



## A DYNAMIC OFFSETTING APPROACH TO TOOL PATH GENERATION FOR MACHINING CONVEX POCKETS

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**Abstract** -- This paper presents a method to automatically generate NC toolpaths with dynamic elimination of machining errors in 2½D arbitrarily shaped convex pockets. The proposed method generates a spiral-like tool path by computing optimal offsets of the pocket boundary contour based on the size of the milling cutter and the pocket contour geometry. This dynamic tool path generating technique is demonstrated herein with an example.

### 1. INTRODUCTION

In an automated manufacturing environment, a large amount of time is spent in transforming a design model to a product model. Most often, the planning time, of which NC part programming accounts for a sizable portion, may exceed the actual execution time. It has been found that in conventional tool path planning methods, offsetting distances are mostly approximated. This is mainly due to inadequate capabilities of CAD programs to determine the included angles in a contour containing circular arc segments. This in turn can result in machining errors, tolerance violations, and increased machining time.

The problem of undercutting has been addressed in a number of tool path planning studies. Some notable work has been carried out by Held *et al* [1] and Tsai *et al* [2] using the concept of Voronoi diagrams. However, the generation of Voronoi diagrams is itself a non-trivial task. In view of these limitations, a tool path generating procedure was developed without a CAD modeller for machining 2½D arbitrarily shaped convex pockets which dynamically eliminates undercutting errors.

### 2. METHODOLOGY

The structure of the proposed tool path planning procedure is detailed in Figure 1, and consists of four main stages. In the first stage the contour elements of the pocket are defined with equations, and classified with developed conventions depending on the offsetting direction. In the second stage the vertex angles between the contour elements are determined. The maximum offset that avoids undercutting is then determined based on the cutter size and the angles between the segments of the pocket contour. Equations of the contour elements at this optimal offset distance are then solved to get the intersection points in the offset loop. This procedure is repeated to generate multiple offsets of the contour. In the third stage the offsetting is stopped when a terminal condition is reached and the surface is checked for any unmachined area. In the final stage the start point of the tool path is first located. The individual offset loops are then combined to form the spiral-like tool path by replacing old intersection points with new intersection points. The output files are then generated. The main aspects are elaborated below.

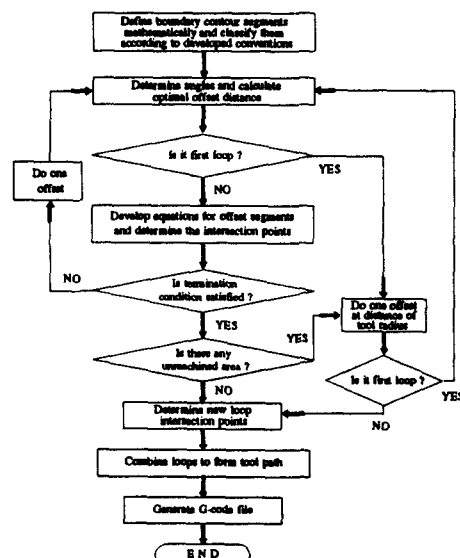


Figure 1 Structure of tool path method

**Stage 1.**

**(a) Definition:** The first step in this tool path planning procedure is to represent the machining surface in a mathematical form. As shown in Figure 2, Straight line segments are defined by equations in the standard form as:  $Ax + By + C = 0$  and circular arc segments are defined by equations in the standard form as:  $x^2 + y^2 + 2Dx + 2Ey + F = 0$

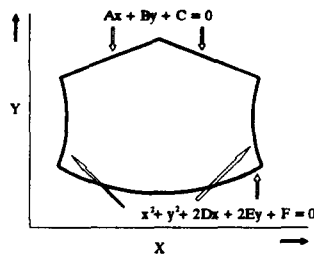


Figure 2 Defining the pocket

**(b) Classification:** In this part programming method all contour segments are first classified and indexed depending on their offsetting direction. These classifications are defined by conventions which were developed based on analytic geometry and hold true for a particular segment over its entire offsetting surface.

**Stage 2.**

**(a) Determining vertex angles:** The critical offset distance is a function of the minimum angle formed between adjacent segments on the boundary contour. Calculation of included angles at vertices (Figure 3), which constitute the contour, irrespective of the type of segments and their combinations are obtained using the general slope formula shown below:

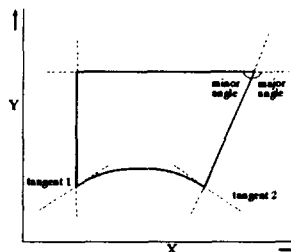


Figure 3 Contour vertex angles

$$\tan \theta = \frac{\tan \alpha_1 - \tan \alpha_2}{1 + \tan \alpha_1 \tan \alpha_2}$$

where  $\alpha_1$  = angle of the first adjacent segment with respect to X axis and  $\alpha_2$  = angle of the second adjacent segment with respect to X axis.

**(b) Calculating optimal offset distance:** When the tool diameter is used as the offset distance it leads to the problem of undercutting in which small triangular shaped areas at the vertices remain unmachined (Figure 4). Based on the geometry of the pocket contour and the size of the cutting tool, this proposed offsetting method calculates the maximum or critical offset distance that can be achieved between parallel passes without leaving any area unmachined. The critical offset distance is derived as:

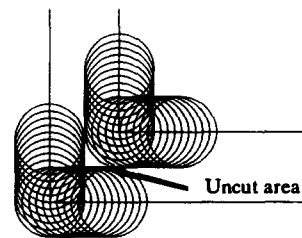


Figure 4 Undercutting errors

$$d = R \left( 1 + \sin \frac{\theta}{2} \right) \text{ where } R = \text{radius of the cutter and } \theta = \text{vertex angle}$$

**(c) Developing equations for offset segments:** To avoid gouging at the boundary walls, the first or initial offset loop is formed at a distance of  $R$ . Other offset loops are formed at a distance of  $d$  the critical offset distance. Equations of offset segments for forming the offset loops are given below.

Line segments  $\frac{Ax + By + C}{\pm \sqrt{A^2 + B^2}} = \pm d$   
 Arc segments  $(x - h)^2 + (y - k)^2 = (a \pm d)^2$

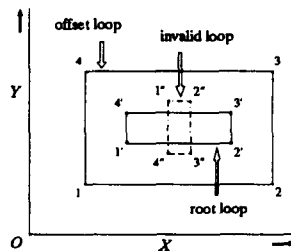


Figure 5 Terminal condition

**Stage 3:**

**(a) Offset termination:** The dynamic offsetting procedure is carried on till a termination condition is reached, i.e., an invalid loop is formed. Every contour vertex is indexed and initialized with inequality conditions. After every offset loop is calculated, the new inequality conditions are checked for orientation. When the orientation does not correspond to the previous loop the offsetting is terminated. This criterion is illustrated in Figure 5.

**(b) Checking for unmachined area:** When the termination criterion is

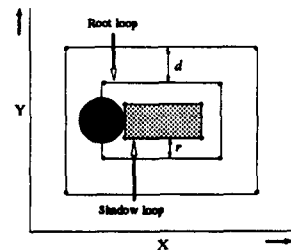


Figure 6 Error checking

satisfied, the following step is carried out to check for unmachined areas. As shown in Figure 6, the final or root loop is offset by the tool radius  $R$  and its orientation is checked with the initialized inequality conditions. If the sequence orientation is satisfied it implies that there is an unmachined area at the centre of the contour. The offset loop enclosing the unmachined area is called a shadow loop, and is then considered as the innermost loop. If the sequence orientation is not satisfied, the root loop is then taken to be the innermost loop of the contour offsets.

**Stage 4:**

**(a) Formation of the spiral-like tool path:**

The proposed path planning method maximizes the use of the linear and circular interpolation commands by combining the individual contour loops to form a single spiral-like shaped tool path. This procedure is carried out by calculating new intersection points and replacing the old intersection points. The centre point is then located and the spiral-like path is generated. This strategy is illustrated in Figure 7.

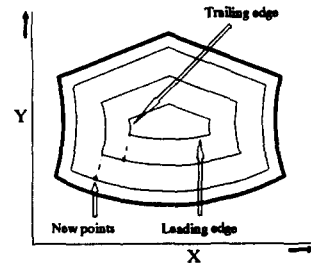


Figure 7 Spiral-like tool path

**(b) Generating the output:**

This tool path planning procedure outputs a standard G-code file. Once the tool path has been verified, it can be run on an NC machine directly.

**3. SYSTEM IMPLEMENTATION**

The tool path generation system based on the above components has been coded in C and implemented and tested on a 486 PC following the structure shown in Figure 1. An example part (Figure 8) consisting of Line-Line, Line-Arc, Arc-Line, and Arc-Arc segment combinations was machined. Line segments are denoted by their starting points and ending points, and circular arc segments are denoted by their starting points, ending points and radius. A flat end milling cutter of one inch diameter was chosen for this example. The main steps of the tool path generating procedure are illustrated below.

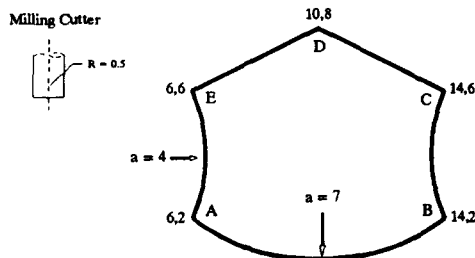


Figure 8 Input data for pocket

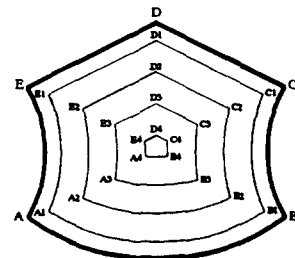


Figure 9 Contour offset loops

The input data available for the pocket is in the form of Cartesian coordinates as shown in Figure 8. An intermediate stage of individual contour loops after dynamic offsetting is shown in Figure 9. The final spiral-like tool path for the pocket is illustrated in Figure 10. The details of dynamic contour offset loops are shown in Table 1. The final tool path sequence and the generated part program (G-code) are given below.

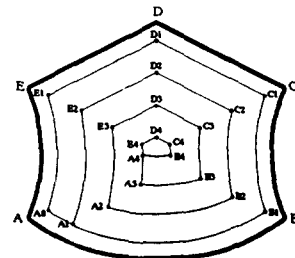


Figure 10 Final tool path pattern

**Final Tool path sequence:**

A4 - B4 - C4 - D4 - E4 - A3 - B3 - C3 - D3 - E3 - A2 - B2 - C2 - D2 - E2 - A1 - B1 - C1 - D1 - E1 - A0 - A1

Contour Loop	Minimum Vertex Angle in Degrees	Optimum Offset Distance	Vertex Point	Cartesian Coordinates
Boundary	86.56	0.5	A	6.00, 2.00
			B	14.00, 2.00
			C	14.00, 6.00
			D	10.00, 8.00
			E	6.00, 6.00
1	93.31	0.8636	A1	6.64, 2.17
			B1	13.35, 2.17
			C1	13.32, 5.77
			D1	10.00, 7.44
			E1	6.67, 5.77
2	98.74	0.8794	A2	7.71, 2.59
			B2	12.28, 2.59
			C2	12.27, 5.34
			D2	9.99, 6.47
			E2	7.72, 5.34
3	97.86	0.8769	A3	8.72, 3.16
			B3	11.27, 3.16
			C3	11.27, 4.85
			D3	9.99, 5.49
			E3	8.72, 4.85
4	94.13	0.8660	A4	9.65, 3.88
			B4	10.34, 3.88
			C4	10.35, 4.33
			D4	9.99, 4.51
			E4	9.64, 4.33

Table 1 Details of dynamic contour offset loops

**Part program (G-code) output for example**

```

N0 G71
N10 G94 F 0.5
N20 G96 S 1500
N30 G00 X9.654985 Y3.880030
N40 G03 X10.345016 Y3.880031 R3.879902
N50 G02 X10.351921 Y4.335663 R7.120098
N60 G01 X10.000000 Y4.511624
N70 G01 X9.648080 Y4.335665 xxx
N80 G02 X9.586231 Y3.005717 R7.120098
N90 G03 X11.277418 Y3.162416 R4.756875
N100 G02 X11.279441 Y4.852389 R6.243125
N110 G01 X10.000000 Y5.492110
N120 G01 X8.720561 Y4.852390
N130 G02 X8.543043 Y2.299759 R6.243125
N140 G03 X12.288153 Y2.593546 R5.636365
N150 G02 X12.270582 Y5.340118 R5.363635
N160 G01 X10.000000 Y6.475410
N170 G01 X7.729419 Y5.340120
N180 G02 X7.418219 Y1.779296 R5.363635
N190 G03 X13.350854 Y2.174843 R6.500000
N200 G02 X13.329493 Y5.776237 R4.500000
N210 G01 X10.000000 Y7.440983
N220 G01 X6.670507 Y5.776237
N230 G02 X6.649146 Y2.174843 R4.500000
N240 G03 X7.418219 Y1.779296 R6.500000
N250 M00

```

**4. CONCLUSION**

A tool path generation procedure for machining  $2\frac{1}{2}$  D arbitrarily shaped convex pockets has been developed. Tool path planning was done without the aid of solid modellers (CAD packages) and their related data exchange formats. This methodology generates a spiral-like tool path by dynamically computing optimal offsets of the pocket contour. As a result, the undercutting error is completely avoided (Figure 11).

Given geometric input data, the part programming system generates a G-code file which can be run on an NC machine directly. This system has achieved improvements with respect to part programming time. Machining time has been reduced by avoiding short segments and tool retractions. A significant improvement in surface finish and tool life can be obtained to a single mode of milling. More details are given in [3].

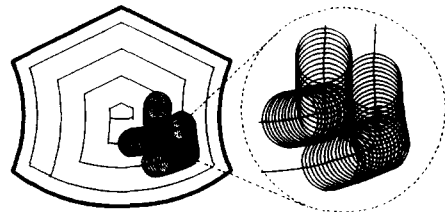


Figure 11 Elimination of undercutting

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- [3] Manuel, D., 1995, CNC tool path generation for finish milling convex pockets. *M.A.Sc., Thesis*. University of Ottawa, Canada.