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Optimal constrained power scheduling in Electricity Market

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***Abstract*--An optimal scheduling of units in an electric spot market presents in this paper. Unit commitment is a non-linear and complex combinatorial optimization problem which is difficult to be solved for large-scale power systems so this study addresses a linear expression of the problem. Proposed approach is a mixed-integer linear programming to minimize the total energy dispatch cost in 24 hours of a day. A system as the same structure as Iranian power market is used to demonstrate the linear expression of the problem. Simulation results compared with another approach. The results shows the applicability of the proposed method.**

***Index Terms*-- optimal scheduling, spot market, mixed integer linear programming**

I. Nomenclature

D_t	Real load demand during period t
DR_i	Ramp-down and shut-down rate limit (MW/min) of generation unit i
DT_i	Minimum down time of generation unit i
$P_i^{\max}$	Maximum real power output of generation unit i
$P_i^{\min}$	Minimum real power output of generation unit i
$P_{t,i}$	Real power output of generation unit i at period t
$P_{t,i,b}$	Real power of block b offered by generation unit i at period t
$P_{t,i,b}^{\max}$	Maximum real power of block b offered by generation unit i at period t
$p^{\bullet}_{t,i,b}$	Price offered by generation unit i at hour t for block b
U_i^0	Time periods that unit i has been on or off at the beginning of the planning horizon (end of hour 0)
UR_i	Ramp-up and start-up rate limit (MW/min) of generation unit i
UT_i	Minimum up time of generation unit i
$u_{t,i}$	Binary variable (0/1) that represents the commitment state of generation unit i at period t in the daily market
$x_{t,i}$	Number of hours that generation unit i has been on (+) or off (−) at the end of hour t

Sets

T	Set of all period indexes in hours
G	Set of indexes of all generation units
B	Set of indexes of energy sale blocks

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II. Introduction

THE unit commitment (UC) has been a subject with increasing interests after 1998. It determines an optimal schedule of units and the amount power generation to be used to meet the demand over a future period. After the liberalization of the electricity industry, in most of countries, the unit commitment problem is solved as a market problem based on bid prices, instead of the cost-based minimization of the classical model. In the simplest formulation, unit commitment can be defined as the problem of finding the best strategy to turn on or switch off generation units in the most economic way taking into account power balance equations and a number of technical limitations. A unit commitment problem is often formulated subject to several constraints that includes minimum up-time and down-time, ramp rate limits, generation constraints, load balances, must-run units, minimum and maximum energy limits, power transmission line capacity and spinning reserve. From the view of mathematics, it is a mixed integer non-linear programming problem to minimize the energy dispatch cost and meet various system constraints.

The presented test system is based on the structure of the Iranian day-ahead pool-based electricity market. Demand-side are not considered in Iran competitive energy market. In Iranian day-ahead energy market, the market participants (producers) submit hourly energy multi-block price bids and the market operator sets the accepted bids. The structure of the energy market is settled in a pay as bid mechanism.

Large-scale, mixed-integer, combinatorial, and nonlinear programming problem is an active research topic because of potential savings in operation costs. As a consequence, several solution techniques have been proposed such as heuristics, Lagrangian relaxation (LR), dynamic programming (DP), mixed-integer linear programming (MILP), Branch and Bound (BB) and Priority List.

Genetic algorithms (GAs) has been employed to solve the unit commitment problem [1]-[6]. However, GAs are time-consuming since it requires binary encoding and decoding to represent each unit operation state and to compute the fitness function, respectively, throughout GA procedures. This huge computation, makes it difficult to apply to large-scale systems. Sishaj in [7] implements movement of ants in the search space and also discusses the accuracy of the solution with respect to the solution time. Simon in [8] has solved UC using ant colony system with its exploration and exploitation ability. Simulated Annealing (SA) is a method to solve the Unit commitment problem in [9] based on the simulation of recrystallizing metal in the process of slow cooling (annealing). EP has been applied to the economic load dispatch problem [10]-[11]. This procedure, need to get a good starting point to converge. The main draw back of all mentioned methods (GA, ant colony, SA, EP, …) is that they don't guarantee the optimality of the solution. Dynamic programming is also used to solve the problem [12]-[13]. A mixed-integer linear programming approach [14] allows a rigorous modeling of non linear minimum up and down time constraints. That approach is based on the formulation stated by Dillon et al. [15]. Although DP and MIP can produce an optimal schedule, they cannot be practically applied even for mid-size systems because of the so-called 'combinatorial explosion' [16]. Branch and Bound solution technique is used to solve UC problem [17]. A straightforward method to find a solution to the unit Commitment problem is the Priority List (PL) method [19]-[20]. More complex models of UC proposed Lagrangian relaxation (LR) [21]-[24]. The LR method is a very effective method. The basic idea of LR is relaxing the system constraints with Lagrangian multipliers and formulating a Lagrangian dual function by appending the relaxed constraints to the primal objective. The process of solution includes major and minor iterations. However, if the problem considered more constraints, the more multipliers will be introduced, which may lead to slow convergence especially when the constraints are highly coupled. Even if the objective function of primal problem is non-convex and if a solution to the dual problem is found, the feasibility is not guaranteed due to the non-convexity of the objective function of the primal (original) problem.

Here a linear expression for UC problem is obtained and a 0/1 mixed integer linear programming is used to solve the problem and to find the optimal schedule of units and the corresponding amount of power generation.

The model has been tested on a system with the same structure of Iranian power market. The proposed method results compared with method described in [14]. The model has been programmed in GAMS, using CPLEX solver to solve the linear mixed-integer programming problem.

The rest of the paper is organized as follows. Section III formulates the unit commitment problem. Section IV described the idea of linearization of nonlinear constraints and formulated the problem as a mixed-integer linear programming formula. Section V provides results and compares it with the result of another approach. Finally, conclusions are stated in Section VI.

III. PROBLEM FORMULATION

The objective function can be stated as the minimization of:

$$\sum_{t\in T}\sum_{i\in G} p^{\bullet}_{t,i,1} u_{t,i} . P_i^{\min} + \sum_{t\in T}\sum_{i\in G}\sum_{\substack{b\in B\\ b\succ 1}} p^{\bullet}_{t,i,b} . P_{t,i,b} \qquad (1)$$

For simplicity, it is assumed that the minimum power of each unit, $P_i^{\min}$, is always offered as the first block and other blocks contains submitted power of each unit that is more than $P_i^{\min}$.

The objective function will be subjected to the following constraints:

Real power limits for any blocks:

$$0 \le P_{t,i,b} \le P_{t,i,b}^{\max} \quad \forall t \in T, \forall i \in G, \forall (b \succ 1) \in B \qquad (2)$$

Real power output limits:

$$u_{t,i} . P_i^{\min} \le P_{t,i} \le u_{t,i} . P_i^{\max} \quad \forall t \in T, \forall i \in G \qquad (3)$$

Which $P_{t,i}$ is the produced real power of unit i at period t and it can be derived by:

$$P_{t,i} = u_{t,i} . P_i^{\min} + \sum_{\substack{b\in B\\ b\succ 1}} P_{t,i,b} \quad \forall t \in T, \forall i \in G \qquad (4)$$

Real power balance:

$$\sum_{i\in G} P_{t,i} \ge D_t \quad \forall t \in T \qquad (5)$$

Ramp rate limits:

$$-DR_i \le P_{t,i} - P_{t-1,i} \le UR_i \quad \forall t \in T, \forall i \in G \qquad (6)$$

Minimum starting up times:

$$(x_{t-1,i} - UT_i)(u_{t-1,i} - u_{t,i}) \ge 0 \quad \forall t \in T, \forall i \in G \qquad (7)$$

Minimum starting down times:

$$(x_{t-1,i} + DT_i).(u_{t,i} - u_{t-1,i}) \le 0 \quad \forall t \in T\ \forall i \in G \qquad (8)$$

The addressed unit commitment problem is a mixed-integer non-linear optimization problem with linear objective function, binary decision variables, continuous variables for operation processes, time couplings and non-linear constraints, such as minimum up and down time constraints. The difficulties related to resolution of non-linear optimization problems with binary variables force to make use of an alternative linear formulation of the problem.

IV. TRANSLATE TO LINEAR MODEL

Mathematic model of UC is stated in the pervious section. This model contains nonlinear constraints such as minimum up time and minimum down time of each unit. This section presents an alternative linear formulation of problem.

A. *Linearization of Minimum Up Time Constraints*

Nonlinear constraints "(7)," are replaced by the equivalent linear constraints "(9)–(11)," below:

To satisfy the minimum up time constraint consider three distinct situations.

- Suppose that the unit was initially in operation less than its minimum up time, following equation will guaranty the minimum up time constraint.

$$\sum_{k=1}^{L_i} \left[1 - u_{k,i}\right] = 0,\ L_i = Min\left[T, (UT_i - U_i^0) u_{0,i}\right] \qquad (9)$$

Clearly, if the unit is initially de-committed or $UT_i \le U_i^0$ constraints "(9)," are not included in the formulation.

- Unit must remain committed at least for UT_i hours at any consecutive time during the day; if it is necessary. Clearly if $L_i + 1 \ge T - UT_i + 1$ these constraints are not be included.

$$\sum_{j=k}^{k+UT_i-1} u_{j,i} \ge UT_i \left[u_{k,i} - u_{k-1,i} \right] \quad (10)$$

$$k = L_i + 1, \ldots, T - UT_i + 1$$

- The unit that its minimum up time is greater than 2 must satisfy the minimum up constraint time in the last $UT_i - 1$ hours. It can be modeled by equation "(11)".

$$\sum_{j=k}^{T} \left[u_{j,i} - (u_{k,i} - u_{k-1,i}) \right] \ge 0 \quad (11)$$

$$k = T - UT_i + 2, \ldots, T$$

B. Linearization of Minimum Down Time Constraints

To replace nonlinear constraints "(8)," by the equivalent linear constraints three distinct situations are considered.

- Suppose that the unit was initially off less than its minimum up time, following equation will guaranty the minimum down time constraint.

$$\sum_{k=1}^{F_i} \left[u_{k,i} \right] = 0, \; F_i = Min\left[T, (DT_i - U_i^0)(1 - u_{0,i}) \right] \quad (12)$$

Clearly if the unit is initially committed or $DT_i \le U_i^0$, constraints "(12)," are not included in the formulation.

- Unit must remain committed at least for DT_i hours at any consecutive time during the day. Clearly if $F_i + 1 \ge T - DT_i + 1$, these constraints are not be included.

$$\sum_{j=k}^{k+DT_i-1} (1 - u_{j,i}) \ge DT_i \left[u_{k-1,i} - u_{k,i} \right] \quad (13)$$

$$k = F_i + 1, \ldots, T - DT_i + 1$$

- The unit that its minimum down time is greater than 2 must satisfy the minimum down time constraint in the last $DT_i - 1$ hours. It can be modeled by equation "(14)".

$$\sum_{j=k}^{T} \left[1 - u_{j,i} - (u_{k-1,i} - u_{k,i}) \right] \ge 0 \quad (14)$$

$$k = T - DT_i + 2, \ldots, T$$

V. TEST SYSTEMS AND RESULTS

The test carried out for the system that its structure is the same as Iranian electricity market. It is programmed in GAMS mathematical modeling language.

In appendix A, table A.1 addresses power demand for each time of the day, for simplicity just five hours reported. Table A.2 presents some characteristics of the unit generations available and the energy offer blocks and their bid prices are shown in table A.3.

Proposed linearization method and 0/1 mixed integer programming is employed to solve unit commitment problem and optimal schedule of units find. In [14] another linearization method presents to convert the nonlinear minimum up time and minimum down time constraints in to linear one. So this method employed and results are reported and compared with the result of our paper.

The test system includes 7 units: two gas plants, two steam units and three combined cycle plants. The total generation capacity amounts to 626 MW. The transmission network constraints are not included in this study. Results and optimal schedule reported only for five hours.

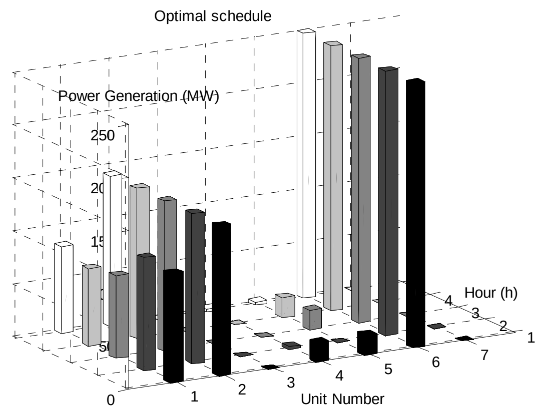

Fig. 1. Optimal scheduling according to presented method

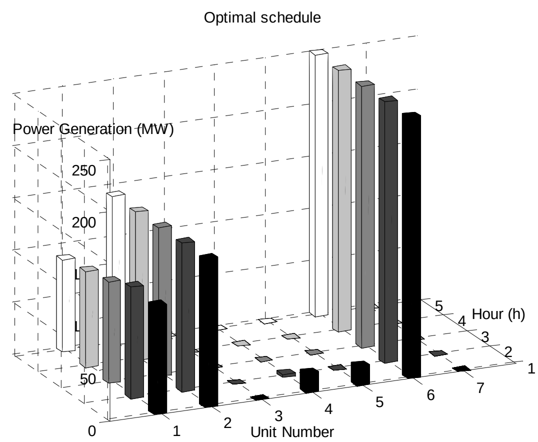

Fig. 2. Optimal scheduling according to [14]

In this section "fig. 1," shows described technique results and "fig. 2," shows results of employing [14] technique. Both "fig. 1," and "fig. 2," suggest the most economic way taking into account power balance equations and technical limitations and the objective function has the same value.

An approach in [14] suggests three sets of binary variable to linearize nonlinear constraints and it becomes problem more complex than one set of binary variable used in this paper. In addition, comparing results shown in "fig. 1," and "fig. 2," clarify that turn on or switch off generation units strategy in figure1 is more fair. Generating units 4 and 5 (G4, G5) are suitable to be used to compare the solution obtained in both papers.

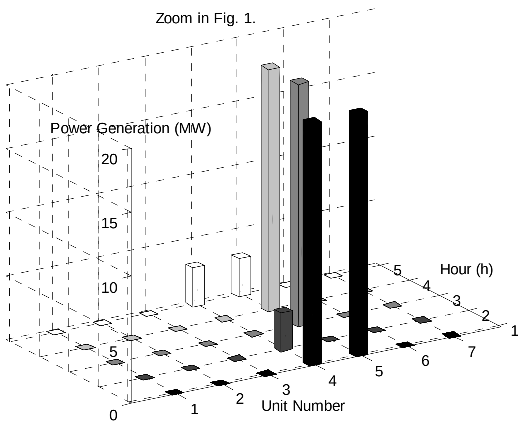

Fig. 3. Zoom in of figure1

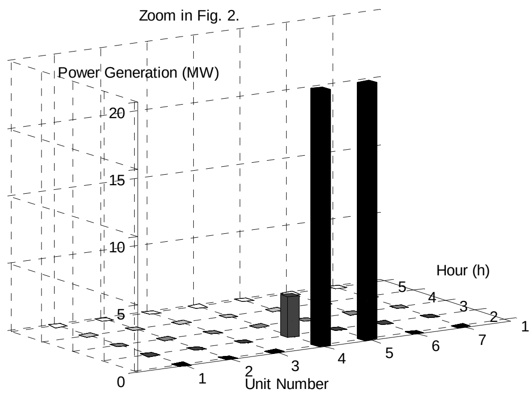

Fig. 4. Zoom in of figure2

"Fig. 3," and "fig. 4," zoom in to study better, and show how G4 and G5 are committed by employing presented linear expression in both papers. By employing presented solution as shown in "fig. 3," unit 5 is committed and it can get money for part of its generation that won in competitive market, but as "fig. 4," shows, market operator meets the hourly demand without unit 5 (G5), so described approach is more fair than in "fig. 4," that, this unit is de-commit and it will be loss.

Table 1 shows the final unit commitment schedule for selected case study. Number '1' underlined in table, indicates that described solution technique commits unit 5 but in [14] this unit is down. It can be seen from table 1 that the supposed optimal schedule is more fair.

Table 1
Final units schedule

Time (h)	G1	G2	G3	G4	G5	G6	G7
1	1	1	0	1	1	1	0
2	1	1	0	1	0	1	0
3	1	1	0	0	1	1	0
4	1	1	0	0	1	1	0
5	1	1	0	1	1	1	0

VI. Conclusion

In this work the unit commitment problem is re-formulated with reference to the electricity market environment being set up in Iran. In this formulation try to reduce non-linear constraint in mathematical model and employ a linearization method to obtain a linear expression of nonlinear minimum up and minimum down time constraints. The objective is to find optimal schedule to minimize the energy dispatch cost. Mixed integer programming technique is selected and employed on a test system. Results from realistic case studies are reported. Comparing the results of this technique with another one, shows that complexity of modeling in this paper is less than another approach. In addition, linear expression in this study leads to fairness.

VII. Appendix

Power demand of network for five hours is shown in table A.1, The features of the generating units are shown in Table A.2 presents the features of generation units and table A.3 presents energy multi-block price bids.

Table A.1
Hourly network demand

Hour (h)	Network demand (MW)
1	534.5
2	502
3	489
4	484.5
5	480

Table A.2
Features of the generating units

Unit	P_i^{min}	P_i^{max}	DT_i	UT_i	UR_i	DR_i	U_i^0	P_i^0
G1	70	142	3	3	11	11	2	125
G2	70	142	4	3	11	11	-72	0
G3	0	38	3	1	40	40	-100	0
G4	3	21	1	2	3	3	-100	0

In table A.2 power unit is (MW), time unit is (h), ramp-up and ramp-down units are (MW/min).

Energy multi-block price bids of the generating units are shown in table A.3. This table dosen't contain the unit that is shut down or dosen't submit their bids.

Table A.3
Energy multi-block price bids

Time,Unit	$\dot{p}_{t,i,b}$ (rial/MWh)			$P_{t,i}$ (MW)		
	1	2	3	1	2	3
1,1	60000	60000	0	70	72	0
1,2	60000	60000	0	70	72	0
1,4	50000	50000	53000	3	10	6
1,5	60000	60000	0	3	16	0
1,6	60000	60000	0	125	125	0
2,1	60000	60000	0	70	72	0
2,2	60000	60000	0	70	72	0
2,4	60100	60100	60500	3	10	6
2,5	60000	60000	0	3	16	0
2,6	60000	60000	0	125	125	0
3,1	60000	60000	0	70	72	0
3,2	60000	60000	0	70	72	0
3,4	60500	60500	61000	3	9	7
3,5	60000	60000	0	3	16	0
3,6	60000	60000	0	125	125	0
4,1	60000	60000	0	70	72	0
4,2	60000	60000	0	70	72	0
4,4	67000	67000	68000	3	10	6
4,5	60000	60000	0	3	16	0
4,6	60000	60000	0	125	125	0
5,1	60000	60000	0	70	72	0

5,2	60000	60000	0	70	72	0
5,4	59000	59000	0	70	72	0
5,5	60000	60000	0	3	16	0
5,6	60000	60000	0	125	125	0

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IX. Biographies

Nadia Zendehdel was born in Mashhad, Iran, on April 1, 1984. She graduated from the Farzanegan School, and studies at M. S. degree in Ferdowsi university of Mashhad.
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