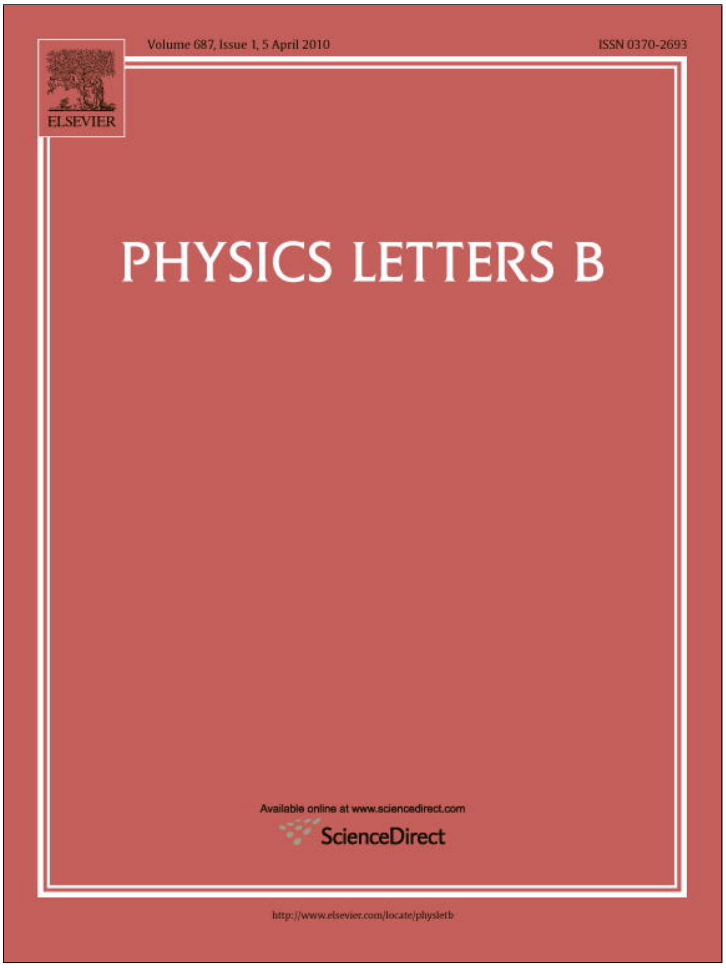
Volume 687, Issue 1, 5 April 2010
ISSN 0370-2693
ELSEVIER
PHYSICS LETTERS B
Available online at www.sciencedirect.com
ScienceDirect
http://www.elsevier.com/locate/physletb

Physics Letters B 687 (2010) 79–83

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The RN/CFT correspondence

Ahmad Ghodsi *, Mohammad R. Garousi

Department of Physics, Ferdowsi University of Mashhad, P.O. Box 1436, Mashhad, Iran

ARTICLE INFO

Article history:
Received 8 December 2009
Received in revised form 20 February 2010
Accepted 24 February 2010
Available online 6 March 2010
Editor: M. Cvetič

Keywords:
RN black holes
Compactification solution
Central charge
CFT dual

ABSTRACT

Recently it has been shown in [S.M. Carroll, et al., arXiv:0901.0931 [hep-th]] that the approach to extremality for the non-extremal Reissner–Nordstrom black hole is not continuous. The non-extremal RN black hole splits into two spacetimes at the extremality: an extremal black hole and a disconnected $AdS_2 \times S^2$ space which has been called the "compactification solution". As a possible resolution for understanding the entropy of extremal RN black hole, it has been speculated that the entropy of the non-extremal black hole may be carried by the latter solution. By uplifting the four-dimensional "compactification solution" with electric charge Q_e to a five-dimensional solution, we show that this solution is dual to a CFT with central charge $c = 6Q_e^3$. The Cardy formula then shows that the microscopic entropy of the CFT is the same as the macroscopic entropy of the "compactification solution".

1. Introduction

One of the most exciting observation in the modern theoretical physics is the holographic dualities that relates a quantum gravity to a quantum field theory without gravity in fewer dimensions [1,2]. The best understood holographic duality is the duality between the ten-dimensional type IIB string theory on background $AdS_5 \times S^5$ with flux and the four-dimensional $\mathcal{N} = 4$ super Yang–Mills theory at the boundary of AdS_5 [3–5]. Recently the idea of the holographic duality has been examined for the more interesting backgrounds using the Brown and Henneaux technique [6]. It has been shown in [7] that there is a two-dimensional CFT dual of quantum gravity on extreme Kerr background. Even though the structure of the CFT is not known, the central charge of the CFT can be found by studying the non-trivial asymptotic symmetry of the extreme Kerr solution. The Cardy formula then gives the microscopic entropy of the CFT to be exactly the same as the macroscopic entropy of the extreme Kerr background [7]. This duality has been extended to other backgrounds in [8–10] (see also [11]).

In this Letter we would like to study the holographic duality for the extreme limit of the Reissner–Nordstrom solution. It has been argued in [12] that the semiclassical method gives zero result for the entropy of any extremal black hole even if its horizon area is non-zero. The reason is that the space outside the horizon of a non-extremal black hole is a manifold with topology $R^2 \times S^2$, whereas, the space outside the horizon of an extremal black hole is a manifold with topology $R \times S^1 \times S^2$, i.e., the horizon is excluded because the physical distance between an arbitrary point and the horizon is infinite. It has been shown in [13] that the approach to extremality for RN black holes is not continuous. The non-extremal RN black hole splits into two spacetimes at the extremality: an extremal black hole and a disconnected AdS space which has been called the "compactification solution". It has been speculated in [13] that the entropy of the non-extremal RN black hole may be carried by the "compactification solution" when one takes the extremal limit.

In this Letter we would like to find the CFT dual of the "compactification solution" by applying the Brown–Henneaux technique. It has been argued in [8] that the gauge symmetry of the extreme Kerr–Newman–AdS black hole may be combined with the geometry of the four-dimensional extreme Kerr–Newman–AdS black hole to write a five-dimensional metric from which the central charge of the extreme RN black hole can be found in the limit $J \to 0$. Using this idea we find a five-dimensional solution which reduces to the four-dimensional "compactification solution" upon compactifying the 5th dimension. The CFT dual of this five-dimensional solution should be also dual to the four-dimensional solution.

The Letter is organized as follows. In the next section we review the non-extremal RN solution of Einstein–Maxwell theory in four dimensions and its extremal limits. In Section 3 we uplift the compactification solution to a five-dimensional Einstein–Maxwell theory. In Section 4, we study the CFT dual of the five-dimensional solution and show that a part of the $U(1)$ isometry of the compactification solution appears at the boundary as Virasoro algebra with a central charge which gives exactly the microscopic entropy after using the Cardy formula.

* Corresponding author.
E-mail addresses: ahmad@mail.ipm.ir (A. Ghodsi), garousi@mail.ipm.ir (M.R. Garousi).

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2010.03.002

2. Review of non-extremal RN solution

In this section we review the non-extremal Reissner–Nordstrom solution of the Einstein–Maxwell theory in four dimensions. The action is given by

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R - \frac{1}{4}\mathcal{F}_{(2)}^2\right\}, \tag{1}$$

where G is the four-dimensional Newton's constant.

The non-extremal Reissner–Nordstrom solution with mass M and electric charge Q_e is given by

$$ds^2 = -\left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)dt^2 + \frac{1}{(1-\frac{r_+}{r})(1-\frac{r_-}{r})}dr^2 + r^2\,d\Omega_2^2,$$
$$\mathcal{F}_{(2)} = \frac{2\sqrt{G}Q_e}{r^2}dt\wedge dr.$$

There are two event horizons located at the coordinate singularities

$$r_\pm = GM \pm \sqrt{G^2M^2 - GQ_e^2}. \tag{2}$$

There are different types of patches

$$\text{Region I:}\quad r_+ < r < \infty, \qquad -\infty < t < \infty,$$
$$\text{Region II:}\quad r_- < r < r_+, \qquad -\infty < t < \infty,$$
$$\text{Region III:}\quad 0 < r < r_-, \qquad -\infty < t < \infty. \tag{3}$$

The distance between an arbitrary point and the outer horizon is finite, hence, the entropy of this solution can be found from the semi-classical method, i.e.,

$$S = \frac{\pi r_+^2}{G}. \tag{4}$$

The Hawking temperature of the black hole which is given by $2\pi T = \sqrt{g^{rr}}\partial_r\sqrt{g_{tt}}$ at the outer horizon is

$$T = \frac{1}{2\pi r_+^2}(r_+ - r_-). \tag{5}$$

The Hawking temperature is zero when $r_+ = r_-$, however, the entropy remains non-zero.

2.1. Extremal limits

It has been shown in [13] that the approach to extremality for RN black holes is not continuous. The non-extremal RN black hole splits into two spacetimes at the extremality: an extremal black hole and a disconnected "compactification solution". The extremal black hole with event horizon at $r = Q_e$ is

$$ds^2 = -\left(1 - \frac{Q_e}{r}\right)^2 dt^2 + \frac{1}{(1-\frac{Q_e}{r})^2}dr^2 + r^2\,d\Omega_2^2,$$
$$\mathcal{F}_{(2)} = \frac{2Q_e}{r^2}dt\wedge dr, \tag{6}$$

where we have set $G = 1$. There are two regions I, III for this solution. The region II disappears in this limit. The physical distance between an arbitrary point and the horizon is infinite[1] so the semiclassical method gives no entropy for this solution [12]. To go to the near horizon in the region I, one introduces the new spacelike coordinate $0 < \lambda < \infty$ and timelike coordinate $-\infty < \sigma < \infty$ as

$$\lambda = \frac{r - Q_e}{\epsilon}, \qquad \sigma = -\frac{t\epsilon}{Q_e^2}, \tag{7}$$

and takes the limit $\epsilon \to 0$ keeping (λ, σ) fixed. The solution for arbitrary λ becomes

$$ds^2 = Q_e^2\left(-\lambda^2\,d\sigma^2 + \frac{1}{\lambda^2}d\lambda^2 + d\Omega_2^2\right),$$
$$\mathcal{F}_{(2)} = -2Q_e\,d\sigma\wedge d\lambda, \tag{8}$$

which is $AdS_2 \times S^2$. Similar geometry for extreme Kerr solution has been found in [16].

The "compactification solution" on the other hand has the three regions I, II, and III. In fact the physical distance between the inner and outer horizons of the non-extremal solution remains non-zero in this case [13]. By appropriate coordinate transformation, the metric of the three regions can be mapped to a global $AdS_2 \times S^2$ solution [13]. For instance, in region II using the new timelike coordinate $0 < \chi < \pi$ and spacelike coordinate $-\infty < \psi < \infty$ via the following coordinate transformation:

$$r = Q_e - \epsilon\cos\chi, \qquad \psi = \frac{\epsilon}{Q_e^2}t, \tag{9}$$

where $\epsilon = \sqrt{M^2 - Q_e^2}$, one finds the metric and the field strength map to

$$ds^2 = Q_e^2\left(-d\chi^2 + \sin^2\chi\,d\psi^2 + d\Omega_2^2\right),$$
$$\mathcal{F}_{(2)} = 2Q_e\sin\chi\,d\psi\wedge d\chi, \tag{10}$$

where we have sent $\epsilon \to 0$. Note that in this limit $r_+ = r_- = Q_e$ and at the same time $r \to Q_e$. Moreover, the physical distance between the outer and the inner horizons remains non-zero at this limit. Using the coordinate transformation

$$\cos\chi = \frac{\cos\tau}{\cos\vartheta}, \qquad \tanh\psi = \frac{\sin\vartheta}{\sin\tau}, \tag{11}$$

the metric (10) transforms to [13]

$$ds^2 = \frac{Q_e^2}{\cos^2\vartheta}\left(-d\tau^2 + d\vartheta^2\right) + Q_e^2\,d\Omega_2^2, \tag{12}$$

which is $AdS_2 \times S^2$. The flux is mapped to

$$\mathcal{F}_{(2)} = -\frac{2Q_e}{\cos^2\vartheta}d\tau\wedge d\vartheta. \tag{13}$$

The metric (12) covers a portion of the global AdS_2. The other portions of the entire manifold are covered by the metric in regions I and III [13]. The boundaries of the global AdS_2 are at $\vartheta = \pm\pi/2$. In terms of new coordinate $u = 1/\cos\vartheta$, the boundaries are at $u \to \infty$. The solution in terms of the u-coordinate is

$$ds^2 = Q_e^2\left(-u^2\,d\tau^2 + \frac{du^2}{u^2 - 1} + d\Omega_2^2\right),$$
$$\mathcal{F}_{(2)} = -\frac{2Q_e u}{\sqrt{u^2 - 1}}d\tau\wedge du. \tag{14}$$

[1] If one consider the Reissner–Nordstrom solution as a solution of the effective theory of the string theory, the situation will change. In that case, it has been argued in [14] that near the horizon, the length of periodic time coordinate approaches to zero and hence the string winding modes become massless or even tachyonic. So one must include these modes to the effective action. It has been speculated in [14] that in the presence of these modes the physical distance between an arbitrary point and the horizon remains finite, hence, the macroscopic entropy of extremal solution of the string theory effective action is non-zero which should be the same as the microscopic entropy of string microstate counting [15].

Near the boundary, $u \to \infty$, it behaves as

$$ds^2 = Q_e^2\left(-u^2\,d\tau^2 + \frac{du^2}{u^2} + d\Omega_2^2\right),$$
$$\mathcal{F}_{(2)} = -2Q_e\,d\tau \wedge du, \tag{15}$$

which is similar to the near horizon geometry of extremal black hole (8).

It has been shown in [13] that in the extremal limit, region II and the near horizon in regions I and III of the non-extremal RN black hole become the compactification solution (14), while the portions of regions I and III with any finite distance away from the horizon form the extremal RN black hole (6).

In the extremal limit the entropy (4) remains non-zero, i.e.,

$$S_{\text{macro}} = \pi Q_e^2, \tag{16}$$

and the Hawking temperature (5) is zero, however, there is another temperature which is conjugate to the electric charge and is defined by $T_e\,dS = dQ_e$. This temperature is

$$T_e = \frac{1}{2\pi Q_e}. \tag{17}$$

The macroscopic entropy (16) should be extracted also from microstates counting. If the extremal RN black hole carries the macroscopic entropy, then the microstates counting of the CFT dual at the boundary of (8) should give the macroscopic entropy. On the other hand, if the compactification solution (14) carries the entropy, then the microstates counting of the CFT dual at the boundary of (14) should give the macroscopic entropy. It has been suggested in [13] that a possible resolution, for having no entropy for the extremal RN black hole in the semiclassical method [12], is that the macroscopic entropy (16) is carried by the compactification solution. In this Letter we would like to study the CFT dual of this solution.

3. Uplifting to five dimensions

To study the two-dimensional CFT dual of the compactification solution using the Brown–Henneaux technique [6] that has been used for the extreme Kerr solution in [7], one should write the metric in a canonical form which has isometry $SL(2,R) \times U(1)$ with off-diagonal metric in the $U(1)$ part. Using this idea, the $U(1)$ gauge symmetry of the extreme Kerr–Newman–AdS black hole has been combined in [8] with the geometry of the four-dimensional extreme Kerr–Newman–AdS black hole to write a canonical five-dimensional metric which has off-diagonal component in the 5th-direction. We note that the new metric must satisfy the equations of motion in order to use the formula for the five-dimensional on-shell generators [17]. We will show that in the present case only a part of the $U(1)$ gauge field (13) can be combined with the metric (12) to write a canonical five-dimensional metric which satisfies the equations of motion. In this section, we uplift the compactification solution to the five dimensions, and then in Section 4 find the CFT dual of the five-dimensional solution.

Consider the following five-dimensional theory:

$$S = \frac{1}{16\pi G^{(5)}}\int d^5x\sqrt{-g}\left\{R - \frac{1}{12}F_{(3)}^2\right\}. \tag{18}$$

The equations of motion are

$$R_{\mu\nu} = \frac{1}{12}\left(3F_\mu{}^{\alpha\beta}F_{\nu\alpha\beta} - \frac{2}{3}g_{\mu\nu}F_{(3)}^2\right),$$
$$\partial_\mu\left(\sqrt{-g}F^{\mu\alpha\beta}\right) = 0. \tag{19}$$

The above equations are satisfied by the following solution:

$$ds_5^2 = \frac{Q_e^2}{\cos^2\vartheta}\left(-d\tau^2 + d\vartheta^2\right) + Q_e^2\,d\Omega_2^2 + (dy + Q_e\tan\vartheta\,d\tau)^2,$$
$$F_{(3)} = \frac{\sqrt{3}Q_e}{\cos^2\vartheta}d\tau \wedge d\vartheta \wedge dy, \tag{20}$$

where y is a fibered coordinate with period 2π. In the above solution, Q_e is a constant which we will take to be the four-dimensional electric charge.

Upon dimensionally reducing the y coordinate as [18]

$$ds_{d+1}^2 = e^{2\alpha\phi}\,ds_d^2 + e^{2\beta\phi}(dy + \mathcal{A})^2, \tag{21}$$

where $\beta = (2-d)\alpha$ and $\alpha^2 = 1/[2(d-1)(d-2)]$, the action (18) reduces to

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{-2(d-1)\alpha\phi}\mathcal{F}_{(2)}^2 - \frac{1}{4}e^{2(d-3)\alpha\phi}F_{(2)}^2\right), \tag{22}$$

where $\mathcal{F}_{(2)} = d\mathcal{A}$, and we have used the fact that the field strength $F_{(3)}$ has component along the y-direction, i.e., $F_{(2)} = dA$ is the reduction of $F_{(3)}$. The equation of motion of the dilaton is

$$D^2\phi = -\frac{2(d-1)\alpha}{4}e^{-2(d-1)\alpha\phi}\mathcal{F}_{(2)}^2 + \frac{2(d-3)\alpha}{4}e^{2(d-3)\alpha\phi}F_{(2)}^2. \tag{23}$$

For the present case that $d = 4$, one finds $\phi = 0$ is a solution of the above equation.

Using the $\phi = 0$, the action (22) reduces to

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - \frac{1}{4}\mathcal{F}_{(2)}^2 - \frac{1}{4}F_{(2)}^2\right), \tag{24}$$

and the five-dimensional solution (20) reduces to the following solution:

$$ds_4^2 = \frac{Q_e^2}{\cos^2\vartheta}\left(-d\tau^2 + d\vartheta^2\right) + Q_e^2\,d\Omega_2^2,$$
$$\mathcal{A} = Q_e\tan\vartheta\,d\tau, \qquad A = \sqrt{3}Q_e\tan\vartheta\,d\tau. \tag{25}$$

The action (24) is invariant under global $SO(2)$ transformation under which $(\mathcal{A}, A)$ is a doublet. Using this symmetry, one can write (24) as (1) and consequently the above solution can be written as the four-dimensional compactification solution, i.e.,

$$ds_4^2 = \frac{Q_e^2}{\cos^2\vartheta}\left(-d\tau^2 + d\vartheta^2\right) + Q_e^2\,d\Omega_2^2,$$
$$\mathcal{A} = 2Q_e\tan\vartheta\,d\tau, \qquad A = 0. \tag{26}$$

It is important to note that, one cannot combine the whole $U(1)$ gauge field $2Q_e\tan\vartheta\,d\tau$ with the metric to write a five-dimensional metric. That would not satisfy the five-dimensional equations of motion.

Using the coordinate transformation $\cos\vartheta = 1/u$ where $1 \leqslant u \leqslant \infty$, the five-dimensional metric (20) becomes

$$ds_5^2 = \rho^2\left\{-u^2\,d\tau^2 + \frac{du^2}{u^2-1} + d\Omega_2^2\right\} + \left(dy + Q_e\sqrt{u^2-1}\,d\tau\right)^2, \tag{27}$$

where $\rho = Q_e$. This metric has the isometry group of $SL(2,R) \times SO(3) \times U(1)$. The Killing vector that generates the rotational $U(1)$ isometry group is

$$\zeta^{(y)} = -\partial_y, \tag{28}$$

the Killing vectors that generate the $SO(3)$ isometry group are the following:

$$\hat{\zeta}_1 = \sin\phi\,\partial_\theta + \cot\theta\cos\phi\,\partial_\phi,$$
$$\hat{\zeta}_2 = -\cos\phi\,\partial_\theta + \cot\theta\sin\phi\,\partial_\phi,$$
$$\hat{\zeta}_3 = -\partial_\phi, \tag{29}$$

and the Killing vectors that generate the $SL(2,R)$ isometry group are the following:

$$\zeta_1 = \frac{2\sin\tau\sqrt{u^2-1}}{u}\partial_\tau - 2\cos\tau\sqrt{u^2-1}\,\partial_u + \frac{2Q_e\sin\tau}{u}\partial_y,$$
$$\zeta_2 = \frac{2\cos\tau\sqrt{u^2-1}}{u}\partial_\tau + 2\sin\tau\sqrt{u^2-1}\,\partial_u + \frac{2Q_e\cos\tau}{u}\partial_y,$$
$$\zeta_3 = 2Q_e\partial_\tau. \tag{30}$$

At the boundary, $u\to\infty$, the above Killing vectors become

$$\zeta_\eta = \eta(\tau)\partial_\tau - \partial_\tau\big(\eta(\tau)\big)u\partial_u, \tag{31}$$

for $\eta(\tau) = 2\sin\tau, 2\cos\tau, 2Q_e$. If one perturbs the background (27), then the Killing vectors will change and hence their values at the boundary will be modified.

4. The CFT dual

We now study the two-dimensional CFT dual of the above five-dimensional solution (27) using the Brown–Henneaux technique [6] which makes use of the asymptotic symmetry group. The asymptotic symmetry group of a spacetime is the group of non-trivial allowed symmetries. A non-trivial allowed symmetry is the one which generates a transformation that obeys the boundary conditions and its associated charge is non-vanishing [7].

Since ∂_τ is the generator whose conjugate conserved charge measures the deviation of the solution from extremality [7], we consider the perturbations that their associated conserved charges commute with ∂_τ. For the fluctuations of the metric (27) we choose the following boundary condition:

$$h_{\mu\nu} \sim \mathcal{O}\begin{pmatrix} u^2 & u & u & 1/u^2 & 1 \\ & 1 & 1 & 1/u & 1 \\ & & 1 & 1/u & 1 \\ & & & 1/u^3 & 1/u \end{pmatrix}, \tag{32}$$

in the basis (τ,ϕ,θ,u,y). At the leading order, the diffeomorphisms which preserve the above boundary condition are

$$\zeta_\epsilon = \epsilon(y)\partial_y - u\epsilon'(y)\partial_u,$$
$$\zeta^{(\tau)} = \partial_\tau,$$
$$\hat{\zeta}_1 = \sin\phi\,\partial_\theta + \cot\theta\cos\phi\,\partial_\phi,$$
$$\hat{\zeta}_2 = -\cos\phi\,\partial_\theta + \cot\theta\sin\phi\,\partial_\phi,$$
$$\hat{\zeta}_3 = -\partial_\phi, \tag{33}$$

where $\epsilon(y)$ is an arbitrary smooth function. The Lie derivative of metric (27) with respect to $\zeta^{(\tau)}$ and $\hat{\zeta}$'s are zero, and with respect to ζ_ϵ is

$$\delta_\epsilon\, ds^2 = 2\bigg(\big(\rho^2 - Q_e^2\big)u^2\epsilon'(y)\,d\tau^2 + \frac{\rho^2}{(u^2-1)^2}\epsilon'(y)\,du^2 + \epsilon'(y)\,dy^2 - \frac{Q_e}{\sqrt{u^2-1}}\epsilon'(y)\,d\tau\,dy - \frac{\rho^2 u}{u^2-1}\epsilon''(y)\,du\,dy\bigg), \tag{34}$$

which is consistent with the boundary condition (32).

Using the periodicity of the y coordinate, one can expand $\epsilon(y)$ in terms of the basis $\epsilon_n(y) = -e^{-iny}$. Defining the generators $\zeta_n \equiv \zeta_{\epsilon_n}$ one finds they satisfy the following Virasoro algebra:

$$i[\zeta_m,\zeta_n]_{L.B.} = (m-n)\zeta_{m+n}. \tag{35}$$

They have zero central charge. To evaluate the central term of the above algebra, one needs to construct the surface charges which generate the asymptotic symmetry (33). For asymptotically AdS spacetimes, the charge differences between $(g_{\mu\nu})$ and $(g_{\mu\nu}+h_{\mu\nu})$ are given by [17] (see [10] for a review)[2]

$$Q_\zeta[g] = \frac{1}{8\pi G}\int_{\partial\Sigma} k_\zeta^{grav}[h;g], \tag{36}$$

where the integral is over the boundary and

$$k_\zeta^{grav}[h,g] = -\frac{1}{2}\frac{1}{3!}\epsilon_{\alpha\beta\gamma\mu\nu}\Big[\zeta^\nu D^\mu h^\sigma_\sigma - \zeta^\nu D_\sigma h^{\mu\sigma} + \zeta_\sigma D^\nu h^{\mu\sigma} + \frac{1}{2}h^\sigma_\sigma D^\nu\zeta^\mu - h^{\nu\sigma}D_\sigma\zeta^\mu + \frac{1}{2}h^{\sigma\nu}\big(D^\mu\zeta_\sigma + D_\sigma\zeta^\mu\big)\Big]dx^\alpha\wedge dx^\beta\wedge dx^\gamma, \tag{37}$$

The covariant derivatives and raised indices are computed using the metric $g_{\mu\nu}$. For the diffeomorphism (33), one finds[3]

$$k_{\zeta_\epsilon}^{grav} = -\frac{Q_e\sin\theta}{4u^2}\Big[2\epsilon(y)u^3\partial_y h_{uy} + \frac{\rho^2+Q_e^2}{\rho^2}u^2\epsilon(y)h_{yy} - 2\epsilon'(y)u^3h_{uy} + \frac{1}{\rho^2}\epsilon(y)h_{\tau\tau}\Big]d\theta\wedge d\phi\wedge dy,$$

where we have discarded total ϕ derivative terms and keep only terms that are non-zero at the boundary $u\to\infty$. We have also included only the terms that are tangent to $\partial\Sigma$.

The algebra of the non-trivial asymptotic symmetries is the Poisson bracket algebra of the charges [17]

$$\{Q_{\zeta_m},Q_{\zeta_n}\}_{P.B.} = Q_{[\zeta_m,\zeta_n]} + \frac{1}{8\pi G}\int_{\partial\Sigma} k_{\zeta_m}^{grav}[\mathcal{L}_{\zeta_n}g,g]. \tag{38}$$

The last term has the structure

$$\frac{1}{8\pi G}\int_{\partial\Sigma} k_{\zeta_m}^{grav}[\mathcal{L}_{\zeta_n},g] = -iA\big(m^3+Bm\big)\delta_{m+n,0}. \tag{39}$$

If one defines the quantum version of the Q's by

$$L_n \equiv Q_{\zeta_n} + \frac{1}{2}(AB+A)\delta_{n,0}, \tag{40}$$

plus the usual rule of $\{.,.\}_{P.B.}\to -i[.,.]$, then the algebra becomes the standard Virasoro algebra

$$[L_m,L_n] = (m-n)L_{m+n} + Am\big(m^2-1\big)\delta_{m+n,0}, \tag{41}$$

with central charge $c = 12A$.

The Lie derivatives of metric (27) at the boundary are

[2] It has been argued in [10] that when metric is in the canonical form the scalars and gauge fields have no contribution to the central charge in four and five dimensions. We have explicitly calculated these contribution and find zero result in the present case.

[3] Note that the Lie derivative of metric with respect to the diffeomorphisms $\zeta^{(\tau)} = \partial_\tau$ and $\hat{\zeta}_1,\hat{\zeta}_2,\hat{\zeta}_3$ are zero, hence, their corresponding charges are zero too.

$$\mathcal{L}_{\zeta_n} g_{\tau\tau} = 2i(\rho^2 - Q_e^2)u^2 n e^{-iny},$$
$$\mathcal{L}_{\zeta_n} g_{\tau y} = -\frac{iQ_e}{u} n e^{-iny},$$
$$\mathcal{L}_{\zeta_n} g_{uy} = -\frac{\rho^2}{u} n^2 e^{-iny},$$
$$\mathcal{L}_{\zeta_n} g_{yy} = 2ine^{-iny},$$
$$\mathcal{L}_{\zeta_n} g_{uu} = \frac{2i\rho^2}{u^4} n e^{-iny}. \tag{42}$$

Inserting the above perturbation into the central term of (38), one finds

$$\frac{1}{8\pi G^{(5)}} \int_{\partial\Sigma} k^{grav}_{\zeta_m}[\mathcal{L}_{\zeta_n} g, g] = -\frac{i}{2} Q_e(m^3\rho^2 + m)\delta_{m+n,0}, \tag{43}$$

where we have used the fact that in five dimension $G^{(5)} = 2\pi$. Therefore, the central charge is

$$c = 6Q_e\rho^2 = 6Q_e^3. \tag{44}$$

This is the central charge of the CFT dual of the background (27) in which the parameter Q_e is the four-dimensional electric charge. This central charge has been also found in [8] by combining the gauge field of extremal Kerr–Newman–AdS black hole with the four-dimensional metric and taking $J \to 0$.

The Cardy formula gives the microscopic entropy of a unitary CFT at large T_e to be

$$S_{\text{micro}} = \frac{\pi^2}{3} c T_e. \tag{45}$$

Using (17) and (44), one finds

$$S_{\text{micro}} = \pi Q_e^2. \tag{46}$$

This exactly reproduces the macroscopic entropy (16).

5. Discussion

In this Letter we have studied the CFT dual of the extremal RN black hole. It has been speculated that there are two extremal limits for the RN black hole [13]: The extremal RN black hole and the compactification solution. The entropy of the RN black hole is speculated in [13] to be carried by the compactification solution at the extremal limit. By uplifting the compactification solution to a five-dimensional solution of the Einstein–Maxwell theory, we have found the central charge of the CFT dual of the compactification solution using the Brown–Henneaux technique, and its microscopic entropy using the Cardy formula. The result is exactly the same as the macroscopic entropy of the RN black hole at the extremal limit.

It has been shown in [8] that the gauge fields have no direct contribution to the central charge when the metric is in the canonical form. We have seen that only a part of the gauge field of the compactification solution (see Eq. (25)) should be combined with the four-dimensional metric to write the five-dimensional metric in the canonical form (20). This indicates that only this part of the gauge field has direct contribution to the central charge. The other part has indirect contribution as it is needed in order the metric satisfies the equations of motion.

We have found the central charge of the CFT dual of the gravity on the background (27) to be given by (44). In [8], the same central charge has been found for the CFT dual of the gravity on four-dimensional background of the extremal Kerr–Newman–AdS black hole fibered with a $U(1)$ gauge field at the limit of $J \to 0$. Moreover, using the Brown–Henneaux technique one can find the central charge for the CFT dual of the gravity on the following four-dimensional metric:

$$ds_4^2 = -Q_e^2 u^2\,d\tau^2 + \left(\frac{Q_e^2}{u^2-1}\right)du^2 + Q_e^2\,d\theta^2 + Q_e^2 \sin^2\theta\left(d\phi - Q_e\sqrt{u^2-1}\,d\tau\right)^2. \tag{47}$$

The result is exactly the same as (44). This may indicate that the CFT dual to the gravity on these different backgrounds have the same central charge, however, other properties of the CFT theories may not be the same. It would be interesting to further study these backgrounds to reveal which one is corresponding to the RN black hole. The first criteria is that they must satisfy the equations of motion. Our five-dimensional solution has been found by requiring it to satisfy the equations of motion.

Acknowledgement

This work was supported by Ferdowsi University of Mashhad under grant p/18(1388/2/14).

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