Arch. Math. 93 (2009), 315–317
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0003-889X/09/040315-3
published online October 8, 2009
DOI 10.1007/s00013-009-0047-y

Archiv der Mathematik

Corrigendum to "On the Gorenstein injective dimension and Bass formula" [Arch. Math. 90 (2008), 18–23]

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Abstract. In this note, we provide a short argument which corrects flaws in the mentioned article.

The proof of Theorem 2.3 in [3], which is our main tool in [3], has an unsensible gap which can not be seen in the first investigations. It is hidden in the last lines of its proof, where we get to $\dim(\frac{\overline{M}}{x\overline{M}}) \leqslant 0$ and we conclude that $\dim(\frac{\overline{M}}{x\overline{M}}) = 0$, while the R-module $\frac{\overline{M}}{x\overline{M}}$ may be trivial. But this flaw can be corrected by an extra condition. One can easily see that in Aghapournahr's counterexample (see [1]) this condition doesn't hold. Moreover, as Aghapournahr has mentioned, one may like to know that each $\mathcal{AF}$ module is an $\mathcal{FA}$ module. Of course, our proof in Theorem 2.3 for $\mathcal{AF}$ modules is just three lines and is too easy or even trivial and the main part of the proof is about $\mathcal{FA}$ modules. Also, in the proof of Theorem 2.5, we used the inequality $\mathrm{Gid}_{R_\mathfrak{p}} M_\mathfrak{p} \leqslant \mathrm{Gid}_R M$ for a prime ideal $\mathfrak{p}$ of R. Although this inequality is so reasonable and is near to being correct, unfortunately it isn't proved up to now. In spite of that, Theorem 2.5, which is our main result of [3], is true in the local case, and it is still a generalization of Bass' formula.

In this regard, we think it is better to rewrite the corrected versions of our results in [3] as follows. Note that all of the proofs are similar to previous ones, so we avoid to rewrite them in this short note.

Theorem 2.3. (Compare [2, Theorem 6.1.4].) *Let $(R,\mathfrak{m})$ be a local ring and M be a non-zero $\mathcal{AF}$ or $\mathcal{FA}$ module of dimension n. Then if*

(1) $n = 0$ *or*

(2) $n > 0$ *and* $\mathfrak{m}M + \Gamma_\mathfrak{m}(M) \neq M$,

then $H^n_\mathfrak{m}(M) \neq 0$.

Corollary 2.4. *Let $(R,\mathfrak{m})$ be a local ring and M be an $\mathcal{AF}$ or $\mathcal{FA}$ module with finite Gorenstein injective dimension such that*

(1) $\dim_R M = 0$ *or*
(2) $\dim_R M > 0$ *and* $\mathfrak{m}M + \Gamma_{\mathfrak{m}}(M) \neq M$.

If $\dim_R M = \dim_R R$, *then* R *is Cohen-Macaulay and* $\mathrm{Gid}_R M = \mathrm{depth} R = \dim_R R$.

Theorem 2.5. *Let* $(R, \mathfrak{m})$ *be a local ring and* M *be a non-zero* $\mathcal{AF}$ *or* $\mathcal{FA}$ *module such that*

(1) $\dim_R M = 0$ *or*
(2) $\dim_R M > 0$ *and* $\mathfrak{m}M + \Gamma_{\mathfrak{m}}(M) \neq M$.

Then we have the following inequalities

$$\dim_R M \leqslant \mathrm{Gid}_R M \leqslant \mathrm{id}_R M.$$

Note that Theorem 2.5 now immediately follows from Theorems 2.1 and 2.3.

Corollary 2.6. *Suppose that* M *is a non-zero Matlis reflexive module over a local ring* $(R, \mathfrak{m})$ *such that*

(1) $\dim_R M = 0$ *or*
(2) $\dim_R M > 0$ *and* $\mathfrak{m}M + \Gamma_{\mathfrak{m}}(M) \neq M$.

Then we have the following inequalities

$$\dim_R M \leqslant \mathrm{Gid}_R M \leqslant \mathrm{id}_R M.$$

Acknowledgements. We would like to thank Professor H. Holm for his careful review on [3] (pointing out the gap of the proof of Theorem 2.5) (see [Zbl 1134.13013]) and Dr. M. Aghapournahr for finding a counterexample for Theorem 2.3. Also, I would thank the referee for his/her patience and good suggestions.

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Received: 30 June 2009