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The effect of blankholder on flange plastic wrinkling of laminated circular plates in deep drawing process

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Abstract

Based on two-dimensional plane stress wrinkling model of a plastic annular plate with blankholder and a bifurcation functional from Hill's general theory of uniqueness for polar coordinate, the critical conditions for plastic wrinkling with blankholder of the flange of a two layered circular blanks during the deep-drawing process are obtained analyticaly. In order to obtain critical conditions for onset of wrinkling, Tresca yield criterion alongwith perfectly plastic manner of material is considered. To have a closed-form solution, deformational theory in plasticity is proposed. Finally from the proposed solution the critical wave numbers and the limitation of drawing for the laminated Steel-Aluminum plate is investigated and the effect of blank holder and also change of it is discussed in detail.

Keywords: Plastic Wrinkling–Bifurcational Functional–Laminated Plates–Tresca Yield Criterion–Blankholder

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Based on two-dimensional plane stress wrinkling model of a plastic annular plate with blankholder and a bifurcation functional from Hill's general theory of uniqueness for polar coordinate, the critical conditions for plastic wrinkling with blankholder of the flange of a two layered circular blanks during the deep-drawing process are obtained analyticaly. In order to obtain critical conditions for onset of wrinkling, Tresca yield criterion alongwith perfectly plastic manner of material is considered. To have a closed-form solution, deformational theory in plasticity is proposed. Finally from the proposed solution the critical wave numbers and the limitation of drawing for the laminated Steel-Aluminum plate is investigated and the effect of blank holder and also change of it is discussed in detail.

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1. Introduction
Wrinkling is one of the major modes of failure in automotive sheet pressing operations. Wrinkling is caused by excessive compressive stresses during forming. In a deep-drawing operation two initially flat round blank is drawn over a die by a cylindrical punch, as shown in Figure 1. The annular part of the blanks are subjected to a radial tensile stress, while in the circumferential direction compressive stress is generated during drawing, Figure 2. For particular drawing-tool dimensions and blank thicknesses, there is a critical blank diameter/thickness ratio as the critical stress causes plastic buckling of the annular part of the blanks so that an undesirable mode of deformation ensues with waves being produced in the flange as shown in Figure. 3. A bifurcation functional was proposed by Hutchinson [1] based on Hill general theory of uniqueness and bifurcation in elastic-plastic solids . This functional is given and explained by [2],

$$F(u,v,w)=\frac{1}{2}\iint (M_{ij}\kappa_{ij}+N_{ij}\varepsilon_{ij}^{0}+t\,\sigma_{ij}w_{,i}w_{,j})\ ds\,. \qquad (1)$$

where s denotes the region of the shell middle surface over which the wrinkles appear, u and v in-plane displacements, w the buckling displacement, t the thickness of the plate, N_{ij} the force resultants, M_{ij} the couple resultants, κ_{ij} the bending strain (or the change of the curvature) tensor and ε_{ij}^{0} the stretch strain tensor. This bifurcation functional is the total energy for wrinkling occurrence. $F=0$

corresponds to the critical conditions for wrinkles to occur for some non-zero displacement fields [1-3]. In our study the functional for a two laminated with considering the energy of the blank holder is proposed from the general form of functional (1), and the critical conditions for onset of wrinkling and the effect of blank holder force is investigated, respectively. Before proposing the general form of functional (1) for the case of two laminated annular plates, we need some definitions in two laminated plates in polar coordinate, Figure 4.

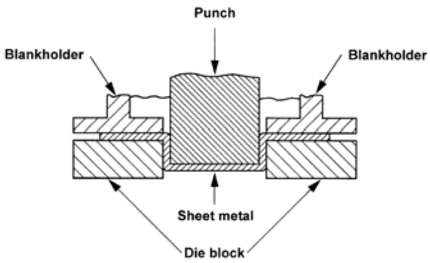

Figure 1. Deep drawing process with cylindrical punch

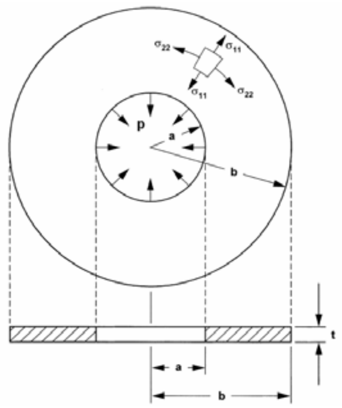

Figure 2. The flange is modeled as two annular plates with radial stress distribution in their inner edges.

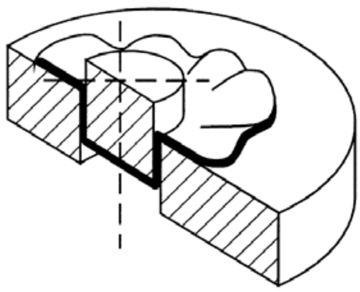

Figure 3. The waves produce in the flange

2. Investigation of plastic flang wrinkling with blankholder

When the coordinate system is set in the middle plane of the undeformed (pre-buckled) laminated plate, which the thickness of each plane is t, the material points in the plate are identified by coordinates r and θ lying in the middle surface of the undeformed body and coordinate z normal to the undeformed middle surface. The displacement fields u, v, w of two layers are the same. Then the bending strain (or the change of the curvature) κ_{ij} in the laminated are created as,

$$\kappa_{ij} = -w_{,ij}\,, \qquad (2)$$

where w is the buckling displacement normal to the middle surface of the plate.
For annular laminated plates and plane stress problem we have,

$$\begin{cases} \kappa_{11} = -\dfrac{\partial^2 w}{\partial r^2}, \\ \kappa_{22} = -\dfrac{1}{r}\dfrac{\partial w}{\partial r} - \dfrac{1}{r^2}\dfrac{\partial^2 w}{\partial \theta^2}, \\ \kappa_{12} = -\dfrac{1}{r}\dfrac{\partial^2 w}{\partial r \partial \theta} + \dfrac{1}{r^2}\dfrac{\partial w}{\partial \theta}. \end{cases} \qquad (3)$$

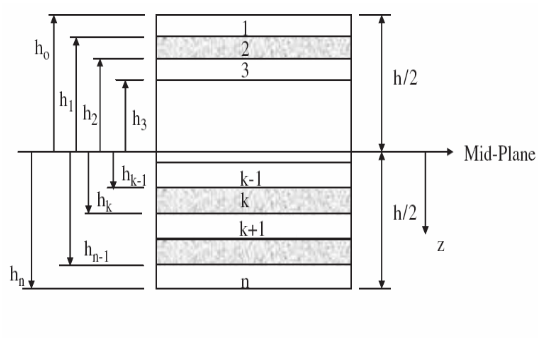

Figure 4. The coordinate of laminated plates

The stretch strain ε_{ij}^0 in plane stress problem and with neglecting nonlinear terms are defined as

$$\varepsilon_{ij}^0 = \frac{1}{2}(u_{i,j} + u_{j,i}). \qquad (4)$$

In polar coordinate the components of ε_{ij}^0 become,

$$\begin{cases} \varepsilon_{rr}^0 = \dfrac{\partial u}{\partial r}, \\ \varepsilon_{\theta\theta}^0 = \dfrac{u}{r} + \dfrac{1}{r}\dfrac{\partial v}{\partial \theta}, \\ \varepsilon_{r\theta}^0 = \dfrac{1}{2}\left(\dfrac{1}{r}\dfrac{\partial u}{\partial \theta} + \dfrac{\partial v}{\partial r} - \dfrac{v}{r}\right), \end{cases} \qquad (5)$$

where u and v are in-plane displacements in the r and θ directions, respectively.
Therefore the Lagrangian strain tensor for any point inside the laminated plates with distances z can be defined as,

$$\varepsilon_{ij} = \varepsilon_{ij}^0 + z\,\kappa_{ij}\,. \qquad (6)$$

In a two laminated plates, which the thickness of each plate is t, the force resultants and couple resultants are defined as,

$$\begin{cases} N_{ij} = \int_{-t}^{0} \sigma_{ij}^{1}\, dz + \int_{0}^{+t} \sigma_{ij}^{2}\, dz, \\ M_{ij} = \int_{-t}^{0} \sigma_{ij}^{1}\, z\, dz + \int_{0}^{+t} \sigma_{ij}^{2}\, z\, dz. \end{cases} \tag{7}$$

The constitutive equation for an elastic-plastic solid for each layer is

$$\sigma_{ij} = L_{ijkl}^{ep}\, \varepsilon_{kl}, \tag{8}$$

where L_{ijkl}^{ep} for perfectly plastic material for each layer is defined as [3,4],

$$L_{ijkl}^{ep} = L_{ijkl}^{e} - \frac{L_{ijkl}^{e} \dfrac{\partial f}{\partial \sigma_{mn}} \dfrac{\partial f}{\partial \sigma_{pq}} L_{pqkl}^{e}}{\dfrac{\partial f}{\partial \sigma_{rs}} L_{rstu}^{e} \dfrac{\partial f}{\partial \sigma_{tu}}}. \tag{9}$$

With using tresca yield criterion $f = \sigma_r - \sigma_\theta - Y = 0$, and Eq. (9), Eq. (8) can be expanded for $i, j = 1,2,3$ and simplified for plane stress conditions (i.e. $\sigma_{33} = \tau_{23} = \tau_{13} = 0$), to obtain L_{ijkl}^{ep}. Now a simple stress-strain relation for each layer can be obtained,

$$\begin{Bmatrix} \sigma_{11}^{1} \\ \sigma_{22}^{1} \\ \tau_{12}^{1} \end{Bmatrix} = \begin{bmatrix} \dfrac{E_1}{2\,(1-\nu_1)} & \dfrac{E_1}{2\,(1-\nu_1)} & 0 \\ \dfrac{E_1}{2\,(1-\nu_1)} & \dfrac{E_1}{2\,(1-\nu_1)} & 0 \\ 0 & 0 & \dfrac{E_1}{2\,(1+\nu_1)} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix}, \tag{10}$$

$$\begin{Bmatrix} \sigma_{11}^{2} \\ \sigma_{22}^{2} \\ \tau_{12}^{2} \end{Bmatrix} = \begin{bmatrix} \dfrac{E_2}{2\,(1-\nu_2)} & \dfrac{E_2}{2\,(1-\nu_2)} & 0 \\ \dfrac{E_2}{2\,(1-\nu_2)} & \dfrac{E_2}{2\,(1-\nu_2)} & 0 \\ 0 & 0 & \dfrac{E_2}{2\,(1+\nu_2)} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix}. \tag{11}$$

where E_1 and E_2 are Young 's modulus of elasticity and ν_1 and ν_2 are Poisson's ratio in each layer.

Substituting Eq. (6) into Eq. (8)) and inserting the result into Eq. (7), force resultant and couple resultant are found as,

$$\begin{cases} N_{ij} = \int_{-t}^{0} \sigma_{ij}^{1}\, dz + \int_{0}^{+t} \sigma_{ij}^{2}\, dz = \int_{-t}^{0} L_{ijkl}^{ep1}\, \varepsilon_{kl}\, dz + \int_{0}^{+t} L_{ijkl}^{ep2}\, \varepsilon_{kl}\, dz \\ = \int_{-t}^{0} L_{ijkl}^{ep1} (\varepsilon_{kl}^{0} + z\,\kappa_{kl})\, dz + \int_{0}^{+t} L_{ijkl}^{ep2} (\varepsilon_{kl}^{0} + z\,\kappa_{kl})\, dz \\ = [\int_{-t}^{0} L_{ijkl}^{ep1}\, z\, dz + \int_{0}^{+t} L_{ijkl}^{ep2}\, z\, dz]\kappa_{kl} + [\int_{-t}^{0} L_{ijkl}^{ep1}\, dz + \int_{0}^{+t} L_{ijkl}^{ep2}\, dz]\varepsilon_{kl}^{0} \\ = \frac{t^2}{2}(L_{ijkl}^{ep2} - L_{ijkl}^{ep1})\kappa_{kl} + t(L_{ijkl}^{ep1} + L_{ijkl}^{ep2})\varepsilon_{kl}^{0}\,, \\ M_{ij} = \int_{-t}^{0} \sigma_{ij}^{1}\, z\, dz + \int_{0}^{+t} \sigma_{ij}^{2}\, z\, dz = \int_{-t}^{0} L_{ijkl}^{ep1}\, \varepsilon_{kl}\, z\, dz \\ + \int_{0}^{+t} L_{ijkl}^{ep2}\, \varepsilon_{kl}\, z\, dz = \int_{-t}^{0} L_{ijkl}^{ep1} (\varepsilon_{kl}^{0} + z\,\kappa_{kl})\, z\, dz + \int_{0}^{+t} L_{ijkl}^{ep2} (\varepsilon_{kl}^{0} + z\,\kappa_{kl})\, z\, dz \\ = [\int_{-t}^{0} L_{ijkl}^{ep1}\, z^2\, dz + \int_{0}^{+t} L_{ijkl}^{ep2}\, z^2\, dz]\kappa_{kl} + [\int_{-t}^{0} L_{ijkl}^{ep1}\, z\, dz + \int_{0}^{+t} L_{ijkl}^{ep2}\, z\, dz]\varepsilon_{kl}^{0} \\ = \frac{t^3}{3}(L_{ijkl}^{ep1} + L_{ijkl}^{ep2}) + \frac{t^2}{2}(L_{ijkl}^{ep2} - L_{ijkl}^{ep1})\varepsilon_{kl}^{0}\,. \end{cases} \tag{12}$$

Finally with substituting these equations into functional (1) and considering the energy of the blank holder with stiffness K, it can be shown that,

$$\begin{aligned} F = & \frac{1}{2}\int_{0}^{2\pi}\int_{a}^{b} \frac{t^3}{3} L_{ijkl}^{ep1} \kappa_{ij}\, \kappa_{kl}\, r\, dr d\theta + \frac{1}{2}\int_{0}^{2\pi}\int_{a}^{b} t\, L_{ijkl}^{ep1} \varepsilon_{ij}^{0}\, \varepsilon_{kl}^{0}\, r\, dr d\theta - \frac{1}{2}\int_{0}^{2\pi}\int_{a}^{b} t^2\, L_{ijkl}^{ep1} \kappa_{ij}\, \varepsilon_{kl}^{0}\, r\, dr d\theta + \\ & \frac{1}{2}\int_{0}^{2\pi}\int_{a}^{b} t\, \sigma_{ij}^{1}\, w_{,i}\, w_{,j}\, r\, dr d\theta + \frac{1}{2}\int_{0}^{2\pi}\int_{a}^{b} \frac{t^3}{3} L_{ijkl}^{ep2} \kappa_{ij}\, \kappa_{kl}\, r\, dr d\theta + \frac{1}{2}\int_{0}^{2\pi}\int_{a}^{b} t\, L_{ijkl}^{ep2} \varepsilon_{ij}^{0}\, \varepsilon_{kl}^{0}\, r\, dr d\theta + \\ & \frac{1}{2}\int_{0}^{2\pi}\int_{a}^{b} t^2\, L_{ijkl}^{ep2} \kappa_{ij}\, \varepsilon_{kl}^{0}\, r\, dr d\theta + \frac{1}{2}\int_{0}^{2\pi}\int_{a}^{b} t\, \sigma_{ij}^{2}\, w_{,i}\, w_{,j}\, r\, dr d\theta + \frac{1}{2} K (w_{\max}^2 + u_{\max}^2 + v_{\max}^2). \end{aligned} \tag{13}$$

The first four integration is related to layer 1 and the second four integration is related to layer 2 and the last term is the energy of the blank holder.

By substituting Eq. (3) and Eq. (5) into functional (13) the functional for a two laminated plate is obtained,

$$
\begin{aligned}
F = & \frac{1}{2}\int_0^{2\pi}\int_a^b \{ \frac{t^3}{3}[L^{ep1}_{1111}(\frac{\partial^2 w}{\partial r^2})^2 + 2L^{ep1}_{1122}(\frac{\partial^2 w}{\partial r^2})(\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}) \\
& + L^{ep1}_{2222}(\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2})^2 + 4L^{ep1}_{1212}(\frac{1}{r}\frac{\partial^2 w}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial w}{\partial\theta})^2]\} \, r\,dr\,d\theta + \\
& \frac{1}{2}\int_0^{2\pi}\int_a^b\{t\,[L^{ep1}_{1111}(\frac{\partial u}{\partial r})^2+2L^{ep1}_{1122}(\frac{\partial u}{\partial r})(\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\theta})+L^{ep1}_{2222}(\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\theta})^2+ \\
& L^{ep1}_{1212}(\frac{1}{r}\frac{\partial u}{\partial\theta}+\frac{\partial v}{\partial r}-\frac{v}{r})^2]\} \, r\,dr\,d\theta - \frac{1}{2}\int_0^{2\pi}\int_a^b\{t^2\,[L^{ep1}_{1111}(-\frac{\partial^2 w}{\partial r^2})(\frac{\partial u}{\partial r})+ \\
& L^{ep1}_{1122}[(-\frac{\partial^2 w}{\partial r^2})(\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\theta})+(-\frac{1}{r}\frac{\partial w}{\partial r}-\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2})(\frac{\partial u}{\partial r})]+L^{ep1}_{2222}(-\frac{1}{r}\frac{\partial w}{\partial r} \\
& -\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2})(\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\theta})+2L^{ep1}_{1212}(-\frac{1}{r}\frac{\partial^2 w}{\partial r\partial\theta}+\frac{1}{r^2}\frac{\partial w}{\partial\theta})(\frac{1}{r}\frac{\partial u}{\partial\theta}+\frac{\partial v}{\partial r}-\frac{v}{r})]\} \\
& r\,dr\,d\theta + \int_0^{2\pi}\int_a^b\{t\,[\sigma_r^1(\frac{\partial w}{\partial r})^2+\sigma_\theta^1(\frac{1}{r}\frac{\partial w}{\partial\theta})^2]\} \, r\,dr\,d\theta + \frac{1}{2}\int_0^{2\pi}\int_a^b\{\frac{t^3}{3}[\\
& L^{ep2}_{1111}(\frac{\partial^2 w}{\partial r^2})^2+2L^{ep2}_{1122}(\frac{\partial^2 w}{\partial r^2})(\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2})+L^{ep2}_{2222}(\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2})^2 \\
& +4L^{ep2}_{1212}(\frac{1}{r}\frac{\partial^2 w}{\partial r\partial\theta}-\frac{1}{r^2}\frac{\partial w}{\partial\theta})^2]\} \, r\,dr\,d\theta+\frac{1}{2}\int_0^{2\pi}\int_a^b\{t\,[L^{ep2}_{1111}(\frac{\partial u}{\partial r})^2+2L^{ep2}_{1122}(\frac{\partial u}{\partial r}) \\
& (\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\theta})+L^{ep2}_{2222}(\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\theta})^2+L^{ep2}_{1212}(\frac{1}{r}\frac{\partial u}{\partial\theta}+\frac{\partial v}{\partial r}-\frac{v}{r})^2]\} \, r\,dr\,d\theta+\frac{1}{2}\int_0^{2\pi}\int_a^b\{t^2\,[L^{ep2}_{1111} \\
& (-\frac{\partial^2 w}{\partial r^2})(\frac{\partial u}{\partial r})+L^{ep2}_{1122}[(-\frac{\partial^2 w}{\partial r^2})(\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\theta})+(-\frac{1}{r}\frac{\partial w}{\partial r}-\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2})(\frac{\partial u}{\partial r})]+L^{ep2}_{2222}(-\frac{1}{r}\frac{\partial w}{\partial r}- \\
& \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2})(\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial\theta})+2L^{ep2}_{1212}(-\frac{1}{r}\frac{\partial^2 w}{\partial r\partial\theta}+\frac{1}{r^2}\frac{\partial w}{\partial\theta})(\frac{1}{r}\frac{\partial u}{\partial\theta}+\frac{\partial v}{\partial r}-\frac{v}{r})]\} \, r\,dr\,d\theta \\
& +\int_0^{2\pi}\int_a^b\{t\,[\sigma_r^2(\frac{\partial w}{\partial r})^2+\sigma_\theta^2(\frac{1}{r}\frac{\partial w}{\partial\theta})^2]\} \, r\,dr\,d\theta+\frac{1}{2}K(w_{\max}^2+u_{\max}^2+v_{\max}^2).
\end{aligned}
\tag{14}
$$

To calculate the critical conditions for onset of wrinkling, displacement fields (u, v, w) which satisfid the geometric boundary conditions must be considered. They can be expressed as a function of the radial coordinate r and the polar angle θ. In this paper, the displacement fields of the flange for a deep drawn cup has the following form [5,6],

$$
\begin{cases}
w(r,\theta) = c\,(r-a)(1+\cos n\theta), \\
u(r,\theta) = d\,r\cos n\theta, \\
v(r,\theta) = e\,r\sin n\theta.
\end{cases}
\tag{15}
$$

where c, d and e are constants. It is obvious that any admissible bifurcation mode in (18) satisfies the kinematical boundary conditions u, $v \neq 0$ and $w = 0$ at the inner edge $r = a$ and the kinematical constraint condition $w(r,\theta) \geq 0$, $u(r,\theta) \geq 0$, $v(r,\theta) \geq 0$ for $a \leq r \leq b$.

In order to find the critical wave number and critical load for a two laminated plate, a pre-buckled stress distribution in each of two annular plates which their inner edges are subjected to a radial stress p are taken into account. Hence, in the case of axisymmetric geometry of laniated plates some calculations are needed. Finally the following expressions will be obtained,

$$\begin{cases} \sigma_r^1 = Y_1 \ln(\frac{b}{r}) + \frac{4\, m^2 (k_1 k_4 - k_2 k_3) p}{[k_1^2 - k_2^2 + k_3^2 - k_4^2 + 2(k_1 k_3 - k_2 k_4)](1-m^2)}, \\ \sigma_\theta^1 = Y_1 [\ln(\frac{b}{r}) - 1] + \frac{4\, m^2 (k_1 k_4 - k_2 k_3) p}{[k_1^2 - k_2^2 + k_3^2 - k_4^2 + 2(k_1 k_3 - k_2 k_4)](1-m^2)}. \end{cases} \quad (16)$$

and

$$\begin{cases} \sigma_r^2 = Y_2 \ln(\frac{b}{r}) - \frac{4\, m^2 (k_1 k_4 - k_2 k_3) p}{[k_1^2 - k_2^2 + k_3^2 - k_4^2 + 2(k_1 k_3 - k_2 k_4)](1-m^2)}, \\ \sigma_\theta^2 = Y_2 [\ln(\frac{b}{r}) - 1] - \frac{4\, m^2 (k_1 k_4 - k_2 k_3) p}{[k_1^2 - k_2^2 + k_3^2 - k_4^2 + 2(k_1 k_3 - k_2 k_4)](1-m^2)}, \end{cases} \quad (17)$$

where $k_1 = L_{1111}^{ep1} = \frac{E_1}{1-\nu_1^2}$, $k_2 = L_{2222}^{ep1} = \frac{\nu_1 E_1}{1-\nu_1^2}$, $k_3 = L_{1111}^{ep2} = \frac{E_2}{1-\nu_2^2}$, $k_4 = L_{2222}^{ep4} = \frac{\nu_2 E_2}{1-\nu_2^2}$.

To calculate the last term in functional (14) The BHF(S) can be considered such as[7],

$$S = K\pi(b^2 - a^2), \quad (18)$$

where a and b are the inner and outer radii of the blank, respectively, therefore we have,

$$\begin{cases} w_{max} = c(r-a)(1+\cos n\theta)\big|_{\substack{r=a \\ \theta=0}} = 2c(b-a), \\ u_{max} = d\, r \cos n\theta\big|_{\substack{r=b \\ \theta=0}} = db, \\ v_{max} = e\, r \sin n\theta\big|_{\substack{r=b \\ \theta=\frac{\pi}{2n}}} = eb. \end{cases} \quad (19)$$

Finally it can be shown that,

$$\frac{1}{2} K (w_{max}^2 + u_{max}^2 + v_{max}^2) = \frac{2}{\pi} S \frac{1-m}{1+m} c^2 + \frac{1}{2\pi} \frac{S}{(1-m^2)} (d^2 + e^2). \quad (20)$$

By substituting these terms into functional (14), it is found that,

$$
\begin{aligned}
F=&\frac{t^3 c^2 E_1 \pi}{24(1-\nu_1^2)} G_1^{ep}(m,n,\nu_1)+\frac{\pi t E_1 b^2}{8(1-\nu_1{}^2)}\{Q_1^{ep}(m,n,\nu_1)d^2+R_1^{ep}(m,n,\nu_1)de+\\
&S_1^{ep}(m,n,\nu_1)e^2\}-\frac{\pi t^2 E_1 b}{4(1-\nu_1^2)}\{T_1^{ep}(m,n,\nu_1)\,cd+U_1^{ep}(m,n)ce\}+\frac{\pi t c^2 b^2}{4}\\
&[p\,\bar{H}_1^{ep}(m,n,E_1,E_2,\nu_1,\nu_2)+Y_1 H_1^{ep}(m,n)]+\frac{t^3 c^2 E_1 \pi}{24(1-\nu_2^2)} G_2^{ep}(m,n,\nu_2)+\frac{\pi t E_2 b^2}{8(1-\nu_2{}^2)}\\
&\{Q_2^{ep}(m,n,\nu_2)d^2+R_2^{ep}(m,n,\nu_2)de+S_2^{ep}(m,n,\nu_2)e^2\}+\frac{\pi t^2 E_2 b}{4(1-\nu_2^2)}\{T_2^{ep}(m,n,\nu_2)\,cd+\\
&U_2^{ep}(m,n)ce\}+\frac{\pi t c^2 b^2}{4}[p\,\bar{H}_2^{ep}(m,n,E_1,E_2,\nu_1,\nu_2)+Y_2 H_2^{ep}(m,n)]+\frac{2}{\pi}\,\frac{1-m}{1+m}\,Sc^2+\\
&\frac{1}{2\pi}\frac{S}{(1-m^2)}(d^2+e^2),
\end{aligned}
\tag{21}
$$

where,

$$
\begin{cases}
G_1^{ep}(m,n,\nu_1)=(1+\nu_1)[-m^2+4m+2\ln(\frac{1}{m})-3]n^4+4[-(1-\nu_1)m^2+(1+\nu_1)\\
(\ln(m)-m)+2]n^2+6\ln(\frac{1}{m})(1+\nu_1),\\
Q_1^{ep}(m,n,\nu_1)=(1-m^2)[(1-\nu_1)n^2+4(1+\nu_1)],\\
R_1^{ep}(m,n,\nu_1)=4(1+\nu_1)(1-m^2)n,\\
S_1^{ep}(m,n,\nu_1)=(1-m^2)(1+\nu_1)n^2,\\
T_1^{ep}(m,n,\nu_1)=2\{[(1-m)(1+\nu_1)+2m\ln(m)]n^2+(m-1)(1+\nu_1)\},\\
U_1^{ep}(m,n,\nu_1)=(1+\nu_1)\{[(\ln(m)-1)m+1]n^3+(m-1)n\},\\
\bar{H}_1^{ep}(m,n,E_1,E_2,\nu_1,\nu_2)=\dfrac{4\ (k_2k_3-k_1k_4)m^2}{[k_2^2-k_1^2+k_4^2-k_3^2+2(k_2k_4-k_1k_3)](1-m^2)}\{[(3+2\ln(\frac{1}{m}))m^2\\
-4m+1]n^2+3(1-m^2)\},\\
H_1^{ep}(m,n)=\{[(\ln(\frac{1}{m}))^2+\ln(\frac{1}{m})+\frac{1}{2}]m^2-\frac{1}{2}\}n^2+[3\ln(m)-\frac{3}{2}]m^2+\frac{3}{2},
\end{cases}
\tag{22}
$$

and

$$
\begin{cases}
G_2^{ep}(m,n,\nu_2)=(1+\nu_2)[-m^2+4m+2\ln(\frac{1}{m})-3]n^4+4[-(1-\nu_2)m^2+(1+\nu_2)\\
(\ln(m)-m)+2]n^2+6\ln(\frac{1}{m})(1+\nu_2),\\
Q_2^{ep}(m,n,\nu_2)=(1-m^2)[(1-\nu_2)n^2+4(1+\nu_2)],\\
R_2^{ep}(m,n,\nu_2)=4(1+\nu_2)(1-m^2)n,\\
S_2^{ep}(m,n,\nu_2)=(1-m^2)(1+\nu_2)n^2,\\
T_2^{ep}(m,n,\nu_2)=2\{[(1-m)(1+\nu_2)+2m\ln(m)]n^2+(m-1)(1+\nu_2)\},\\
U_2^{ep}(m,n,\nu_2)=(1+\nu_2)\{[(\ln(m)-1)m+1]n^3+(m-1)n\},\\
\bar{H}_2^{ep}(m,n,E_1,E_2,\nu_1,\nu_2)=-\dfrac{4\ (k_2k_3-k_1k_4)m^2}{[k_2^2-k_1^2+k_4^2-k_3^2+2(k_2k_4-k_1k_3)](1-m^2)}\{[(3+2\ln(\frac{1}{m}))m^2\\
-4m+1]n^2+3(1-m^2)\},\\
H_2^{ep}(m,n)=\{[(\ln(\frac{1}{m}))^2+\ln(\frac{1}{m})+\frac{1}{2}]m^2-\frac{1}{2}\}n^2+[3\ln(m)-\frac{3}{2}]m^2+\frac{3}{2}.
\end{cases}
\tag{23}
$$

The matrix form of Eq. (21) is,

$$
F=\{c\quad d\quad e\}\begin{bmatrix} M_{11} & M_{12} & M_{13}\\ M_{21} & M_{22} & M_{23}\\ M_{31} & M_{32} & M_{33}\end{bmatrix}\begin{Bmatrix} c\\ d\\ e\end{Bmatrix}=u^T M\,u,
\tag{24}
$$

where,

$$\begin{cases} M_{11} = \frac{\pi t^3}{24}[\frac{E_1}{1-\nu_1^2}G_1^{ep}(m,n,\nu_1)+\frac{E_2}{1-\nu_2^2}G_2^{ep}(m,n,\nu_2)]+\frac{2}{\pi}\frac{1-m}{1+m}S+\frac{\pi t b^2}{2}H^{ep}(m,n)Y, \\ M_{12}=M_{21}=\frac{1}{2}\frac{\pi t^2 b}{4}[\frac{E_2}{1-\nu_2^2}T_2^{ep}(m,n,\nu_2)-\frac{E_1}{1-\nu_1^2}T_1^{ep}(m,n,\nu_1)], \\ M_{13}=M_{31}=\frac{1}{2}\frac{\pi t^2 b}{4}[\frac{E_2}{1-\nu_2^2}U_2^{ep}(m,n,\nu_2)-\frac{E_1}{1-\nu_1^2}U_1^{ep}(m,n,\nu_1)], \\ M_{22}=\frac{\pi t b^2}{8}[\frac{E_1}{1-\nu_1^2}Q_1^{ep}(m,n,\nu_1)+\frac{E_2}{1-\nu_2^2}Q_2^{ep}(m,n,\nu_2)]+\frac{1}{2\pi}\frac{S}{(1-m^2)}, \\ M_{23}=M_{32}=\frac{1}{2}\frac{\pi t b^2}{8}[\frac{E_1}{1-\nu_1^2}R_1^{ep}(m,n,\nu_1)+\frac{E_2}{1-\nu_2^2}R_2^{ep}(m,n,\nu_2)], \\ M_{33}=\frac{\pi t b^2}{8}[\frac{E_1}{1-\nu_1^2}S_1^{ep}(m,n,\nu_1)+\frac{E_2}{1-\nu_2^2}S_2^{ep}(m,n,\nu_2)]+\frac{1}{2\pi}\frac{S}{(1-m^2)}. \end{cases} \quad (25)$$

The critical conditions for onset of wrinkling is [2-4],

$$\begin{cases} F=0 \ \ or \ \ Det(M_{ij})=0, \\ \frac{\partial F}{\partial n}=0 \ \ or \ \ \frac{\partial [Det(M_{ij})]}{\partial n}=0. \end{cases} \quad (26)$$

With considering these conditions and takeng $\psi_1=\frac{S}{t^3}, \psi_2=\frac{S}{b^2 t}$. it is found that,

$$Y=\frac{1}{4}\frac{t^2}{b^2}\frac{K^{ep}(m,n,E_1,E_2,\nu_1,\nu_2,\psi_1,\psi_2)}{H^{ep}(m,n)}. \quad (27)$$

This equation can be shown in the following form,

$$\sqrt{\frac{1}{Y}}\frac{t}{b}=2\sqrt{\frac{H^{ep}(m,n)}{K^{ep}(m,n,E_1,E_2,\nu_1,\nu_2,\psi_1,\psi_2)}}. \quad (28)$$

where

$$\sqrt{\frac{1}{Y}}\frac{t}{b}<2\sqrt{\frac{H^{ep}(m,n)}{K^{ep}(m,n,E_1,E_2,\nu_1,\nu_2,\psi_1,\psi_2)}}. \quad (29)$$

Wrinkling will occure.

3. Results and Discussion

By considering the mechanical properties of steel ($E=200\ Gpa, \nu=0.3$) and properties of aluminum ($E=70\ Gpa, \nu=0.25$) and substituting these values in Eq. (29) , the first condition in (26) yields to $\frac{\partial Y}{\partial n}=0$. This equation has no exact solution so we first, draw the Eq. (28) for different n values. Then, from intersection of these curves one can obtaion the curve n_{cr} by least square method. After finding n_{cr}, they have to be inserted in Eq. (29) to find the values for wrinkling. In Figure () the wrinkling limitation of St-St, Al-Al and St-Al laminated have been compared and it can be deduced that the wrinkling limitation of St-St >St-Al>Al-Al.

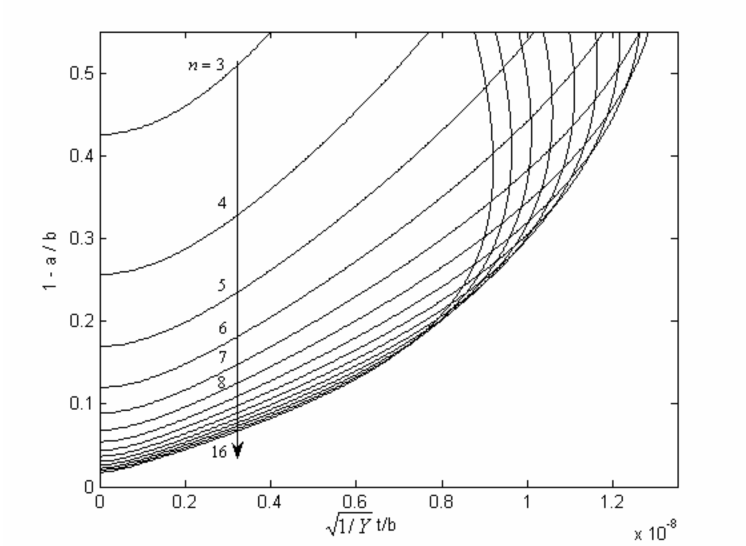

Figure 5. Onset of winkling for different n's.

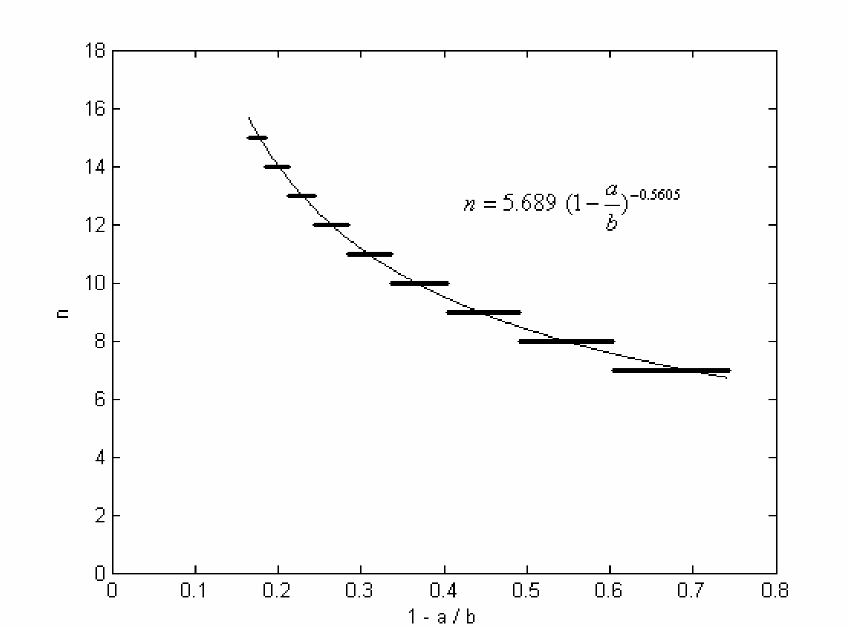

Figure 6. Onset of wrinkling for laminated, St-St, Al-Al, St-Al

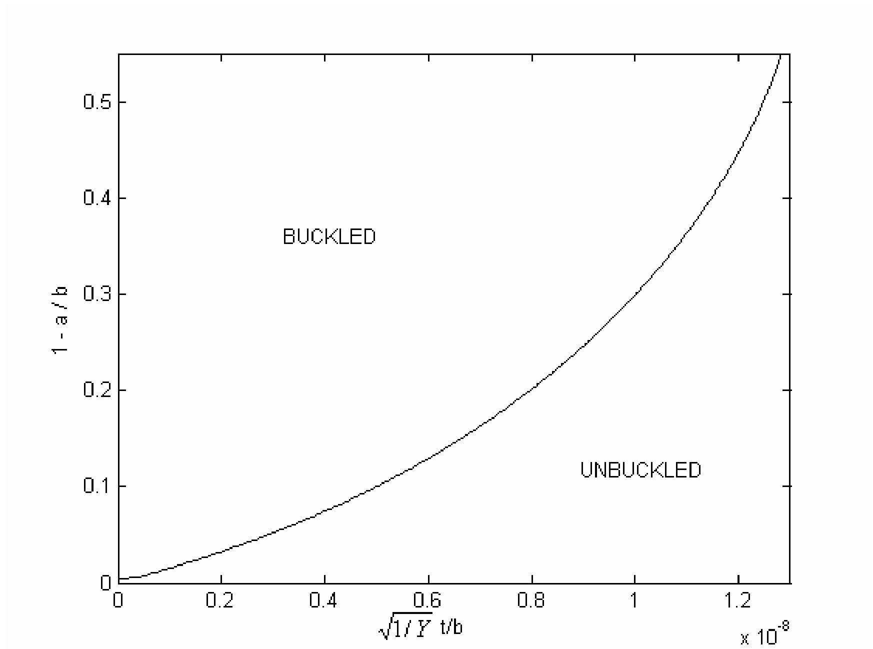

Figure 7. Wrinkling rea for laminated, St-St, Al-Al, St-Al

In Figs 6. and 7. the wave number generated and also onset of wrinkling are compared for different blank holder forces. The effect of blank holder force on the generated wave number and onset of wrinkling can be found by taking different blank holder forces in three cases such as, case 1: $\psi_1 = 10\times10^{16}, \psi_2 = 6.25\times10^{13}$, case 2: $\psi_1 = 15\times10^{16}, \psi_2 = 9.375\times10^{13}$ and case 3: $\psi_1 = 20\times10^{16}, \psi_2 = 12.5\times10^{13}$, Figures, 8. As it can be seen, increasing the BHF increase the safe area for deep drawing.

In Figure 9, the onset of wrinkling of laminated St-St, Al-Al and St-Al is investigated and it is deduced that St-St>St-Al>Al-Al.

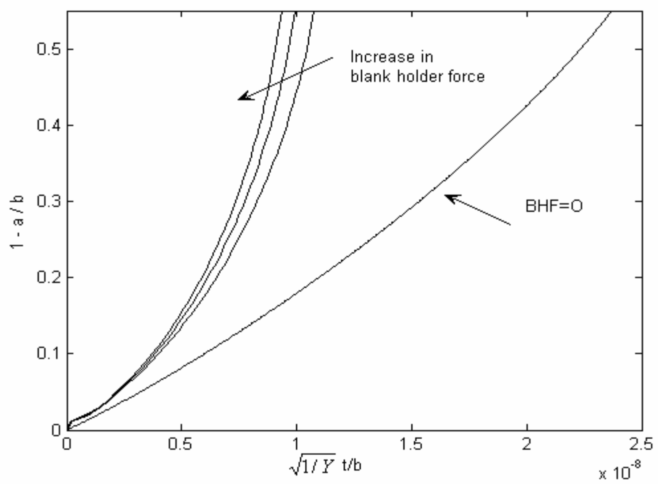

Figure 8. The Effect of changing blankholder force on onset of wrinkling

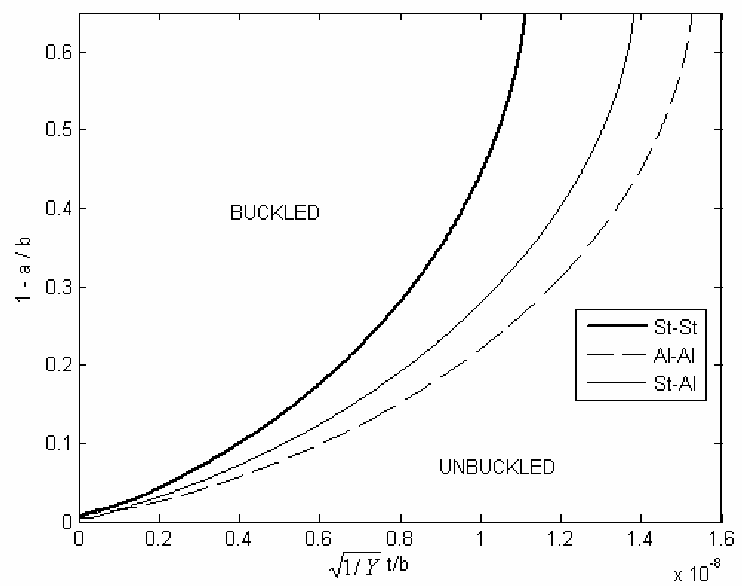

Figure 8. Onset of wrinkling of laminated, St-St, Al-Al and St-Al

4. Conclusions

The plastic wrinkling of flange with blank holder in deep drawing have been studied analytically by proposing a new functional for a two laminated plates. The critical wave number of flange, the limitation of wrinkling, the effect of blank holder force on onset of wrinkling were investigated, and finally limitations of drawing of laminated St-St, Al-Al and St-St were compared.

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