computer simulations in Section IV and Section V contains conclusions.

II. System Model

We consider a MIMO communication system consisting of N transmitter antennas and M receiver antennas with a Rayleigh flat fading channel. When $\mathbf{s}(k) = [s_0(k),..., s_{N-1}(k)]^T$ is the transmitted signal vector, the received signal vector, $\mathbf{x}(k) = [x_0(k),..., x_{M-1}(k)]^T$, can be expressed as

$$\mathbf{x}(k) = \boldsymbol{H}\mathbf{s}(k) + \mathbf{n}(k) \tag{1}$$

where $\boldsymbol{H}$ is the $M \times N$ channel matrix, $\mathbf{n}(k)$ is the $M \times 1$ additive white Gaussian noise (AWGN) vector with zero mean and autocorrelation matrix $R_{\mathbf{n}}(l) = \sigma_n^2 \boldsymbol{I}_{\boldsymbol{M}} \delta(l)$, while $\boldsymbol{I}_{\boldsymbol{M}}$ is the $M \times M$ identity matrix. The SVD of the channel matrix, $\boldsymbol{H}$, can be given as

$$\boldsymbol{H} = \boldsymbol{U}\Sigma\boldsymbol{V}^H \tag{2}$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are $M \times P$ and $N \times P$ unitary matrices, respectively. Note that P is the rank of $\boldsymbol{H}$ where $P \le \min(M, N)$. $\Sigma = \operatorname{diag}(\sigma_1, \sigma_2, \cdots, \sigma_P)$ is a diagonal matrix containing the singular values of the channel matrix and $(.)^H$ denotes transposed complex conjugate.

Our aim is to estimate $\boldsymbol{U}$, $\boldsymbol{V}$ and Σ matrices directly at the receiver based on a training sequence. The estimation procedure can be developed by minimizing the mean square error (MSE) criterion as follows.

$$J = E\left[\left\|\mathbf{x}(k) - \boldsymbol{U}\Sigma\boldsymbol{V}^H\mathbf{s}(k)\right\|^2\right] \tag{3}$$

Due to the unitary property of $\boldsymbol{U}$ and $\boldsymbol{V}$ matrices, minimization of the criterion in (3) should be attained under nonlinear constraints $\boldsymbol{U}^H\boldsymbol{U} = \boldsymbol{I}_{\boldsymbol{P}}$ and $\boldsymbol{V}^H\boldsymbol{V} = \boldsymbol{I}_{\boldsymbol{P}}$. Estimation of the $\boldsymbol{U}$, $\boldsymbol{V}$ and Σ matrices under the nonlinear constraints is a very complicated procedure that seems less practical. In the next section, we will propose an iterative approach that leads to a linear constraint for estimating the SVD of $\boldsymbol{H}$ based on the MMSE criterion.

III. Iterative SVD Estimation

The SVD of the channel matrix can be written as follows.

$$\boldsymbol{H} = \boldsymbol{U}\Sigma\boldsymbol{V}^H = \boldsymbol{W_1}\boldsymbol{V}^H = \boldsymbol{U}\boldsymbol{W_2}^H \tag{4}$$

where

$$\boldsymbol{W_1} = \boldsymbol{U}\Sigma \tag{5}$$

$$\boldsymbol{W_2} = \boldsymbol{V}\Sigma \tag{6}$$

while the diagonal elements of $\Sigma = \operatorname{diag}(\sigma_1, \sigma_2, \cdots, \sigma_P)$

matrix are positive values. Defining $\mathbf{u}_i$ and $\mathbf{v}_i$ as the *i*th columns of $\boldsymbol{U}$ and $\boldsymbol{V}$, respectively, we have

$$\mathbf{w}_{1i} = \boldsymbol{H}\mathbf{v}_i = \sigma_i \mathbf{u}_i \tag{7}$$

$$\mathbf{w}_{2i}^{H} = \mathbf{u}_i^{H}\boldsymbol{H} = \sigma_i \mathbf{v}_i^{H} \tag{8}$$

where $\mathbf{w}_{1i}$ and $\mathbf{w}_{2i}$ are the *i*th columns of $\boldsymbol{W_1}$ and $\boldsymbol{W_2}$, respectively. By using (7) and (8), one can write

$$\mathbf{Y}_1(k) = \mathbf{x}(k)\mathbf{s}(k)^H \boldsymbol{V} = \boldsymbol{H}\mathbf{s}(k)\mathbf{s}(k)^H \boldsymbol{V} + \boldsymbol{Z}_1(k) \tag{9}$$

$$\mathbf{Y}_2(k) = \boldsymbol{U}^H \mathbf{x}(k)\mathbf{s}(k)^H = \boldsymbol{W_2}^H \mathbf{s}(k)\mathbf{s}(k)^H + \boldsymbol{Z}_2(k) \tag{10}$$

where $\boldsymbol{Z}_1(k) = \boldsymbol{n}(k)\mathbf{s}(k)^H \boldsymbol{V}$ and $\boldsymbol{Z}_2(k) = \boldsymbol{U}^H \boldsymbol{n}(k)\boldsymbol{s}(k)^H$. Assuming the training sequence is an independent and identically distributed (iid) signal such that $E[\mathbf{s}(k)] = 0$, $R_s = E[\mathbf{s}(k)\mathbf{s}(k)^H] = \sigma_s^2 \boldsymbol{I_N}$ and $E[\mathbf{s}(k)\mathbf{n}(l)^H] = \mathbf{0}$ for all k and l, it is straightforward to show that $E[\boldsymbol{Z}_1(k)] = \mathbf{0}$, $E[\boldsymbol{Z}_2(k)] = \mathbf{0}$, $E[\boldsymbol{Z}_1(k)^H \boldsymbol{Z}_1(k)] = M\sigma_s^2\sigma_n^2 \boldsymbol{I_P}$ and $E[\boldsymbol{Z}_2(k)\boldsymbol{Z}_2(k)^H] = N\sigma_s^2\sigma_n^2 \boldsymbol{I_P}$. Without loss of generality we assume that $\sigma_s^2 = 1$. Thus, from (9) and (10), one can show that

$$\boldsymbol{W_1} = E[\mathbf{Y}_1(k)] \tag{11}$$

$$\boldsymbol{W_2} = E[\mathbf{Y}_2(k)]^H \tag{12}$$

To achieve our objective, the SVD of $\boldsymbol{H}$ is estimated in two steps in an iterative manner based on (11) and (12). In the first step, from (11) the columns of $\boldsymbol{W_1}$ matrix are estimated by assumption that the $\boldsymbol{V}$ estimation is available and in the second step, the columns of $\boldsymbol{W}_2$ are estimated from (12) based on the previous estimation of $\boldsymbol{U}$.

A. Step I

The estimation criterion of $\boldsymbol{W_1}$ based on the MMSE can be given as

$$J_1 = E\left[\left\|\mathbf{E}_1(k)\right\|_F^2\right] = E\left[\left\|\boldsymbol{W_1} - \mathbf{Y}_1(k)\right\|_F^2\right] \tag{13}$$

where $\|\cdot\|_F$ denotes Frobenius norm and $\mathbf{E}_1(k) = [\boldsymbol{e}_{11}(k), \cdots, \boldsymbol{e}_{1P}(k)]$. If it is assumed that the $\hat{\boldsymbol{V}}^{(l-1)}$ is the estimation of $\boldsymbol{V}$ at the (*l-1*)th iteration, the estimation criterion of $\boldsymbol{W_1}$ at the *l*th iteration becomes

$$J_1^{(l)} = E\left[\left\|\mathbf{E}_1^{(l)}(k)\right\|_F^2\right] = E\left[\left\|\boldsymbol{W_1}(k) - \mathbf{Y}_1^{(l-1)}(k)\right\|_F^2\right] \tag{14}$$

where $\mathbf{Y}_1^{(l-1)}(k) = \mathbf{x}(k)\mathbf{s}(k)^H \hat{\boldsymbol{V}}^{(l-1)}$. Since $E[\mathbf{z}_{1i}(k)^H \mathbf{z}_{1j}(k)] = 0$ for $i \neq j$, while $\mathbf{z}_{1i}(k)$ is the *i*th column of $\boldsymbol{Z}_1(k)$, the criterion of (14) can be modified as

$$J_1^{(l)} = \sum_{i=1}^{P} E\left[\left|\boldsymbol{e}_{1i}^{(l)}(k)\right|^2\right] = \sum_{i=1}^{P} E\left[\left|\mathbf{w}_{1i}(k) - \mathbf{y}_{1i}^{(l-1)}(k)\right|^2\right] \tag{15}$$

where $\mathbf{w}_{1i}$ and $\mathbf{y}_{1i}^{(l-1)}(k)$ are the *i*th columns of $\boldsymbol{W_1}$ and $\mathbf{Y}_1^{(l-1)}(k)$, respectively. From (9), $\mathbf{y}_{1i}^{(l-1)}(k)$ can be given as

$$\mathbf{y}_{1i}^{(l-1)}(k) = \mathbf{x}(k)\mathbf{s}(k)^H \hat{\mathbf{v}}_i^{(l-1)} \tag{16}$$

where $\hat{\mathbf{v}}_i^{(l-1)}$ is the estimation of the *i*th column of $\boldsymbol{V}$ at the (*l-1*)th iteration. Thus the minimizing of $J_1^{(l)}$ can be achieved by minimizing the $E\left[\left|\boldsymbol{e}_{1i}^{(l)}(k)\right|^2\right]$ for $i = 1, \cdots, P$ separately. Moreover, $J_1^{(l)}$ should be minimized under the constraint of $\mathbf{w}_{1i}^H \mathbf{w}_{1j} = 0$ for all $i \neq j$. Using the Lagrange multiplier method in order to satisfy the constraint, the criterion of estimating $\mathbf{w}_{1i}$ at the *l*th iteration is given as

$$\begin{aligned}\hat{\mathbf{w}}_{1i}^{(l)} &= \arg\min_{\mathbf{w}_{1i}}\{\zeta_{1i}^{(l)}\}\\ &= \arg\min_{\mathbf{w}_{1i}}\left\{E\left[\left|\boldsymbol{e}_{1i}^{(l)}(k)\right|^2\right] + \boldsymbol{\lambda}_{1i}^{(l)H}\hat{\boldsymbol{W}}_{1i}^{(l)H}\mathbf{w}_{1i}\right\}\\ &\qquad \text{for } i = 1, \cdots, P\end{aligned} \tag{17}$$

where $\lambda_{1i}^{(l)} = [\lambda_{11}^{(l)}, \cdots, \lambda_{1i-1}^{(l)}]^T$ is the Lagrange multiplier vector and $\hat{\boldsymbol{W}}_{1i}^{(l)} = [\hat{\mathbf{w}}_{11}^{(l)}, \cdots, \hat{\mathbf{w}}_{1i-1}^{(l)}]$ is an $M \times (i-1)$ matrix. Thus, in order to satisfy the orthogonality property of the columns of $\boldsymbol{W_1}$, the $\zeta_{1i}^{(l)}$ is minimized under the following constraint.

$$\hat{\boldsymbol{W}}_{1i}^{(l)H}\mathbf{w}_{1i} = 0 \qquad \text{for } i = 1, \cdots, P \tag{18}$$

By taking the derivation of $\zeta_{1i}^{(l)}$ with respect to $\mathbf{w}_{1i}$ and doing some manipulations, it can be shown

$$\hat{\mathbf{w}}_{1i}^{(l)} = \mathbf{P}\hat{\mathbf{v}}_i^{(l-1)} - \hat{\boldsymbol{W}}_{1i}^{(l)}\lambda_{1i}^{(l)} \qquad \text{for } i = 1, \cdots, P \tag{19}$$

where $\mathbf{P} = E[\mathbf{x}(k)\mathbf{s}(k)^H]$. By substituting $\hat{\mathbf{w}}_{1i}^{(l)}$ in (18) and applying the constraint, we have

$$\lambda_{1i}^{(l)} = (\hat{\boldsymbol{W}}_{1i}^{(l)H}\hat{\boldsymbol{W}}_{1i}^{(l)})^{-1}\hat{\boldsymbol{W}}_{1i}^{(l)H}\mathbf{P}\hat{\mathbf{v}}_i^{(l-1)} \qquad (20)$$

After substituting $\lambda_{1i}^{(l)}$ from (20) in (19), the estimation of $\mathbf{w}_{1i}$ at the lth iteration becomes

$$\hat{\mathbf{w}}_{1i}^{(l)} = \left(\boldsymbol{I}_M - \hat{\boldsymbol{W}}_{1i}^{(l)}(\hat{\boldsymbol{W}}_{1i}^{(l)H}\hat{\boldsymbol{W}}_{1i}^{(l)})^{-1}\hat{\boldsymbol{W}}_{1i}^{(l)H}\right)\mathbf{P}\hat{\mathbf{v}}_i^{(l-1)} \quad \text{for } i = 1,\cdots,P \qquad (21)$$

Thus, the $\boldsymbol{W}_1$ matrix can be estimated based on the constrained MMSE criterion at the lth iteration by computing $\hat{\mathbf{w}}_{1i}^{(l)}$ from (21) for $i = 1,\cdots,P$. The estimation of the ith column of $\boldsymbol{U}$ at the lth iteration, $\hat{\mathbf{u}}_i^{(l)}$, can be obtained by

$$\hat{\mathbf{u}}_i^{(l)} = \hat{\mathbf{w}}_{1i}^{(l)}(\hat{\mathbf{w}}_{1i}^{(l)H}\hat{\mathbf{w}}_{1i}^{(l)})^{-\frac{1}{2}} \quad \text{for } i = 1,\cdots,P \qquad (22)$$

B. Step II

Similar to step *I*, the $\boldsymbol{V}$ can be estimated by estimating $\boldsymbol{W}_2$ matrix. The MMSE criterion at the lth iteration is given as

$$J_2^{(l)} = E\left[\left\|\mathbf{E}_2^{(l)}(k)\right\|_F^2\right] = E\left[\left\|\boldsymbol{W}_2(k) - \mathbf{Y}_2^{(l)}(k)^H\right\|_F^2\right] \qquad (23)$$

where $\mathbf{E}_2^{(l)}(k) = [\boldsymbol{e}_{21}^{(l)}(k),\cdots,\boldsymbol{e}_{2P}^{(l)}(k)]$ and $\mathbf{Y}_2^{(l)}(k)^H = \mathbf{s}(k)\mathbf{x}(k)^H\hat{\boldsymbol{U}}^{(l)}$. Since $E[\boldsymbol{Z}_2(k)\boldsymbol{Z}_2(k)^H]$ is a diagonal matrix, the criterion of (23) can be given as

$$J_2^{(l)} = \sum_{i=1}^{P} E\left[\left|\boldsymbol{e}_{2i}^{(l)}(k)\right|^2\right] = \sum_{i=1}^{P} E\left[\left|\mathbf{w}_{2i}(k) - \mathbf{y}_{2i}^{(l)}(k)^H\right|^2\right] \qquad (24)$$

where $\mathbf{w}_{2i}$ is the ith column of $\boldsymbol{W}_2$ and $\mathbf{y}_{2i}^{(l)}(k)^H$ is the ith column of $\mathbf{Y}_2^{(l)}(k)^H$ that is defined as

$$\mathbf{y}_{2i}^{(l)}(k)^H = \mathbf{s}(k)\mathbf{x}(k)^H\hat{\mathbf{u}}_i^{(l)} \qquad (25)$$

Similar to $J_1^{(l)}$ in step *I*, $J_2^{(l)}$ should be minimized under the constraint of $\mathbf{w}_{2i}^H\mathbf{w}_{2j} = 0$ for all $i \neq j$. Defining $\boldsymbol{\lambda}_{2i}^{(l)} = [\lambda_{21}^{(l)},\cdots,\lambda_{2i-1}^{(l)}]^T$ as the Lagrange multiplier vector and $\hat{\boldsymbol{W}}_{2i}^{(l)} = [\hat{\mathbf{w}}_{21}^{(l)},\cdots\hat{\mathbf{w}}_{2i-1}^{(l)}]$, the estimation criterion of $\mathbf{w}_{2i}$ at the lth iteration is given as

$$\hat{\mathbf{w}}_{2i}^{(l)} = \arg\min_{\mathbf{w}_{2i}}\{\zeta_{2i}^{(l)}\}$$

$$\hat{\mathbf{w}}_{2i}^{(l)} = \arg\min_{\mathbf{w}_{2i}}\left\{E\left[\left|\boldsymbol{e}_{2i}^{(l)}(k)\right|^2\right] + \boldsymbol{\lambda}_{2i}^H\hat{\boldsymbol{W}}_{2i}^{(l)H}\mathbf{w}_{2i}\right\} \quad \text{for } i = 1,\cdots,P \qquad (26)$$

By minimizing $\zeta_{2i}^{(l)}$ with respect to $\mathbf{w}_{2i}$ under the following constraint

$$\hat{\boldsymbol{W}}_{2i}^{(l)H}\mathbf{w}_{2i} = 0 \quad \text{for } i = 1,\cdots,P \qquad (27)$$

one can show

$$\hat{\mathbf{w}}_{2i}^{(l)} = \mathbf{P}^H\hat{\mathbf{u}}_i^{(l)} - \hat{\boldsymbol{W}}_{2i}^{(l)}\lambda_{2i}^{(l)} \quad \text{for } i = 1,\cdots,P \qquad (28)$$

$\lambda_{2i}^{(l)}$ can be obtained by substituting $\hat{\mathbf{w}}_{2i}^{(l)}$ in (27) and applying the constraint.

$$\lambda_{2i}^{(l)} = (\hat{\boldsymbol{W}}_{2i}^{(l)H}\hat{\boldsymbol{W}}_{2i}^{(l)})^{-1}\hat{\boldsymbol{W}}_{2i}^{(l)H}\mathbf{P}^H\hat{\mathbf{u}}_i^{(l)} \qquad (29)$$

By replacing $\lambda_{2i}^{(l)}$ from (29) in (28), the estimation of $\mathbf{w}_{2i}$ at the lth iteration becomes

$$\hat{\mathbf{w}}_{2i}^{(l)} = \left(\boldsymbol{I}_N - \hat{\boldsymbol{W}}_{2i}^{(l)}(\hat{\boldsymbol{W}}_{2i}^{(l)H}\hat{\boldsymbol{W}}_{2i}^{(l)})^{-1}\hat{\boldsymbol{W}}_{2i}^{(l)H}\right)\mathbf{P}^H\hat{\mathbf{u}}_i^{(l)} \quad \text{for } i = 1,\cdots,P \qquad (30)$$

The estimation of $\boldsymbol{W}_2$ at the lth iteration can be obtained based on the constrained MMSE criterion by using (30) to estimate $\mathbf{w}_{2i}$ for $i = 1,\cdots,P$. Also from (30) the estimation of the ith column of $\boldsymbol{V}$ at the lth iteration, $\hat{\mathbf{v}}_i^{(l)}$, becomes

$$\hat{\mathbf{v}}_i^{(l)} = \hat{\mathbf{w}}_{2i}^{(l)}(\hat{\mathbf{w}}_{2i}^{(l)H}\hat{\mathbf{w}}_{2i}^{(l)})^{-\frac{1}{2}} \quad \text{for } i = 1,\cdots,P \qquad (31)$$

Meanwhile, the ith diagonal element of Σ at the lth iteration can be estimated by

$$\hat{\sigma}_i^{(l)} = (\hat{\mathbf{w}}_{2i}^{(l)H}\hat{\mathbf{w}}_{2i}^{(l)})^{\frac{1}{2}} \quad \text{for } i = 1,\cdots,P \qquad (32)$$

Note that due to the orthogonality property, $\hat{\boldsymbol{W}}_{1i}^{(l)H}\hat{\boldsymbol{W}}_{1i}^{(l)}$ and $\hat{\boldsymbol{W}}_{2i}^{(l)H}\hat{\boldsymbol{W}}_{2i}^{(l)}$ are diagonal matrices and computing the inverse of them needed in (21) and (30) are straightforward.

C. Algorithm procedure

After choosing the initial value of $\boldsymbol{V}$, say $\hat{\boldsymbol{V}}^{(0)}$, the iterative algorithm can be implemented by employing step *I* and step *II* for $i = 1,\cdots,P$ in order to estimate $\mathbf{u}_i$ and $\mathbf{v}_i$ at each iteration. The procedure of the iterative algorithm can be summarized as follows.

1) Determination of $\mathbf{P} = E[\mathbf{x}(k)\mathbf{s}(k)^H]$

For $i = 1,\cdots,P$ $\{$

For $l = 1,2,\cdots\{$

2) Step *I*:

$$\hat{\mathbf{w}}_{1i}^{(l)} = \left(\boldsymbol{I}_M - \hat{\boldsymbol{W}}_{1i}^{(l)}(\hat{\boldsymbol{W}}_{1i}^{(l)H}\hat{\boldsymbol{W}}_{1i}^{(l)})^{-1}\hat{\boldsymbol{W}}_{1i}^{(l)H}\right)\mathbf{P}\hat{\mathbf{v}}_i^{(l-1)}$$

$$\hat{\mathbf{u}}_i^{(l)} = \hat{\mathbf{w}}_{1i}^{(l)}(\hat{\mathbf{w}}_{1i}^{(l)H}\hat{\mathbf{w}}_{1i}^{(l)})^{-\frac{1}{2}}$$

3) Step *II*:

$$\hat{\mathbf{w}}_{2i}^{(l)} = \left(\boldsymbol{I}_N - \hat{\boldsymbol{W}}_{2i}^{(l)}(\hat{\boldsymbol{W}}_{2i}^{(l)H}\hat{\boldsymbol{W}}_{2i}^{(l)})^{-1}\hat{\boldsymbol{W}}_{2i}^{(l)H}\right)\mathbf{P}^H\hat{\mathbf{u}}_i^{(l)}$$

$$\hat{\mathbf{v}}_i^{(l)} = \hat{\mathbf{w}}_{2i}^{(l)}(\hat{\mathbf{w}}_{2i}^{(l)H}\hat{\mathbf{w}}_{2i}^{(l)})^{-\frac{1}{2}}$$

$$\hat{\sigma}_i^{(l)} = (\hat{\mathbf{w}}_{2i}^{(l)H}\hat{\mathbf{w}}_{2i}^{(l)})^{\frac{1}{2}}$$

$\}$

$\}.$

The iterative algorithm for estimating $\mathbf{u}_i$ and $\mathbf{v}_i$ can be terminated when at the l'th iteration the following condition is satisfied

$$\left\|\hat{\boldsymbol{H}}_i^{(l')} - \hat{\boldsymbol{H}}_i^{(l'-1)}\right\|_F^2 \le \varepsilon_i \tag{33}$$

where ε_i is a small positive value and $\hat{\boldsymbol{H}}_i^{(l')}$ is defined as

$$\hat{\boldsymbol{H}}_i^{(l')} = \hat{\mathbf{u}}_i^{(l')}\hat{\sigma}_i^{(l')}\mathbf{v}_i^{(l')H} \tag{34}$$

III. SIMULATIONS

A MIMO communication system that consists of N transmitter antennas and M receiver antennas with a channel matrix, $\boldsymbol{H}$, has been considered for simulations in order to evaluate the performance of the proposed iterative SVD estimation algorithm. The channel matrix has been modeled for $N = M = 2,4$ and 8 in system simulations. At each model, five hundred channel matrices have been generated randomly such that the elements of each $\boldsymbol{H}$ are mutually independent complex Gaussian random variables with zero-mean and variance one. A sequence of independent and identically distributed (iid) 16QAM training signal vector, $\mathbf{s}(k)$, is sent from transmitter antennas such that $R_{\mathbf{s}} = \boldsymbol{I}_N$. The power of the noise vector, $\mathbf{n}(k)$, with zero-mean and $R_{\mathbf{n}} = E[\mathbf{n}(k)\mathbf{n}(k)^H] = \sigma_n^2\boldsymbol{I}_M$ is adjusted in order to achieve the SNR defined as follows:

$$\text{SNR} = \frac{E[|\boldsymbol{H}\mathbf{s}(k)|^2]}{E[|\boldsymbol{n}(k)|^2]} \tag{35}$$

We use the normalized mean-square error (NMSE) as the estimator performance criterion that is defined as

$$\text{NMSE }(\hat{\boldsymbol{H}}) = \frac{E\left[\left\|\hat{\boldsymbol{H}} - \boldsymbol{H}\right\|_F^2\right]}{E\left[\left\|\boldsymbol{H}\right\|_F^2\right]} \tag{36}$$

where $\hat{\boldsymbol{H}}$ is the estimation of $\boldsymbol{H}$. Note that the MIMO channel $\boldsymbol{H}$ is estimated based on its SVD estimation from following relation.

$$\hat{\boldsymbol{H}} = \hat{\boldsymbol{U}}\hat{\Sigma}\hat{\boldsymbol{V}}^{\boldsymbol{H}} \tag{37}$$

The performance of the iterative MIMO channel SVD estimation algorithm based on the normalized MSE criterion defined in (36) for *N*=*M*=2,4 and 8 channel types are shown in Fig. 1, Fig. 2 and Fig. 3, respectively. In these figures, the performance is shown for two different training sequence lengths containing *L*=100 and 500 symbol vectors when the number of iterations, NI=1,2 and 4. As can be seen, the performance of the estimator improves significantly in the second iteration in comparison with the first iteration. Also, the NMSE of the channel estimation decreases by increasing the training sequence length. However, as the figures show, increasing the number of the transmitter and receiver antennas reduces the estimator performance.

Figure 4 shows the performance of the estimation algorithm based on the number of iterations, NI. The performance improvement is insignificant after the fourth iteration. The impact of training sequence length, *L*, on the performance of the estimation algorithm is evaluated in Fig. 5 for *N*=*M*=2, 4 and 8 at SNR=30dB. Although increasing the length of the training sequence up to five hundred symbol vectors increases the performance of estimator substantially, the trend does not remain for training sequence length beyond five hundred symbol vectors. Meanwhile, it should be noted that the estimated $\boldsymbol{U}$ and $\boldsymbol{V}$ matrices always satisfy the unitary property when the number of transmitted symbol vectors is more than the maximum number of transmitter/receiver antennas.

V. CONCLUSIONS

An iterative algorithm has been developed in this paper to estimate the SVD of the MIMO channels. The algorithm has been derived based on the constrained minimum mean square error (MMSE) criterion. Computer simulations have shown the algorithm achieves a good performance after the first iteration when the channel matrix has been estimated based on the SVD estimation. Also, the estimation algorithm satisfies the unitary property of the SVD matrices perfectly when the number of the transmitted symbol vectors is more than the number of the transmitter or receiver antennas, whichever has more antennas.

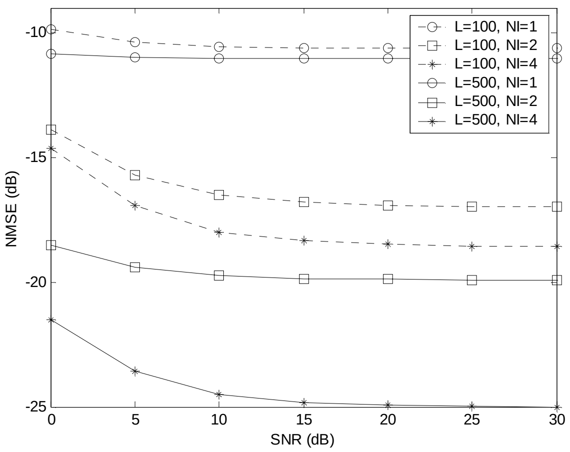

Fig. 1. The normalized MSE of the channel matrix estimation, ***H***, using the iterative SVD estimation algorithm versus SNR for different number of iterations (NI) and training length (*L*) when *N*=*M*=2.

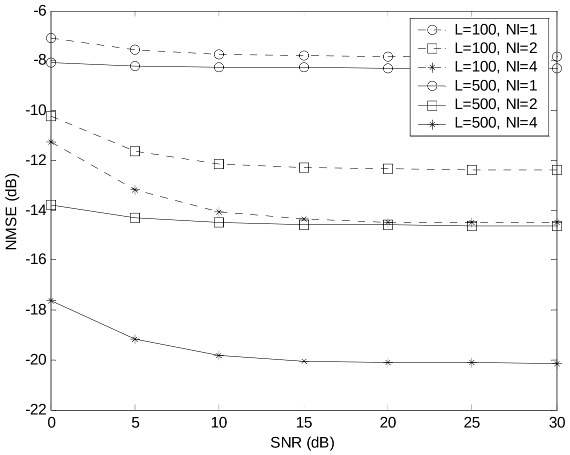

Fig. 2. The normalized MSE of the channel matrix estimation, ***H***, using the iterative SVD estimation algorithm versus SNR for different number of iterations (NI) and training length (*L*) when *N*=*M*=4.

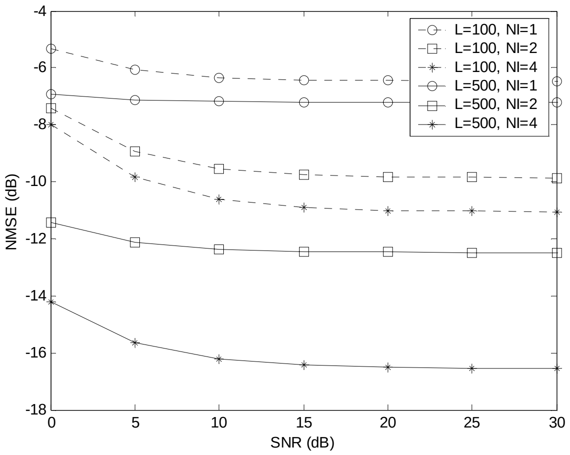

Fig. 3. The normalized MSE of the channel matrix estimation, ***H***, using the iterative SVD estimation algorithm versus SNR for different number of iterations (NI) and training length (*L*) when *N*=*M*=8.

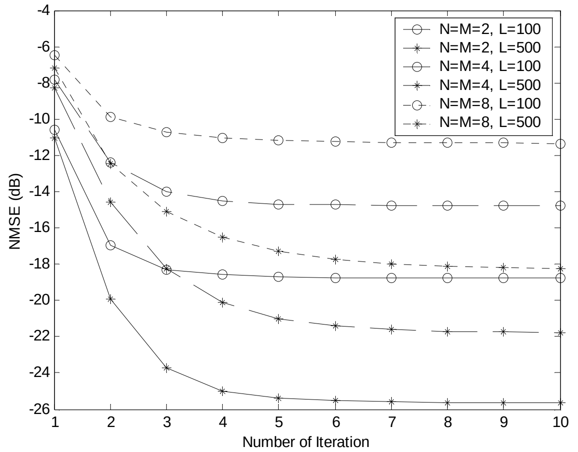

Fig. 4: The normalized MSE of the channel matrix estimation, ***H***, using the iterative SVD estimation algorithm as a function of the number of iterations for *N*=*M*=2,4and 8 at SNR=30dB with training lengths, *L*=100 and 500 symbol vectors.

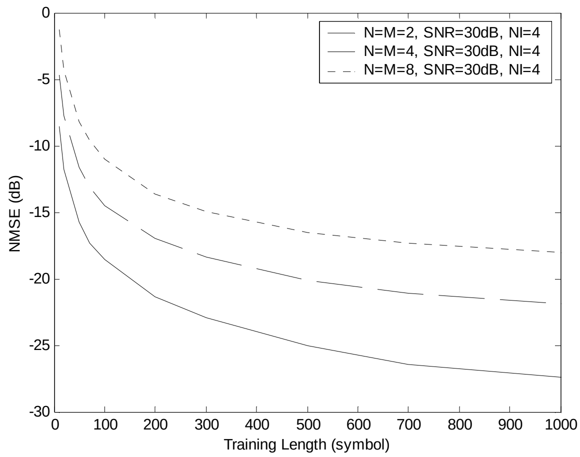

Fig. 5: The normalized MSE of the channel matrix estimation, ***H***, using the iterative SVD estimation algorithm based on different training lengths for *N*=*M*=2,4 and 8 at SNR=30dB when the number of iterations NI=4.

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