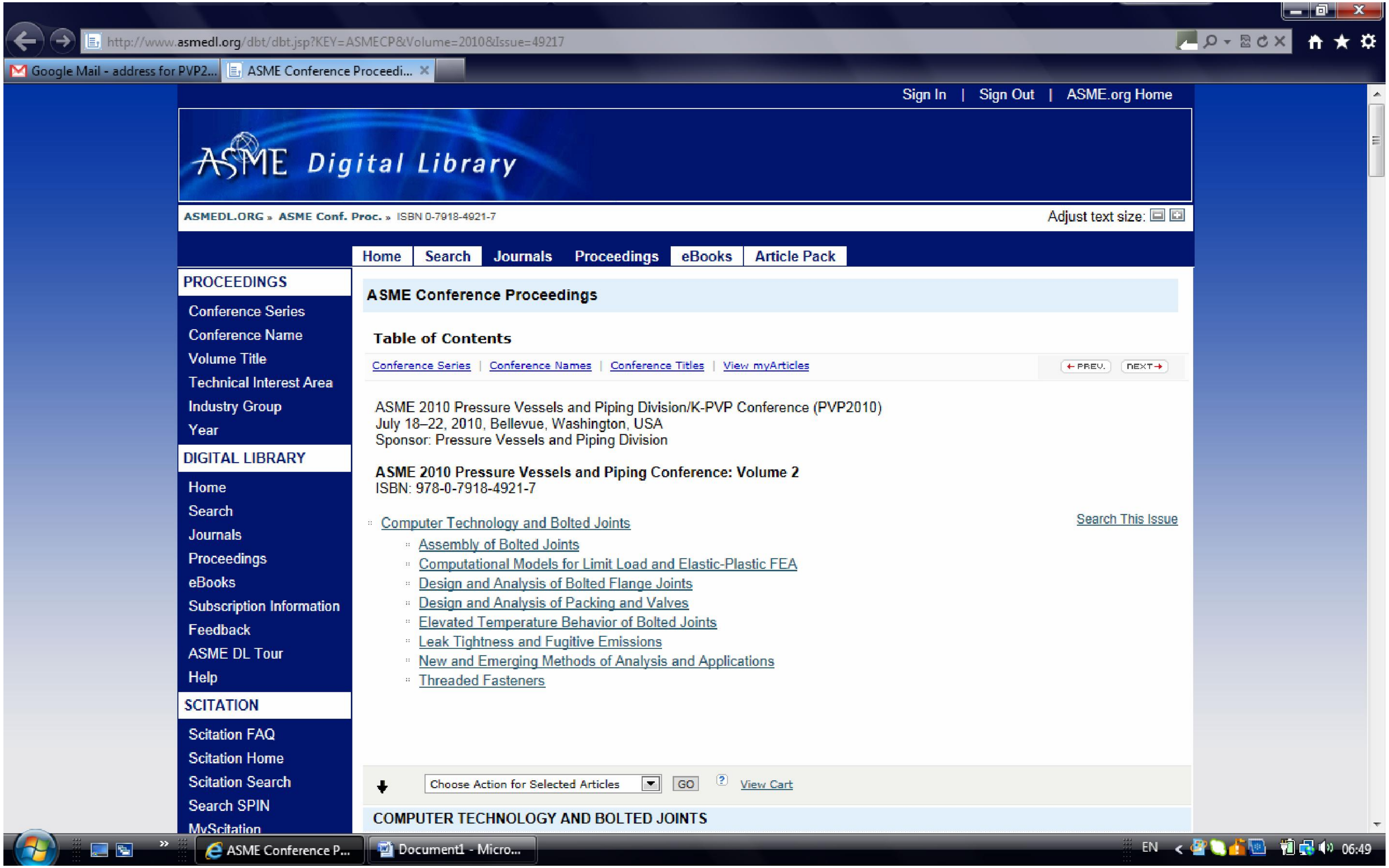

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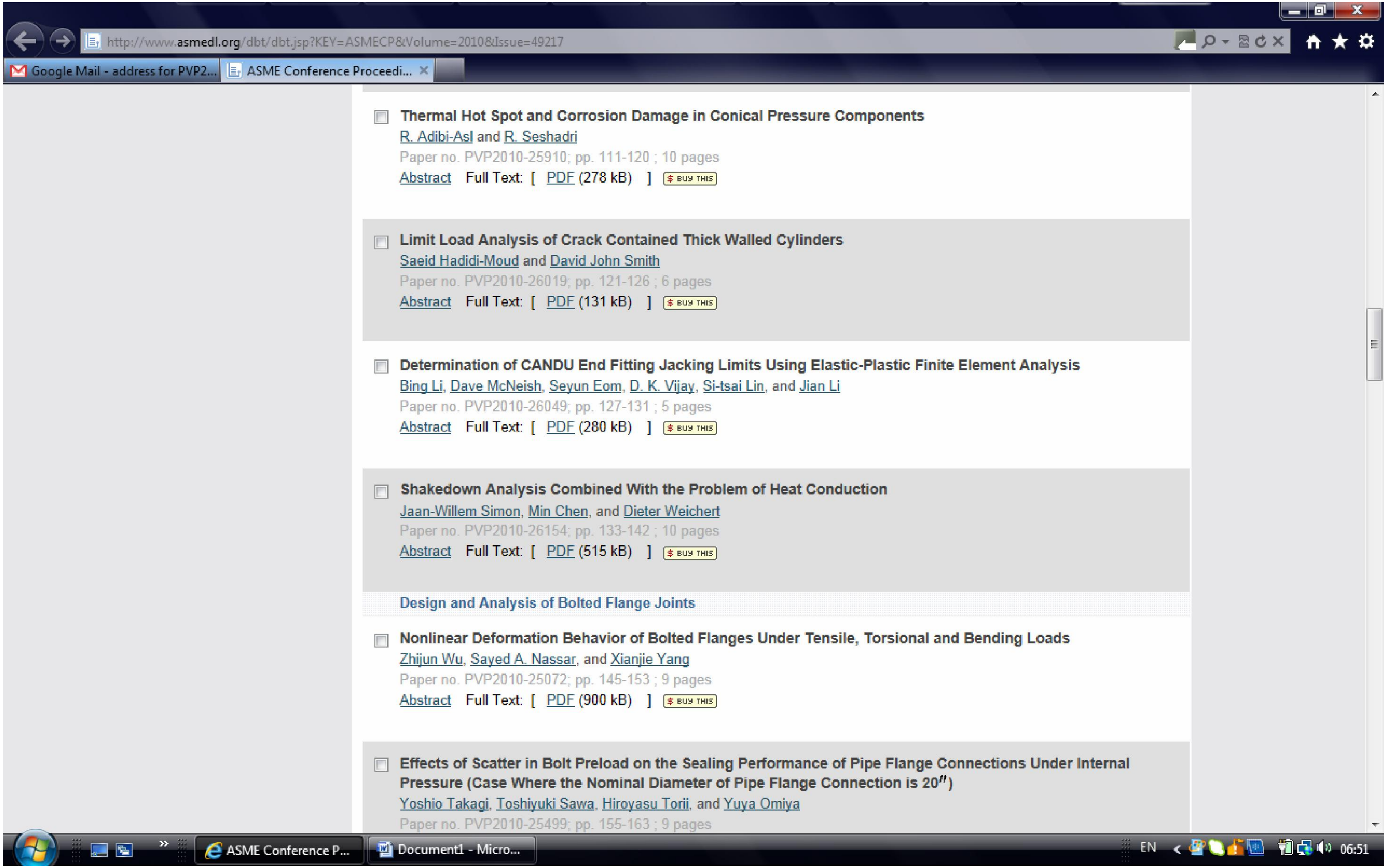

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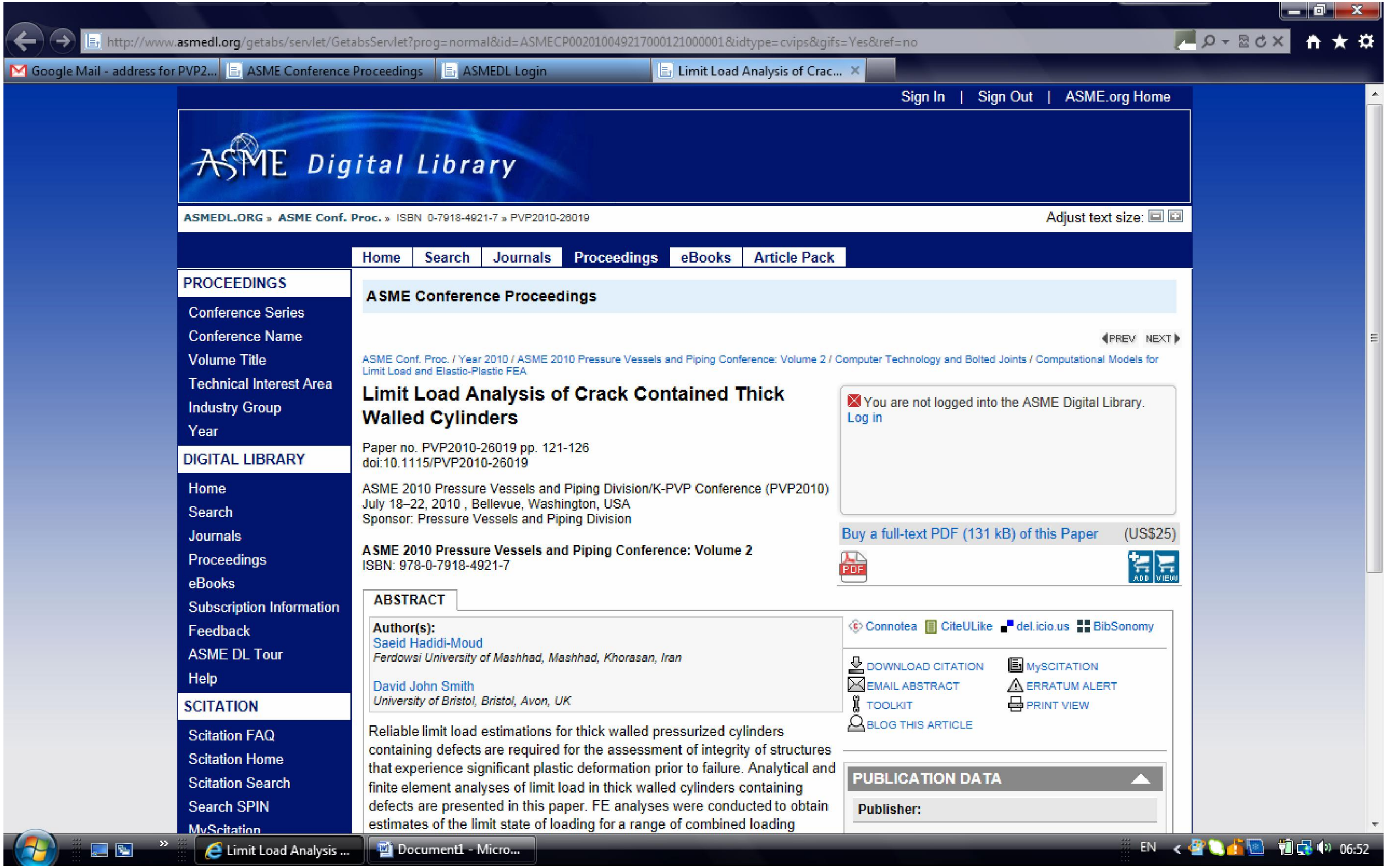

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**ABSTRACT**

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Reliable limit load estimations for thick walled pressurized cylinders containing defects are required for the assessment of integrity of structures that experience significant plastic deformation prior to failure. Analytical and finite element analyses of limit load in thick walled cylinders containing defects are presented in this paper. FE analyses were conducted to obtain estimates of the limit state of loading for a range of combined loading

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# LIMIT LOAD ANALYSIS OF CRACK CONTAINED THICK WALLED CYLINDERS

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## ABSTRACT

Reliable limit load estimations for thick walled pressurized cylinders containing defects are required for the assessment of integrity of structures that experience significant plastic deformation prior to failure. Analytical and finite element analyses of limit load in thick walled cylinders containing defects are presented in this paper. FE analyses were conducted to obtain estimates of the limit state of loading for a range of combined loading schemes and loading sequences for open-end and closed-end cylinder. Part through shallow and deep hoop cracks in the cylinder for uniform radial, uniform axial and combined loading were examined. The results suggest that adjustments to the estimates of limit loads obtained from conventional methods reported in literature are needed in order to reflect the role of material response, crack configuration and boundary conditions on the limit loads of defected thick walled pipes and cylinders. These findings are very important and should be noted carefully, especially in the context of treatment of hoop and axial residual stresses in the integrity assessment of pipelines containing part through cracks.

## INTRODUCTION

Proper estimation of limit load for thick cylindrical steel vessels under combined (internal pressure and axial tension) loading requires careful consideration of conditions experienced in practice. These include presence of cracks and /or defects around the weld lines with various shape, size and orientation, residual stresses arising from fabrication, welding process and loading history, the applied boundary conditions and the material stress/strain behavior.

Whereas in service conditions influence the load carrying capacity of structures procedures followed by integrity assessment codes are based on simplifying assumptions and modifications are therefore often needed to interpret such estimates. Analytical and finite element analyses of limit load in thick walled cylindrical vessels containing defects are presented in this paper. Approximate solutions available for radial, hoop and axial stresses in an un-cracked cylinder subjected to combined internal pressure and axial load are used for model validation. Limit load is represented by combinations of the normalized internal pressure and the normalized axial tension.

Application of combined loading to a defect free cylinder would result in two situations;

a) The structure remains fully elastic under the applied internal pressure and reaches its limit state by application of additional axial force

b) The internal pressure is sufficient to introduce plasticity but application of additional axial force is required to provide sufficient increase in the “reference” stresses in the cylinder to reach its limit state.

In either case the limit state condition may be characterized by incorporating the stresses due to additional axial force and the stresses due to the assumed internal pressure. Gao et. al. [1] derived approximate limit load solutions for a thick walled cracked cylinder with a fully circumferential outer crack by following similar procedures to the case of un-cracked cylinder.

A range of limit load solutions are reported in literature including the extensive collection by Miller [2] who provided analytical solutions for a range of geometries. There exists numerous research output in literature on limit load solutions for un-cracked cylinders that provide approximate estimates for thin and thick walled cylinders. For example Ainsworth [3] proposed a solution for un-cracked cylinder. For thin walled cylinder Lei and Budden [4] proposed simple analytical solutions for limit load estimation. As noted approximate analytical and finite element solutions for limit load in thick-walled cylinders with fully circumferential cracks under combined internal pressure and axial tension were recently reported by Gao et. al. [1]. Recently Budden and Lei [5] derived solutions for limit load of thick walled cylinders with full circumferential crack subjected to axial tension.

This work presents a series of axi-symmetric and 3D FE analyses conducted to generate limit load solutions for a range of combined loading schemes and loading sequences in un-cracked and crack contained cylinders. ABAQUS/CAE version 6.8 was used for all FE modeling and analyses. Finite element based estimations for extreme cases of axial uniform applied displacement only and pure radial uniform displacement were performed to verify the existing analytical solutions proposed by Gao et. al. [1]. Both cases of open-end and closed-end cylinder were simulated. Crack modeling procedures were chosen depending on the crack configuration and symmetry planes and in most cases cracks were modeled by excluding the presumed crack surface from the symmetry conditions. In a series of 2-step FE analyses for combined loading the axi-symmetric models were used for all un-cracked and fully extended crack configurations. It was assumed that a radial displacement was first applied and then propagated. This was followed by application of an additional axial displacement in the second step of FE analyses that was continued to increase until the limit condition was reached. Limit load analysis for un-cracked, shallow crack and deep crack for uniform radial, uniform axial and combined loading was performed. Using axi-symmetric models combined loading in the form of application of pressure on the internal surface of the open-end cylinder followed by further application of axial uniform displacement was also simulated.

The paper closes by comparison of limit load finite element based estimates in the thick walled cylinder for crack free and for fully circumferential external shallow and deep cracks. Specifically, the role of boundary conditions and material response on the limit load estimations is addressed using finite element analyses for various crack/loading/BC combinations.

## NOMENCLATURE

$a$ = crack depth (length)
$n_p, n_N$ = normalized pressure, normalized axial load
$N$ = applied axial load
$p$ = applied internal pressure
$p_L, N_L$ = limit pressure, limit axial load
$r_i, r_o, r_c$ = internal, external and crack tip radii
$r_p$ = radius corresponding to tip of plastic region
$t$ = wall thickness of cylinder
$\sigma_y$ = yield stress (elastic limit) of material

## ANALYSIS PROCEDURS AND MODEL DESCRIPTION

The geometry of the cylinder considered in all analyses is shown in Table 1. Approximate solutions available for radial, hoop and axial stresses in an un-cracked cylinder subjected to internal pressure and combined internal pressure and axial load, representing open and closed end cylinder respectively have been used for model validation. These solutions and their derivation are widely available in literature. To avoid elongation such solutions are not repeated here. The material used for the analysis was M316H stainless steel for which material properties were available [6]. Normalization indicates that the form of limit load combinations of pressure and axial load are independent of the specific material data and theses are only meaningful when evaluation of the load carrying capacity of the structure is required. For an un-cracked cylinder:

At the start of plasticity, i.e. $r_p = r_i$:

$$p = \frac{\sigma_y}{\sqrt{3}}\left(1 - \frac{r_i^2}{r_o^2}\right) \tag{1}$$

At plastic collapse, i.e. $r_p = r_o$:

$$p = \frac{\sigma_y}{\sqrt{3}}\left(2\ln\frac{r_o}{r_i}\right) \tag{2}$$

The latter equation gives the pressure corresponding to plastic collapse under internal pressure loading only whereas the former refers to the onset of plasticity hence representing the elastic limit pressure, i.e. the maximum internal pressure below which the whole cylinder remains elastic.

Table 1. Geometry of cylinder considered for FE analyses

| $r_i$; mm | $r_o$; mm | $r_c$; mm | a; mm | t; mm | L; mm |
|---|---|---|---|---|---|
| 225 | 275 | 235 ~ 265 | 10 ~ 40 | 50 | 700 |

For the case of the un-cracked thick walled cylinder under combined internal pressure and axial force, as noted earlier, two situations are likely to occur. Under the internal pressure the cylinder may remain fully elastic in which case the cylinder reaches the limit state when the additional axial force is applied. Alternatively the internal pressure portion may be sufficient to introduce some plasticity in which case the axial force provides the additional stress required for the cylinder to reach the limit state. Both cases were considered in the analyses performed for the un-cracked cylinder. The extreme state is where the applied internal pressure causes the cylinder to reach the limit state and no additional axial load is required.

If the cylinder is elastic under internal pressure and added axial force takes the structure to the limit state, the radial and hoop stress components for elastic cylinder remain unchanged whereas the axial stress under combined loading increases. At the limit state using this axial stress and the elastic stress state due to internal pressure in Mises yield criterion the required additional axial stress will be obtained from which the axial force component, $N_L$ for the limit state is derived.

Some further derivation and integration leads to the relation between the normalized internal pressure and normalized axial load ($n_N$ and $n_p$).

In the case where the cylinder exhibits elastic-plastic response under internal pressure, additional axial force will result in limit state. Similarly, using the stress components for the elastic region into Mises criterion, the additional axial force for limit state is obtained. Further integration and derivation gives the relation between the two normalized loads, $n_N$ and $n_p$, that is implicit. Based on these relationships the normalized limit load solutions, for thick-walled cylinder under combined internal pressure and axial force for un-cracked cylinder are obtained.

Following similar procedures to the un-cracked problem it is also possible to derive approximate limit load solution for thick walled "*cracked*" cylinder with a fully circumferential part through crack on the inner or outer surface of the cylinder. Again, as for the case of un-cracked problem the internal pressure on its own may or may not result in introducing plasticity. In either case additional axial load is needed to reach the limit state. Following similar procedures to the case of un-cracked cylinder the relation between $n_N$ and $n_p$ can be derived. It should be noted that in presence of a crack only the case where the crack tip under internal pressure is in the elastic region needs to be considered. Stress distributions in the plastic region are those given for the un-cracked cylinder. Hoop and radial stresses in the elastic region under internal pressure also follow the expressions given for the un-cracked cylinder.

## LIMIT LOAD ESTIMATION USING FINITE ELEMENT ANALYSIS

Due to ambiguity in literature, before going into details of procedures followed for limit load estimation a definition should be provided. Traditionally the term "limit load" that corresponds to the "net section yield load" has been determined based on the perfect plasticity as the response of material when reached the elastic limit. Furthermore it is often assumed that the limit state is the same as the plastic collapse condition i.e. when the whole structure has become plastic. This point was noted by Zerbst et. al. [7]. In the present work the load at which first indication of extended plasticity across the pipe wall thickness occurs is referred to as the limit load.

Using elastic-perfectly plastic material model it is possible to estimate limit loads for un-cracked and cracked structures. An incremental input to the analysis may be continued until the whole structural section reaches the elastic limit (onset of plasticity) as the elastic perfectly plastic material response does not allow for any increase in the Mises stress anywhere in the structure (assuming Mises yield criterion is applied) collapse condition is reached at the corresponding input level. FE analysis may be used to estimate limit conditions for any combined loading scheme and with any loading sequence. Using the same loading scheme to un-cracked and cracked structure would result in estimating the ratio of limit loads of the two configurations independent from the value of yield stress used.

The estimated normalized limit loads based on FE simulations for un-cracked cylinder are shown in Figure 1. Finite element based estimations for extreme cases of axial uniform applied displacement only and pure radial uniform displacement were used to validate the approximate analytical solutions proposed in literature.

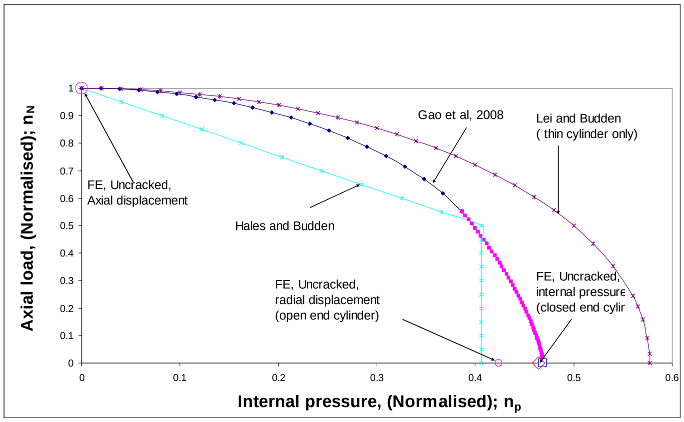

Figure 1. FE verification of limit load solutions for un-racked pipe proposed in Literature

Figure 2 also indicates FE estimates for a range of fully circumferential external cracks with crack depths from 10 to 40 mm (i.e. depth to thickness ratios of 0.2 to 0.8). For $n_N$=0 (i.e. no axial force applied) both cases of open end and closed end cylinder were simulated. Gao et. al. [1] formulation is appropriate for closed end cylinder meaning that even where there is no axial applied force an axial component corresponding to the internal pressure applied to a closed end cylinder is present. This case was simulated using two different 3-D FE models, a cylinder closed with rigid end blocks for which the FE deformed model and the loading are shown in Figure 3.a and an open cylinder, Figure 3.b, that was subjected to pressure on the internal surface plus an axial component that simulated the equivalent axial load due to pressure in a closed end cylinder. The two loading schemes resulted in similar values for $n_p$ equal to approximately 0.468 that is almost identical to the solution of Gao et. al. [1]. However for the open-end model FE analyses in which no axial effect due to applied internal pressure was included, provided a lower $n_p$ equal to 0.42. The observed difference is obviously due to the absence of the axial stress component associated with internal pressure loading of a closed end cylinder. Similar FE simulations for the case of no internal pressure were carried out for both un-cracked and 10 mm crack models. For the un-cracked closed end cylinder (blocked ends) the result was $n_N$=1 as expected and agreed with the theoretical estimate. This was also the case for open end 3-D cylinder subjected to an axial input only (load or displacement). Theoretical calculation for cracked case was based on the net section stress at the crack plane ligament. Results agreed with those presented by Gao et. al. [1] in which details of analytical solutions was provided. These are not repeated here although it is worth noting that few corrections (most likely due to typing mistakes) to those equations were necessary.

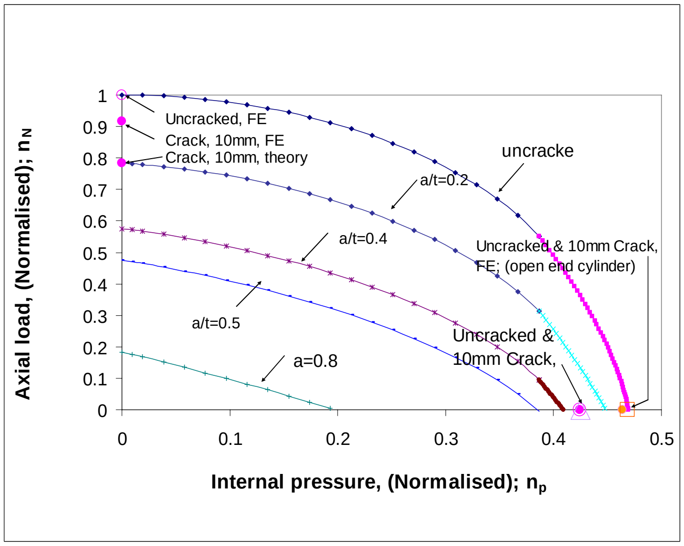

Figure 2. Assessment of closed form limit load solutions of Gao et. al. [1] for thick-walled cylinder under combined loading for un-cracked, shallow and deep circumferential part through external cracks

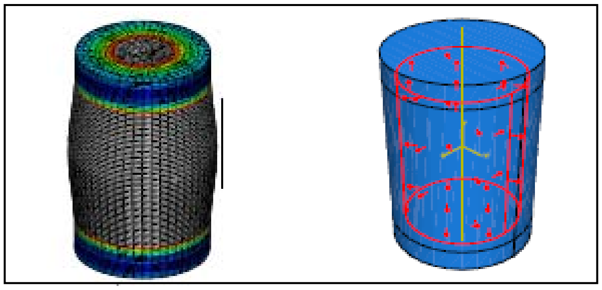

Figure 3.a. Geometry, FE mesh, loading and Mises stress contours for internally pressurized closed-end cylinder

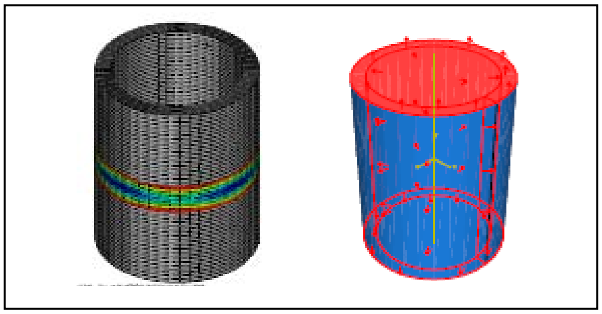

Figure 3.b. Geometry, FE mesh, loading and Mises stress contours for internally pressurized open-end cylinder

## LIMIT LOAD UNDER RADIAL AND AXIAL UNIFORM DISPLACEMENTS

In the series of two-step analyses for combined loading the axi-symmetric and 3D FE models with same mesh refinements were used for all un-cracked simulations as well as shallow and deep crack configurations. Together with mesh sensitivity tests these procedures were followed to ensure reliable and consistent results from FE simulations. Furthermore all these analyses assumed that a radial displacement was first applied (step 1). Then in the second step of analyses the conditions of step 1 were propagated and an additional axial displacement was introduced. The axial displacement of step 2 was continued to increase in small increments and the extension of progressing plasticity was monitored until the limit condition was reached. It is evident that the accuracy of estimated limit load depends on the level of controls over the loading increments as well as the element size within the front head of the plastic region across the thickness of the cylinder wall section. In all FE simulations for cracked cases independent axial and radial uniform displacements were applied for all loading combinations and the above procedures were followed to spot the limit load. For the case of $n_p$=0 using the open end axi-symmetric model the estimated $n_N$ from FE simulation was higher than the estimation of Gao et. al. and the theoretical simplified solutions for all un-cracked and cracked cases. This may be explained as follows: For the extreme case of no applied radial displacement (representing the internal pressure) the radial applied displacement in step 1 was set to zero. Propagating this condition to step 2 implied that reduction in diameter due to axial loading (in step2) was prevented.

Limit load analyses (using elastic -perfectly plastic material model) for un-cracked, shallow crack (*a*/*t*=0.2) and deep crack (*a*/*t*=0.8) for various loading schemes i.e. uniform radial, uniform axial and combined loading were also performed. As described there was no axial effect associated with the pressure in these analyses in any case meaning that the two loading components were completely independent. Yet for comparison purposes the normalizing procedure used was that applied by Gao et. al. (based on the internal pressure in closed end cylinder corresponding to limit state in the un-cracked cylinder). Results of FE simulations for open end cylinder under combined loading for un-cracked and cracked configurations are plotted in Figure 4. As explained above for the extreme case of $n_p$=0 the results of these series of analyses are expected to deviate from Gao et. al. and from FE simulations of un-cracked 3-D models as illustrated earlier.

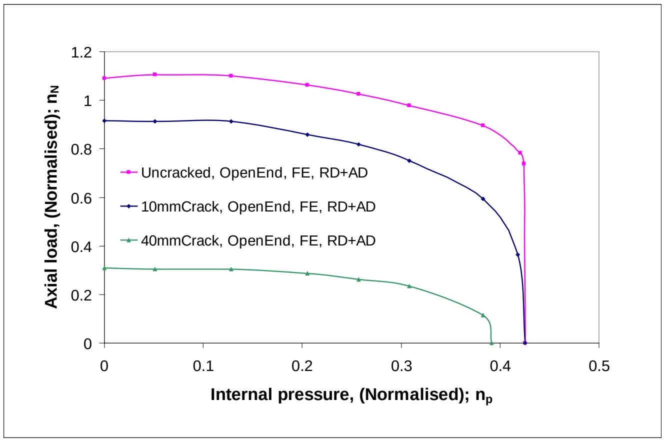

Figure 4. Limit load analysis of cracked and un-cracked open end cylinder using FE simulations

## LIMIT LOAD USING INTERNAL PRESSURE AND UNIFORM AXIAL LOADING

Real loading conditions for a cylinder subjected to internal pressure and axial force with or without the presence of additional stress components such as residual stresses is not practically represented by the displacement controlled combinations of applied radial and axial uniform displacements as used in the analyses reported in previous section.

In this section use of combined loading in the form of application of distributed load or pressure on the internal surface of the open end cylinder followed by further application of axial uniform displacement is presented. The FE models used for these analyses were the same axi-symmetric models used in previous section but alternative loading scheme was used. Similar cases of un-cracked and cracked models were analyzed for a selection of combined loading cases and similar plots to those in Figure 4 were obtained. Results of this series of analyses were expected to be representative of limit load solutions for open end cylinder under combined loading. These were also expected to agree with the extreme solutions of un-cracked 3-D analyses as well as for cracked configurations at $n_p$=0. The results at $n_N$=0 and at combined selections were expected to differ from the analytical solution of Gao et. al. [1] but agree with other FE 3-D results at $n_N$=0. Figure 5 shows the limit load solutions obtained from the loading schemes and confirms that these expectations are met.

Interestingly the results suggest that in the absence of the axial effect of internal pressure, i.e. in the case of open end pressurized cylinder, the subsequent axial loading that follows the applied pressure would initially reduce the equivalent Mises stress before it eventually contributes to the plastic failure conditions being reached. As a result $n_N$ initially increases beyond 1 and then reduces when the applied pressure approaches its limit level. This was not the case for deep crack (40 mm). Figure 5 also suggests that with the exception of un-cracked problem for the extreme case of axial load only ($n_p$=0) the normalized axial load, $n_N$ from FE analyses is generally higher that the estimation of Gao et. al. All analyses indicate significant impact of hoop crack on limit state for axial loading. In contrast shallow hoop crack (10mm) shows no reduction in the normalized pressure (at $n_N$=0) compared with the un-cracked and only little influence is shown for the deepest crack. This is very important to notice in the context of treatment of hoop and axial residual stresses in a pipe containing hoop external cracks.

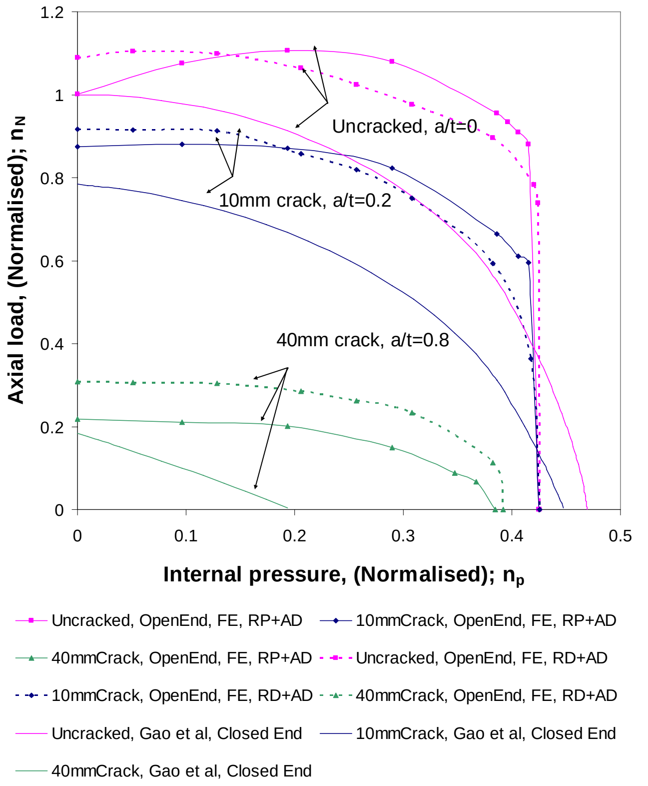

Figure 5. Summary of results for comparison; contains the finite element results for open end and closed end conditions and the proposed solution of Gao et. al. [1]

## CONCLUSIONS AND FURTHER WORK

FE simulations of thick walled cylinder suggested that in the case of open-end pressurized cylinder, the subsequent axial loading that follows the applied pressure initially reduces the equivalent Mises stress before it eventually contributes to the plastic failure conditions being reached. Results also indicated that the normalized axial load from FE analyses is higher than the estimations reported in literature. For all cracks analyses indicated significant impact of hoop cracks on limit state for axial loading. In contrast shallow hoop crack showed no reduction in the normalized pressure compared with the un-cracked and only little influence was observed for the case of the deepest crack. These findings are very important to notice in the context of treatment of hoop and axial residual stresses in a pipe containing hoop external part through cracks.

Standard solutions of limit load aim at providing simple design tools to achieve fast estimates. However the user should be aware of the significant influence of practical aspects of Loading, material behavior and the boundary conditions on the limit load. Access to powerful finite element modeling tools in modern engineering design practice provide means to achieve accurate yet time efficient data by taking the real loading conditions into account in FE simulations. To highlight the significance of such considerations a comparison is presented in Figure 6 for typical results obtained from FE simulations as reported in this paper with the approximate estimates based on R6 integrity assessment procedures. Work is in progress to examine the impact of practical aspects including the loading conditions, material hardening and boundary conditions on the integrity of thick walled steel cylinders containing cracks of various length , depth, shape and orientation as observed under in service conditions. Specifically it is worth mentioning that the influence of boundary conditions on the limit load evaluation of pressurized vessels with limited length or distance between the supports may be significant in practice.

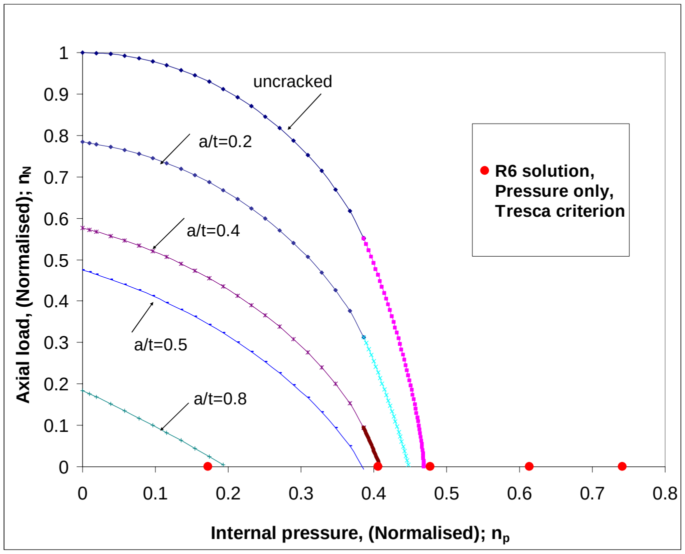

Figure 6. Comparison of limit load solutions of cracked and un-cracked open end cylinder using FE simulations with R6 limit load approximations for pressure only loading

The present work does not provide experimental data to support the estimated limit loads for cracked cylinders. It rather verifies the finding based on analytical solutions. In brief available analytical solutions for specific cases have been used as benchmark models in the FE analyses to ensure that the analysis procedures were appropriate. This was achieved by examination of the FE analyses results against reliable analytical solutions and adjusting the simulation details including the mesh refinement, the loading procedures, the element type and controls (i.e. fully structured mesh of 3D 8-noded elements with reduced integration scheme) and the boundary conditions so that excellent agreement wit the analytical solutions were obtained. The verified details through such examinations were then used in the finite element simulation of other cases where either no analytical solutions are available or the solutions were based on simplifying assumptions.

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