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Measurement of Strong Coupling Constant Using Different QCD Models

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Abstract – *We have studied hadronic events from e^+e^- annihilation data at different centre-of-mass energies. The operation of the AMY collaboration offers a unique opportunity to test QCD by considering different observables. The main results concern the measurement of the strong coupling constant, α_s, from hadronic event shapes. The coupling constant, α_s, is measured by two different methods, first by employing the jet clustering algorithm introduced by the JADE group. By using AMY data , the value of α_s is determined to be 0.123 ± 0.004. Next, from the thee jet distribution, we extract the strong coupling constant, α_s, at Leading(LO), Next to Leading(NLO) and Next to Next Leading Order (NNLO) from AMY data. The results are more consistent with the running of α_s, expected from QCD predictions at NNLO corrections. We will discuss all these features in this paper.* **Copyright © 2009 Praise Worthy Prize S.r.l. - All rights reserved.**

Keywords: *Altarelli-Parisi, Three-Jet Rate, NNLO*

I. Introduction

The precision of the strong coupling constant determined from three jet rate has been limited up to three years ago largely by the scale uncertainty of the perturbative NLO calculation. But recently some papers reported on the first calculations of NNLO corrections to event shape variables and three jet observable, and discussed their phenomenological impact [1] - [2].

Three-jet production at tree-level is induced by the decay of a virtual photon (or other neutral gauge boson) into a quark-antiquark-gluon final state. At higher orders, this process receives corrections from extra real or virtual particles. The individual partonic channels that contribute through to NNLO are cited in ref [1]. For a given partonic final state, the event shape observable y is computed according to the same definition as in the experiment, which is applied to partons instead of hadrons.

At leading order, all three final state partons must be well separated from each other, such that y differs from the trivial two-parton limit. At NLO, up to four partons can be present in the final state, two of which can be clustered together, whereas at NNLO, the final state can consist of up to five partons, and as many as three partons can be clustered together. The more partons in the final state, the better one expects the matching between theory and experiment to be [3].

The data from e^+e^- annihilation provide us with one of the cleanest ways of probing our quantitative understanding of QCD and hence giving us an opportunity to measure the strong coupling between quarks and gluons. Three-jet production cross sections and related event shape distributions in e^+e^- annihilation processes are classic hadronic observables which can be measured very accurately and provide an ideal proving ground for testing our understanding of strong interactions. Jet observables in electron-positron annihilation can be used to extract the value of the strong coupling constant α_s [4], [5], [6]. This applies in particular to three-jet observables, where the leading-order parton process is proportional to α_s. In order to extract the numerical value from the AMY L data, precise theoretical calculations are necessary, calling for a next-to next-to-leading order (NNLO) calculation.

In this paper by studying dependence of the longitudinal and transverse normalized inclusive cross-section on the center of mass energy we try to find the proof for gluon emission which leads to scaling violation and calculation of strong coupling constant. Then we measure the strong coupling constant, α_s, considering the Altarelli-Parisi-Weizsacker-Williams technique by using AMY data [7]. Finally we will measure the coupling constant by using three jet observable at NNLO from AMY data [7] - [9].

The reason for doing such measurement is that in ref [7]; the strong coupling, α_s, has been determined from the measured moments of particle momenta within jets and three jet rates using Next to Leading order.

One might anticipate a **priori** that this would yield a less precise determination of α_s than the differential distribution of jet rate, because the theory lacks resummation of large logarithms, and the moments include regions of phase space where hadronization effects are large [8]. Nevertheless, the comparison of α_s determined in this way with that obtained from the jet distribution should provide an illumination test of the adequacy of QCD in this area.

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II. Scaling Violation

The differential cross sections $\frac{1}{\sigma_{tot}}\frac{d\sigma}{dp}$ for inclusive charged particle productions is given in Fig. 1, for AMY data [7] as well as the results obtained at lower energies[9].

The cross section decreases steeply with momentum. Furthermore, the distribution becomes broader as the C.M energy increases. We conclude that the scaling is violated with increasing energy.

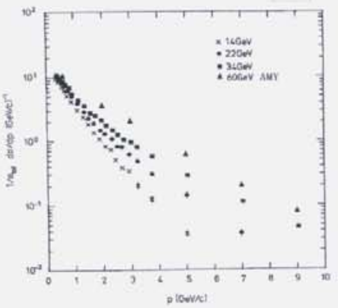

Fig. 1. The differential cross section $\frac{1}{\sigma_{tot}}\frac{d\sigma}{dp}$ for charged particles

To clarify further the situation, we display the normalized differential cross section $\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx_{II}}$ and $\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx_T}$ in Figs. 2 and 3 separately, where x_{II} is defined as $x_{II}=\frac{2p_{II}}{W}$ and x_T defined as $x_T=\frac{2p_T}{W}$. (p_T and p_{II} are measured with respect to the original parton, namely jet axis) [9]-[12].

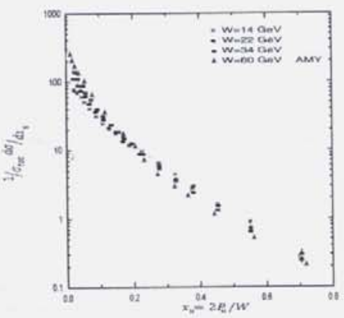

Fig. 2. The normalized cross section $\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx_{II}}$

for inclusive charged Particles in W=14-60GeV

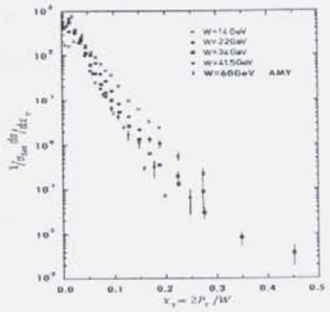

Fig. 3. The normalized cross section $\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx_T}$ for inclusive charged particles in W=14-60GeV

The important feature in Fig. 2, is that the scaling is valid for the normalized cross section with respect to x_{II} variable .On the other hand, the normalized cross section $\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx_T}$ in Fig. 3 doesn't scale. The lower energy data are above the higher energies. This may be due to the fact that in reaction $e^+e^- \to q\bar{q}g$ (at the parton level) the emission of a gluon leads to the scaling violation at high energies. We display in Fig. 4 the normalized cross section $\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx_p}$ (x_p is the fractional particle momentum, $x_p=\frac{2p}{W}$) for our AMY data as well as the data from ref. [9]. As the figure indicates the cross sections fall steeply with x_p. At small x_p, $x_p<0.1$ a rapid rise with W is seen which corresponds to the observed growth of the multiplicity.

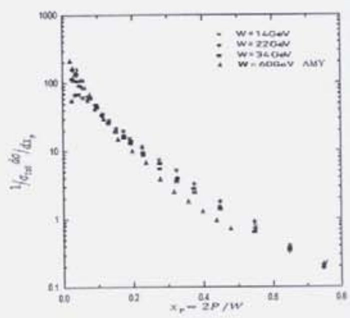

Fig. 4. The normalized cross section $\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx_p}$ for inclusive charged particles in W=14-60GeV

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For $x_p > 0.2$ the data show a slow but significant decrease with W.

This is more clearly seen in Fig. 5 where $\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx_p}$ is plotted versus $\sqrt{s}$.

Going from W=14 to the highest energies, $\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx_p}$ on the average is reduced by 25%.

This is another manifestation of the scale breaking which is discussed in detail in ref. [7], [9]-[13].

The data in Fig. 5 are also compared to the fitted NLO predictions [14]-[16].

Overall, a good agreement between data and theory is found in the region of the fit (full lines).

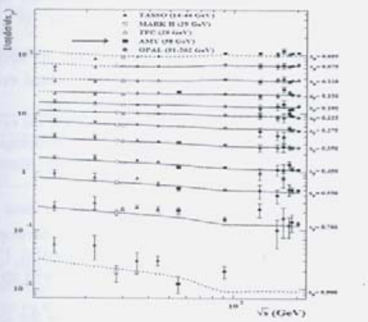

Fig. 5. Inclusive charged particle rate, as a function of energy compared to NLO predictions

III. JADE Algorithm

We separate two and three jet events by employing the jet clustering algorithm introduced by the JADE group [17].

In this algorithm the scaled mass spread defined as $Y_{ij} = \frac{m_{ij}^2}{E_{vis}^2}$ with $m_{ij}^2 = 2E_iE_j\left(1-\cos\theta_{ij}\right)$ is calculated for each pair of particles in the event.

If the smallest of the Y_{ij} values is less than a parameter Y_{cut}, the corresponding pair of particles is combined into a cluster by summing the four momenta.

This process is repeated, using all combinations of clusters and remaining particles, until all the Y_{ij} values exceed Y_{cut}.

The clusters remaining at this stage are defined as the jets. The distribution of jet multiplicities obtained by these clustering algorithms depends on the jet defining parameter Y_{cut}.

For small Y_{cut}, many jets are found because of the hadronization of fluctuation process, whereas for large Y_{cut}, mostly two jet events are found and the $q\bar{q}g$ -events are not resolved.

However, Monte Carlo studies show that there is a range of cluster parameters, for which QCD effects can be resolved and the fragmentation effects are sufficiently small.

In the following, the parameter Y_{cut}=0.04 is used which is found to be a reasonable cut [17]. In Fig. 6, we show 3 jet fraction for different Y_{cut}.

The decrease of the 3-jet rate at large Y_{cut} is clearly visible.

Our results are completely consistent with the results obtained by the JADE scheme [17], [18].

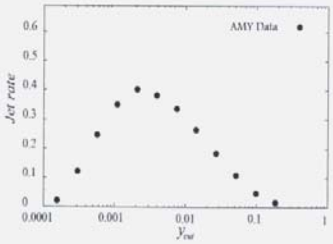

Fig. 6. Three jet fraction for different Y_{cut}s

In order to calculate the α_s by this method, we work with the transverse momentum of the partons as in Fig. 7 scaled to the e^+(and e^-) beam energy.

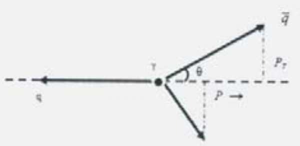

Fig. 7. The process $e^+e^- \to q\bar{q}g$ in the center of mass energy

Calculating the cross section corresponding to the three jet events, and using the Altarelli-Parisi-Weizsacker-Williams technique [19], we find:

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} = \frac{4\alpha_S}{3\pi}\frac{1}{p_T^2}Log\left(\frac{Q^2}{4p_T^2}\right) \tag{1}$$

Where α_s is the quark – gluon coupling strength of QCD (first order).

Fitting this expression after integration, to the experimental P_T^2 distribution as in Fig. 8, one obtains the value of α_s equal to 0.123±0.004.

The value of α_s for different Y_{cut}s with respect to the P_T^2 is listed in Table I.

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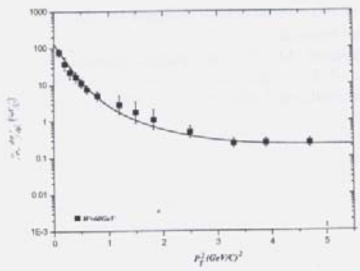

Fig. 8. Normalized cross section for the square of the transverse component of momentum for w = 60 GeV

TABLE I

A$_s$ VALUES FOR DIFFERENT Y_{CUT}S ACCORDING TO THE DIFFERENTIAL CROSS SECTION OF THREE JET EVENTS WITH RESPECT TO THE P_T^2

Y_{cut}	α_s
0.02	0.130±0.004
0.025	0.128±0.005
0.03	0.127±0.002
0.035	0.124±0.006
0.04	0.123±0.004
0.045	0.125±0.002
0.05	0.126±0.003

The p_T^2 distribution (eq. (1)) can be translated into an acollinearity distribution $\frac{d\sigma}{d\theta}$ where θ is the angle between the q and $\bar{q}$ jet direction defined in Fig. 7 [19]:

$$\frac{1}{\sigma}\frac{d\sigma}{d\theta} = \frac{8\alpha_S}{3\pi}\frac{1}{\theta}Log\left(\frac{1}{\theta^2}\right) \tag{2}$$

Fitting this expression after integration, to the experimental θ distribution as in Fig. 9, we obtain the value of α_s equal to 0.122±0.007.

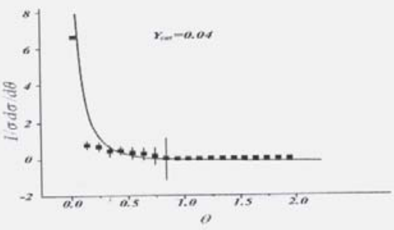

Fig. 9. Differential cross section of three jet events with respect to θ. Data are compared with the prediction of QCD (solid line)

The value of α_s for different Y_{cut}s with respect to the θ is listed in Table II.

TABLE II

A$_s$ VALUES FOR DIFFERENT Y_{CUT}S ACCORDING TO THE DIFFERENTIAL CROSS SECTION OF THREE JET EVENTS WITH RESPECT TO THE Θ

Y_{cut}	α_s
.02	.119±.004
.025	.121±.006
.03	.125±.003
.035	.115±.008
.04	.122±.005
.045	.124±.006
.05	.120±.003

IV. Three Jet Observables at NNLO

Up to now, the precision of the strong coupling constant determined from event shape data has been limited largely by the scale uncertainty of the perturbative NLO calculation.

We report here on the calculation of NNLO corrections to the 3-jet cross section and related event shape variables.

The knowledge of the NNLO corrections to the event shape distributions has important phenomenological impact on the extraction of strong coupling constant from AMY data [7], [26].

The calculation of the α_s^3 corrections for three jet production is carried out using a recently developed parton-level event generator program EERAD3 [20] which contains the relevant matrix elements with up to five external partons.

Besides explicit infrared divergences from the loop integrals, the four-parton and five-parton contributions yield infrared divergent contributions if one or two of the final state partons become collinear or soft.

In order to extract these infrared divergences and combine them with the virtual corrections,the antenna subtraction method [21] was extended to NNLO level [22] and related event-shape variables [23], [24] into EERAD3.

The three-jet cross section is expanded as:

$$\frac{\sigma_{3-jet}}{\sigma_{tot}} = \frac{\alpha_s}{2\pi}A_{3-jet} + \left(\frac{\alpha_s}{2\pi}\right)^2 B_{3-jet} + \left(\frac{\alpha_s}{2\pi}\right)^3 C_{3-jet} \tag{3}$$

where σ_0 is the LO cross section for $e^+e^- \to hadrons$.

The coefficients A_{3-jet}, B_{3-jet} and C_{3-jet} are calculated for $\mu^2 = Q^2$ [25]. The measured three-jet cross-sections are shown in graphical form in Fig. 10 for AMY data [7] and compared with different QCD models [25]. By comparing the three QCD plots, we observe that the agreement for each of the jet rates becomes systematically better as the order of perturbation theory increases.

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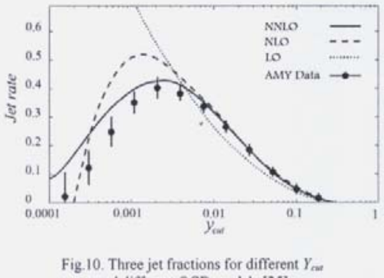

Fig.10. Three jet fractions for different Y_{cut} and different QCD models [25]

Also the figure indicates the AMY results are more consistent with NNLO corrections. By fitting our data with eq. (3), the strong coupling constant α_s is derived. The values of α_s at LO,NLO,NNLO are tabulated in Table III. We conclude that our results are more consistent with the NNLO QCD calculations.

TABLE III
A_s VALUES FOR 60 GEV AT LO, NLO AND NNLO

	α_s
LO	0.121±0.008
NLO	0.123±0.005
NNLO	0.124±0.003

V. Conclusion

In this paper we considered dependence of the longitudinal and transverse normalized inclusive cross-section on the center of mass energy to give the proof for gluon emission which leads to scaling violation and calculation of strong coupling constant. Then we have presented measurements of the Altarelli-Parisi-Weizsacker-Williams technique and three jet rate for hadronic events produced at AMY collaboration. The predictions of the NNLO are found to be in good agreement with the measured distributions. In general, NNLO provides the best description of the data and LO the least good. The coupling constant, α_s, is measured by two different methods, first by employing the the Altarelli-Parisi-Weizsacker-Williams technique. The value of α_s is determined to be 0.123± 0.004. Next, from the three jet distributions, we extract the strong coupling constant, α_s at Leading (LO), Next to Leading (NLO) and Next to Next Leading Order (NNLO). Our results are more consistent with the running of α_s, expected from QCD predictions at NNLO corrections.

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