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Optimal Stabilizable Switching Law for Switched Linear Systems Using GA

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Abstract: *This paper investigates the optimal switching stabilizability problem for a class of continuous time switched linear systems. The proposed method is based on a theorem with necessary and sufficient condition for stability of switched linear systems. An optimization problem is produced and solved using Genetic algorithm to find the optimal switching rule. Although many cost functions can be used as the optimality's target, here the smallest convergence time which is noticeable in most of optimal switching problems is used as optimal target.*

Keywords: Genetic algorithm, Optimal switching rule, Switched linear system.

1. Introduction

Many systems have dynamics that are described by a set of continuous time differential equations in conjunction with a discrete event process. Such systems are usually referred to as switched or hybrid systems.

If the individual subsystems are given, then the behaviour of a switched system depends on the switching signal. Usually, different switching strategies produce different system behaviours and hence lead to different system performances. A well-known example is the switched server system which is able, not only to produce regular stable behaviour, but also to produce highly unstable behaviour such as chaos and multiple limit circles. In this situation, the choice of a suitable switching law to optimize certain performance index becomes an important and well-motivated problem [4].

Optimization over switching signals is indeed a challenging problem [5-6]. As a switching signal is a discontinuous function of time and possibly highly nonlinear, the optimization is extremely intricate and non-convex in nature.

In this paper we investigated a problem of finding the optimal stabilizable switching law for switched linear system with no control input. The subsystems are continuous time linear time invariant (LTI) systems,

$$x^{\cdot}(t) = A_\sigma x(t) \qquad t \in R^+, \sigma \in I$$

Most of switching rules that stabilize the switched linear systems are based on the existence of a stable convex combination of the subsystems [1-5,7]. The existence of such stable convex combination is just a sufficient condition, so although checking of that existence is not simple and straightforward, if there is not any stable convex combination of the subsystems the instability of switched systems can not be concluded.

Example 1: Consider the following continuous time switched linear system:

$$x^{\cdot}(t) = \begin{cases} A_1 x(t) & ,\sigma(t) = 1 \\ A_2 x(t) & ,\sigma(t) = 2 \end{cases}$$

with

$$A_1 = \begin{bmatrix} 0 & 10 \\ 0 & 0 \end{bmatrix}$$

and

$$A_2 = \begin{bmatrix} 1.5 & 2 \\ -2 & -0.5 \end{bmatrix}.$$

This switching system was used in [3] to show that switching between two unstable systems may exhibit stable behavior and it was also used in [2] as an example of satisfying the following theorem.

The eigenvalues of A_1 are two zeroes and the eigenvalues of A_2 are $0.5 \pm \sqrt{3}i$, which means that both systems are unstable.

It can be seen that there is not any stable convex combination of these two subsystems, because these two systems convex combination is as follows:

$$\omega A_1 + (1-\omega) A_2 = \begin{bmatrix} 1.5 - 1.5\omega & 8\omega + 2 \\ -2 + 2\omega & -0.5 + 0.5\omega \end{bmatrix},$$

$0 \le \omega \le 1$.

And this combination eigenvalues are

$$\lambda_1, \lambda_2 = 0.5 - 0.5\omega \pm \sqrt{-3 - 14\omega + 17\omega^2}$$

The square part ($-3 - 14\omega + 17\omega^2$) is negative for any $0 \le \omega \le 1$, which means the eigenvalues are complex for all acceptable ω and their real part is $0.5 - 0.5\omega$, which is positive for $0 \le \omega \le 1$. Figure 1 shows the eigenvalues loci for $0 \le \omega \le 1$, it shows that the convex combination of two systems is always unstable, so the sufficient condition _for stability_ is not satisfied, but it will be shown that the switched system is stable by introducing the appropriate switching rule. □

2. Description

Recently, Lin and Antsaklis [2] proposed a necessary and sufficient condition for the existence of a switching control law for asymptotic stabilization of continuous-time switched linear systems as a theorem.

2.1 Antsaklis theorem

Theorem: [2] assume that there is no sliding motion in the closed loop switched system. The continuous-time switched linear system can be globally asymptotically stabilized, if and only if

1) There exist a full row rank $L_i \in R^{m_i \times n}$, where $m_i < n$, such that the auxiliary system for i-th subsystem, i.e.

$$\theta^{\cdot}(t) = L_i A_i R_i \theta(t) \qquad , t \in R^+ \qquad (1)$$

is asymptotically stable. Here $R_i \in R^{n \times m_i}$ is a right inverse of L_i, it means $L_i R_i = I_{m_i \times m_i}$, such that the matrix

$$\begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_N \end{bmatrix} \in R^{\sum m_i \times n} \qquad (2)$$

has n linear independent row vectors.

2) Let Ω^i stands for conic cones induced through the intersection of these polyhedral lyaponov-like functions' level sets, and Ω^i be required to be contained in the range space of R_i. These induced conic cones cover the whole state space, i.e.

$$\bigcup \Omega^i = R^n$$

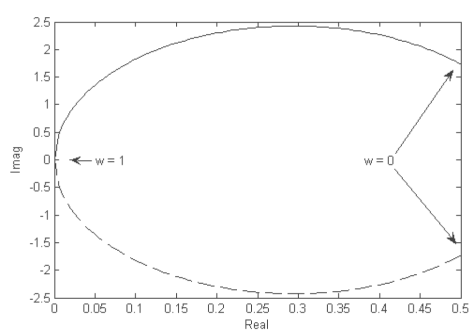

Fig. 1: Eigenvalues loci for convex combination of two systems of example 1.

2.2 Explanation of the theorem

The first condition is straightforward, because there always exist L and R satisfying the above assumption (1), except when $A = \lambda I_n$ for some positive real $\lambda > 0$. Here I_n is the identity matrix of dimension n.

The first condition can be interpreted as considering a linear combination of the states of the original subsystems as auxiliary systems, which is asymptotically stable. The auxiliary systems evolve in the lower dimensional state-space to which the original systems can be projected for stability. Note that even when all parts of the states of the original system are unstable, there may exist L satisfying the assumption (1), as below example.

Example 2: Consider a continuous time system,

$$x^{\cdot}(t) = \begin{bmatrix} 0 & 10 \\ 0 & 0 \end{bmatrix} x(t)$$

The above continuous time system is obviously unstable, but the auxiliary system $\theta^{\cdot}(t) = LAR\theta(t)$ with $L = \begin{bmatrix} 0 & 1 \end{bmatrix}$ and $R = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is asymptotically stable, because

$$LAR = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 10 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = -10 < 0$$

and

$$LR = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1$$

Note that R can be any matrix of the form $\begin{bmatrix} 1 \\ -r \end{bmatrix}$ with $r > 0$ for the selected L. □

In the second condition, according to the converse lyapunov theorem for linear time variant systems, the asymptotic stability of the auxiliary system implies the existence of a polytopic lyapunov function, which can be represented as

$$\phi_i(\theta) = \max_{1 < j < s_i} \{ f_j \theta \}$$

where $f_i \in R^{1 \times m_i}$ is a nonzero row vector and s_i is an integer greater than m_i. $F_i \in R^{s_i \times m_i}$ is taken to be the matrix with $f_i \in R^{1 \times m_i}$ as its rows. $\phi_i(\theta)$ must satisfy the conditions represented in [2].

The basic idea is that a polyhedral lyapunov-like function $\psi(x)$ can be constructed for each subsystem by transforming the corresponding polyhedral lyapunov function of its auxiliary system as follows. Denote $F_i L_i \in R^{s_i \times n}$ as H_i, and h_i as its row vector of H_i. Then the polyhedral lyapunov-like function $\psi_i(x)$ can be defined as

$$\psi_i(x) = \max_{1 < j < s_i} \{ h_j x, 0 \}$$

Considering the intersection of their level sets, i.e.

$$S = \bigcap_{i \in I} S_i = \bigcap_{i \in I} \{ x \in R^n | \psi_i(x) \le 1 \}$$

where S_i is the level set of ith subsystem's polyhedral lyapunov-like function.

Let Ω^i stands for conic cones induced through the intersection of these polyhedral lyaponov-like functions' level sets, and Ω^i be required to be contained in the M_i, which is the range space of R_i, i.e.

$$M_i = \bigcup_{R_i} image(R_i)$$

These induced conic cones must cover the whole state space, i.e.

$$\bigcup \Omega^i = R^n$$

Ω^i is the state space partition where i-th subsystem is active. The partitioning of two dimensional state space based on the above theorem is illustrated in figure 2.

2.2 Optimal stabilizable switching law

The necessary and sufficient conditions of discussed theorem were proved in [2], but checking of those conditions are not easy because it requires to parameterized all matrix L_i and R_i which satisfy (1). The calculation of such L_i and R_i for each subsystem could be tedious. Fortunately, it is always possible to restrict the search to the vector case, i.e., $m_i = 1$, $L_i \in R^{1\times n}$, and $R_i \in R^{n\times 1}$. This makes it is possible to formulate the determination of L_i and R_i into an optimization problem. There maybe a lot of L_i and R_i which satisfy the theorem's conditions, but just one of them tends to optimal results of the switching system with respect to the aim or cost function. Here the goal is minimization of the convergence time which corresponds to norm of states will be less than a specified error.

In this paper genetic algorithm is used to solve this problem, because the GA tends to find the global solution of the problem.

The GA is a stochastic optimization algorithm that was originally motivated by the mechanisms of natural selection and evolution of genetics. The underlying principles of the GA were first proposed by Holland in 1962 [9], whereas the mathematical framework was developed in the late 1960s and was presented in Holland's pioneering book [8].

The search space is a vector space with length Nn, where *N* is the number of subsystems and *n* is subsystems dimension, so each chromosome has the following formatting:

$$L = [L_1 \quad L_2 \quad \cdots \quad L_N].$$

Another important part in the GA is fitness function, which is declared as convergence time that tends to norm of states will be less than a specified error. Many of vectors L may not satisfy the theorem condition and so not feasible. Fitness value of these infeasible population members sets to a large number.

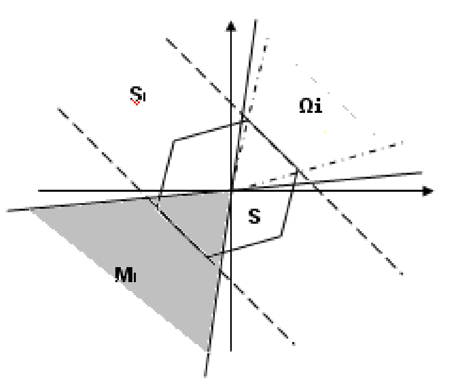

Fig. 2: Illustration of patitioning of two dimentional state space.

3. Simulation Results

The algorithm is set to find the best switching rule for switched linear systems which is used in example 1 to show that there was not any stable convex combination. i.e.,

$$A_1 = \begin{bmatrix} 0 & 10 \\ 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.5 & 2 \\ -2 & -0.5 \end{bmatrix}$$

The phase plane portraits of these two systems are illustrated in figure 3. The instability of both systems is indicated in figure 3.

Using genetic algorithm with population size of 100 and 50 iterations, following vector L is found.

L= [-11.4423 98.4065 -27.9669 -65.6957]

This vector L results two following switching lines, which is illustrated in figure 4.

$$x_2 = -0.1007 x_1$$

and

$$x_2 = 1.2048 x_1$$

Figure 5 shows the phase plane portrait of switched system with respect to above switching lines. Also the switching signal is shown in figure 6. Finally, switched system response is illustrated in figure 7 for initial condition equal to [-5 -5]. Here, the norm of states will be less than 0.1 after 3.95 seconds and less than 0.001 after 8.12 seconds.

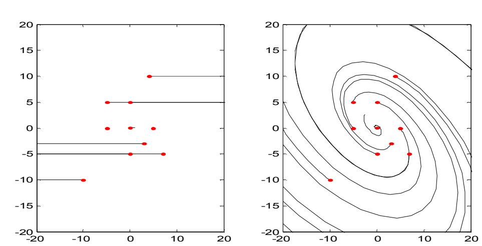

Fig. 3: The phase plane portrait of systems.

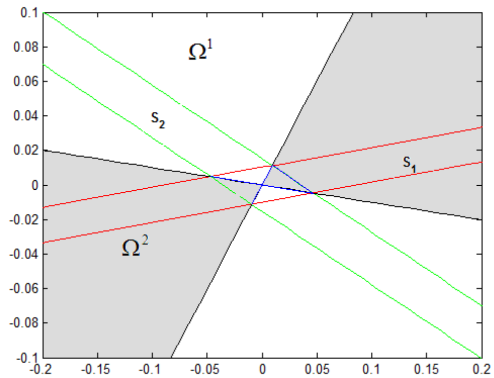

Fig. 4: Partiotioning of state space and switching line.

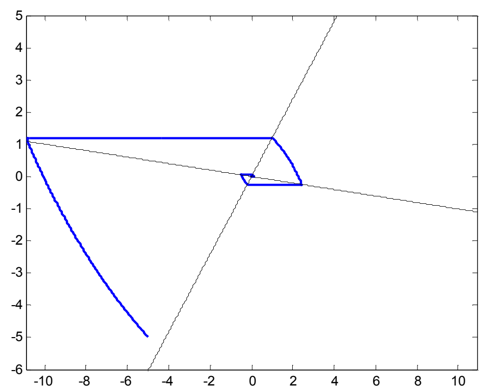

Fig. 5: phase plane portrait of switched system.

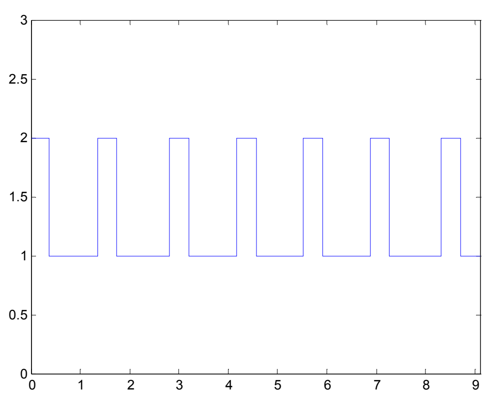

Fig. 6: The resulted switching signal.

3. Conclusion

Finding an optimal switching law for continuous time switched linear systems is considered in this paper. An optimization problem is produced based on Lin and Antsaklis [2] theorem. The optimal solution is found using GA. The explained theorem is very powerful,

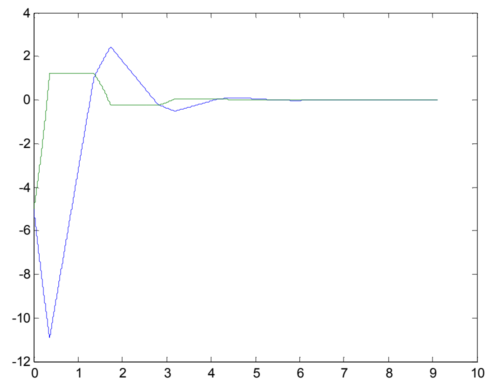

Fig. 7: Switched system response.

because of necessary and sufficient conditions for stability of switched linear system. This theorem is also applicable for a class of continuous time switched linear systems with uncertain parameters. So, further studies is needed to contain this kind of systems as an optimization problem.

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