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The Characteristics of Ion-Acoustic Solitons in Electron-Positron-Ion Magnetized Plasma

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Abstract – *In this paper, the propagation of ion-acoustic waves in electron-positron-ion magnetized plasmas is analyzed numerically. The effect of positron density on the amplitude of ion-acoustic waves is also studied. Corresponding to a fixed positron density, it is shown that the speed of the wave can change in certain parameter range.* ***PACS:*** *52.35.Mw, 52.35.Sb, 52.25.Xz, 52.27.Aj, 52.35.Fp.*

Keywords: *Ion-Acoustic, Solitons, Magnetized Plasma*

Nomenclature

φ	The electrostatic potential.
$\omega_{ci} = \dfrac{eB}{m_i c}$	The ion gyrofrequency.
v_i	The ion fluid velocity.
e	The magnitude of the electron charge.
m_i	The mass of the ion.
n_α	The perturbed density.
n_α	The unperturbed density of α species, where $\alpha = e, p$ stands for electrons and positrons respectively.
$T_e\,(T_p)$	The electron (positron) temperature.
M	The constant speed of the localized nonlinear struc-ture in the moving frame.
K_x, K_z	The direction cosines along the x and z-axis, respectively.

I. Introduction

The basic natural modes of magnetized plasmas such as those that occur in molecular clouds, cometary's plasmas and stellar atom-spheres are of great interest. The linear approximation to these waves breaks down at large amplitudes where nonlinear effects become important in their propagation. One such large amplitude wave is a magneto-acoustic wave, which modifies the background magnetic field in an oscillatory fashion, and so can be considered as a pump wave that drives other waves nonlinearly. Such large amplitude pump waves may occur in conditions such as seen in solar and space plasmas. For example, solar shock waves can set up large amplitude standing magneto-acoustic waves in coronal loops or magnetic flux tubes [1]. Ion acoustic waves propagating in plasma are nearly similar to ordinary sound waves in neutral gas they are longitudinal waves consisting of compressions and rarefactions progressing in the medium. The role of ions is the same of neutral atoms in ordinary sound waves. A difference is that, unlike sound waves, ion acoustic waves can also propagate in collision less medium, because the charged ions interact at long distances via their electromagnetic field. The nonlinear structures in magnetized electron-positron-ion plasmas have also been investigated [2]-[11]. The aim of this paper is to study the characteristics of ion-acoustic solitons in electron-positron-ion magnetized plasma. This paper is organized as follows details of the governing equations for nonlinear ion-acoustic waves in electron-positron-ion magnetized plasma are stated in section 2.In section 3, the conditions for existence of localized solutions are presented and finally the results are discussed in sections 4.

II. Formulation

We consider three component electron-positron-ion (e-p-i) plasma in the presence of a static and uniform magnetic field $\vec{B} = B_z\hat{k}$. The phase velocity of the ion acoustic wave is assumed to be much larger than the ion thermal velocity and much less than the electron and positron thermal velocities, i.e. $v_{ti} \ll \dfrac{w}{k} \ll v_{te}, v_{tp}$ (where $v_{tj} = \left(\dfrac{T_j}{m_j}\right)^{\frac{1}{2}}$ is the thermal speed of the jth species while $j = e, p, i$). Therefore the set of equations for cold fluid ions are the ion continuity equation:

$$\frac{\partial n_i}{\partial t} + \nabla\left(n_i \vec{v}_i\right) = 0 \qquad (1)$$

and the ion momentum equation:

$$\frac{\partial \vec{v}_i}{\partial t} + \left(\vec{v}_i \vec{\nabla}\right)\vec{v}_i = \frac{-e}{m_i}\vec{\nabla}\varphi + \omega_{ci}\vec{v}_i \times \hat{z} \qquad (2)$$

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the two terms in the r.h.s of the above equation are electric force, $\vec{F} = e\vec{E}$, and magnetic force, $\vec{F} = e\vec{v} \times \vec{B}$, respectively. If we consider the electrostatic potential as a perturbation, the hot electrons and positrons densities follow the Boltzmann distributions:

$$n_e = n_{e\circ} \exp\left(\frac{e\varphi}{T_e}\right) \qquad n_p = n_{p\circ} \exp\left(-\frac{e\varphi}{T_p}\right) \tag{3}$$

Equilibrium requires:

$$n_{i\circ} + n_{p\circ} = n_{e\circ} \tag{4}$$

and for the perturbed densities, the quasi-neutrality is defined as:

$$n_i + n_p \approx n_e \tag{5}$$

III. The Condition for Existence of Solitonic Solutions

Let us consider the two-dimensional perturbation in the xz-plane. To obtain the solitonic solutions for the above equations, we introduce a moving coordinate as:

$$\xi = K_x x + K_z z - Mt \tag{6}$$

So we have $K_x^2 + K_z^2 = 1$.

We assume that all the variables depend on ξ and M. From the quasi-neutrality condition, we obtain the ion number density [2]:

$$n_i = \frac{1}{1-p}\left(n_e - pn_e^{-\alpha}\right) \tag{7}$$

where $\alpha = \frac{T_e}{T_p}$, $p = \frac{n_{p\circ}}{n_{e\circ}}$, and $n_j = n_j / n_{j\circ}$ (where j = e, p, i). Note that the above equation holds for ($\circ < p < 1$) in e-p-i plasmas. By changing the variable ξ in the equations (1), (2) and using equations (7) and (8) and then integrating, we obtain:

$$\frac{d^2}{d\xi^2}\left[\frac{M_a^2(1-p)^2}{2\left(n_e - pn_e^{-\alpha}\right)^2} + Ln\, n_e\right] = -1 - \frac{\left(n_e - pn_e^{-\alpha}\right)}{1-p}$$
$$\left[\frac{K_z^2}{M_a^2}\frac{\left(n_e + {}^{p}\!/_{\alpha}\, n_e^{-\alpha}\right)}{1-p} - \frac{K_z^2}{M_a^2}\left(\frac{1 + {}^{p}\!/_{\alpha}}{1-p}\right) - 1\right] \tag{8}$$

Here, ξ and M normalized by ρ_s and c_s, respectively, where $c_s = \left(\frac{T_e}{m_i}\right)^{1/2}$ is the ion-sound speed in two-component electron-ion plasma and $\rho_s = \frac{c_s}{\omega_{ci}}$ is the ion Larmor radius at electron temperature. We have also defined $M_a = \frac{M}{c_s}$. In order to find localized solution we have used the boundary conditions; $n_e \to 1$ as well as $\left.\frac{\partial n_e}{\partial \xi}\right|_{n_e=1} \to \circ$ and $\left.\frac{\partial^2 n_e}{\partial \xi^2}\right|_{n_e=1} \to \circ$ at $\xi \to \pm\infty$. By introducing u as:

$$u = \left[\frac{M_a^2(1-p)^2}{2\left(n_e - pn_e^{-\alpha}\right)^2} + Ln\, n_e\right] \tag{9}$$

and multiplying both sides of equation (8) by $\frac{du}{d\xi}$, we obtain 'the energy law':

$$\frac{1}{2}\left(\frac{\partial n_e}{\partial \xi}\right)^2 + U\left(n_e, K_z, M_a, p\right) = 0 \tag{10}$$

where the effective Sagdeev potential U reads as (eq. (11):

$$U\left(n_e, K_z, M_a, p\right) = \left[\frac{1}{n_e} - \frac{M_a^2(1-p)^2\left(1 + p\alpha n_e^{-\alpha-1}\right)^2}{\left(n_e - pn_e^{-\alpha}\right)^3}\right]^{-2}$$
$$\left[\frac{M_a^2}{2} + \frac{K_z^2\left(1 + \frac{p}{\alpha}\right)}{2M_a^2(1-p)^2} + \frac{1 + \frac{p}{\alpha}}{1-p} + \left(1 - K_z^2\right) \ln n_e + \right.$$
$$\frac{K_z^2\left(n_e + \frac{p}{\alpha} n_e^{-\alpha}\right)^2}{2M_a^2(1-p)^2} + \frac{M_a^2(1-p)^2}{2\left(n_e - pn_e^{-\alpha}\right)^2} +$$
$$-\left(\frac{1}{1-p} + \frac{K_z^2\left(1 + \frac{p}{\alpha}\right)}{M_a^2(1-p)^2}\right)\left(n_e + \frac{p}{\alpha} n_e^{-\alpha}\right) - \frac{M_a^2}{\left(n_e - pn_e^{-\alpha}\right)}$$
$$\left.\left(\frac{K_z^2\left(1 + \frac{p}{\alpha}\right)}{M_a^2} + (1-p)\right) + K_z^2 \frac{\left(n_e + \frac{p}{\alpha} n_e^{-\alpha}\right)}{\left(n_e - pn_e^{-\alpha}\right)}\right]$$

Equation (10) is a well known equation in the form of an oscillating particle of unit mass, with velocity $\frac{\partial n_e}{\partial \xi}$ and position n_e in a potential well $U\left(n_e, K_z, M_a, p\right)$ [12].

The conditions corresponding to the existence of a localized solution of equation (10) are:

$$U(n_e = 1) = U(n_e = N) = \left.\frac{\partial U}{\partial n_e}\right|_{n_e=1} = 0$$
$$\left.\frac{\partial^2 U}{\partial n_e^2}\right|_{n_e=1} < 0 \quad (12)$$

namely the potential must have a maximum at the point of $n_e = 1$. $n_e = N$ is a solution of equation $U(N, K_z, M_a, p) = 0$ which determines the maximum amplitude n_e of the soliton (as a function of M_a, p and K_z). The next section is devoted to the results of numerical analysis of equation (10).

IV. Results

IV.1. The Solitonic Solutions

The profiles of the Sagdeev potentials for two cases, namely (i) $p = 0.5$, $K_z = 0.3$ and $M_a = \sqrt{0.08}$ and (ii) $p = 0.7$, $K_z = 0.7$ and $M_a = \sqrt{0.05}$ are plotted in Figs. 1 and 2, respectively. To obtain the electron density hump $n_e(\xi)$ for the above two cases, we have solved numerically equation (10) by fourth-order Rung-Kutta method [13]. In conclusion, a localized solution can be found only in case (i), which is shown in Fig. 3. For case (ii) the condition of equation (12) is invalid. We have found that our results completely agree with those obtained in reference [2].

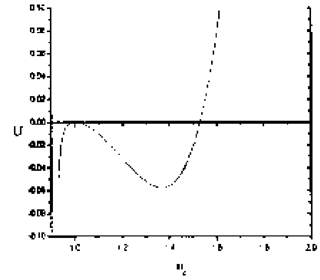

Fig. 1. The profile of Sagdeev potential U versus electron density n_e for $p = 0.5$, $K_z = 0.3$ and $M_a = \sqrt{0.08}$

The profile of the variation of $\left.\frac{\partial^2 U}{\partial n_e^2}\right|_{n_e=1}$ against M_a for different values of p, k_z and $T_e = T_p$ have been shown in Fig. 4. It can be seen that by using the condition (12) the speed of the soliton M_a is restricted to $M_a < 1$. Therefore, the only subsonic ion-acoustic solitons, propagate in magnetized e- p-i plasma.

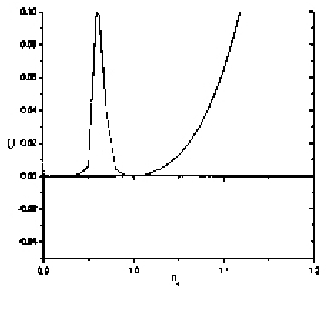

Fig. 2. The profile of Sagdeev potential U versus electron density n_e for $p = 0.7$, $K_z = 0.7$ and $M_a = \sqrt{0.05}$

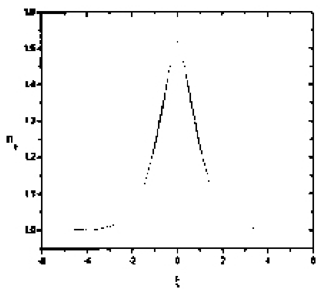

Fig. 3. The profile of ion-acoustic solitary wave n_e versus ξ for $p = 0.5$, $K_z = 0.3$ and $M_a = \sqrt{0.08}$

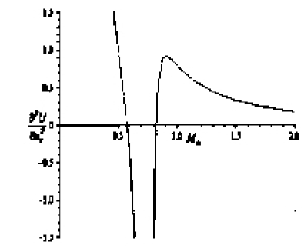

Fig. 4. The variation of $\frac{\partial^2 U}{\partial n_e^2}$ against M_a for $p = 0.2$, $K_z = 0.7$ and $T_e = T_p$

By plotting $\left.\frac{\partial^2 U}{\partial n_e^2}\right|_{n_e=1}$ and $\left.\frac{\partial^3 U}{\partial n_e^3}\right|_{n_e=1}$ for different values of p and K_z, we get the bounds on M_a. Note that the range of M_a is a consequence of the inequalities

$$\left.\frac{\partial^2 U}{\partial n_e^2}\right|_{n_e=1} < 0 \text{ and } \left.\frac{\partial^3 U}{\partial n_e^3}\right|_{n_e=1} > 0 .$$

The results of these simulation for $p = 0.2$, $K_z = 0.7$ and $T_e = T_p$ is presented in Figs. 4 and 5. Furthermore the bounds on M_a for various values of p and K_z are shown in Table I.

We have also verified these results by simulation of the equation (10).

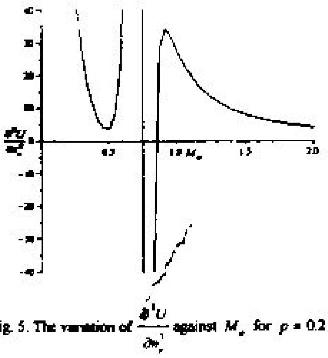

Fig. 5. The variation of $\frac{\partial^3 U}{\partial n_e^3}$ against M_a for $p = 0.2$, $K_z = 0.7$ and $T_e = T_p$

TABLE I
BOUNDS ON M_a FOR VARIOUS VALUES OF K_z, p

$\alpha = \frac{T_e}{T_p}$	$p = \frac{n_p}{n_e}$	K_z	M_a
1	0.2	0.7	$0.571 < M_a < 0.745$
1	0.4	0.7	$0.456 < M_a < 0.552$
1	0.6	0.7	$0.349 < M_a < 0.394$
1	0.8	0.7	$0.233 < M_a < 0.247$

IV.2. Effect of the Variation of Different Parameters on the Solitons

Fig. 6 displays the dependence of the Mach number on the fractional number p for a fixed values of $K_z = 0.6$, $n_e = 1.1$ and $T_e = T_p$.

We see that the existence of the ion-acoustic solitons of such amplitude is possible for $M < 1$. Furthermore, for the case $T_e = T_p$, the solitons with the amplitude $n_e = 1.1$ can exist only if $p \leq 0.5$. We see, the amplitude $n_e = 1.1$ is the maximum possible one in a plasma with $p \leq 0.5$ and $T_e = T_p$.

This figure shows that the speed of the solitons of a given amplitude decreases as the fractional number p increases.

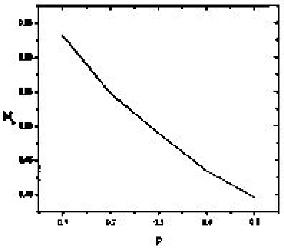

Fig. 6. The dependence of speed of the ion-acoustic soliton M_a on the fractional number p for a given values of $K_z = 0.6$, $n_e = 1.1$ and $T_e = T_p$

We have also seen from the numerical analysis that the amplitude of ion-acoustic wave due to the presence of positrons in the magnetized plasma, resulting from small K_Z is decreasing and the amplitude of electron density hump increases with the concentration of positrons associated with the larger K_Z is increasing. Besides, we note that the width of ion-acoustic solitons decrease with an increase in the positrons concentration. Figs. 7(a) and 7(b) depict change of n_e for two cases (i) $K_z = 0.2$, $M_a = \sqrt{0.05}$ and (ii) $K_z = 0.6$, $M_a = \sqrt{0.4}$ for different values of p.

Fig. 8 displays the dependence of the electron density on the fractional number p for a fixed values of K_Z, M_a and $T_e = T_p$. Obviously this profile shows that for small K_Z the amplitudes of solitons are drastically reduced in the presence of significant fraction of the positrons.

We have also plotted the electron density hump versus K_Z for a given values p, M_a and $T_e = T_p$. The result is shown in Fig. 9. We have found that the amplitude of the solitons are rapidly reduced by increasing K_Z.

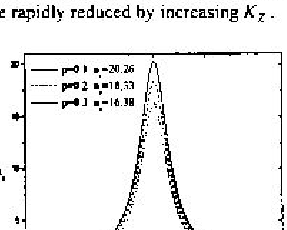

Fig. 7(a) The electron density hump n_e versus ξ for $K_z = 0.2$, $M_a = \sqrt{0.5}$ and the different values of p

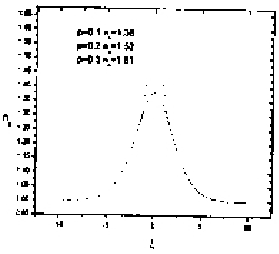

Fig. 7(b) The electron density hump n_e versus ξ for $K_z = 0.6$, $M_e = \sqrt{0.4}$ and the different values of p

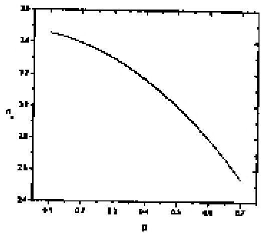

Fig. 8. The variation of electron density n_e against ξ for $K_z = 0.2$, $M_e = \sqrt{0.1}$ and $T_e = T_p$

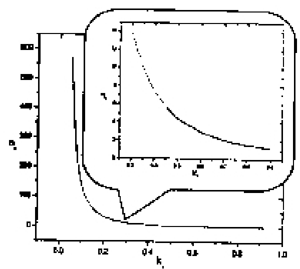

Fig. 9. The variation of electron density n_e against K_z for $p = 0.1$, $M_e = \sqrt{0.8}$ and $T_e = T_p$.

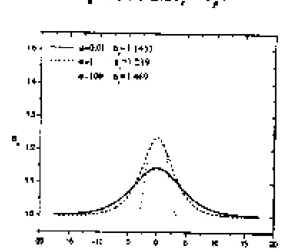

Fig. 10. The electron density hump n_e versus ξ for $p = 0.1$, $K_z = 0.1$, $M_e = \sqrt{0.01}$ and the different values of α

The variation of amplitude and width of solitons for different values of α is shown in Fig. 10 for $p = 0.1$, $K_z = 0.1$ and $M_a = 0.01$. It can be seen that the amplitude and the width of the solitons increase with increasing the value of α.

V. Conclusion

We have solved numerically the nonlinear equations of e-p-i plasma in the presence of a constant and uniform magnetic field and obtained the localized solutions. Then we have studied some properties of these solutions. It has been found that depending on the direction of propagation, the increasing positron density in some cases decreases the amplitude of ion- acoustic wave solitons, while increase in the other cases, although in the unmagnetized e-p-i plasma the increasing positron density reduces the amplitude of ion-acoustic wave solitons [14]. Comparing to the recent work, which has been done in reference [2], the present some difference with our results. We have also showed that for fixed electron density and propagation direction, the presence of positron can result reduce the speed of soliton.

Our main result is that the speed of the solitons of a given propagation direction and positron density, is limited to maximum and minimum values. Since the propagation of ion acoustic wave in plasmas plays important role, we believe that our results should be useful in understanding the properties of localized waves that may appear in early universe, active galactic nuclei and the pulsar magnetospheres.

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