Controlling Nonlinear Processes, Using Laguerre Functions Based Adaptive Model Predictive Control (AMPC) Algorithm

Nasser Saghatoleslami
Department of Chemical Engineering
Ferdowsi University of Mashhad
Mashhad, Iran
slami@um.ac.ir

Masood Khaksar Toroghi
Department of Chemical Engineering
Ferdowsi University of Mashhad
Mashhad, Iran
Masood_khaksar2004@yahoo.com

Abstract— **Laguerre function has many advantages such as good approximation capability for different systems, low computational complexity and the facility of on-line parameter identification. Therefore, it is widely adopted for complex industrial process control. In this work, Laguerre function based adaptive model predictive control algorithm (AMPC) was implemented to control a nonlinear process. Simulation result reveals that AMPC has a good performance in set-point tracking and load rejection. For comparison purposes, a nonlinear model predictive control based on Laguerre- wiener model was also applied to the process. Simulation result demonstrates that the two controllers have the same performance in set point tracking and load rejection problem.**

Keywords-***Laguerre function; predictive control; nonlinear process; Laguerre-Wiener model.***

I. INTRODUCTION

Model Predictive Control (MPC) refers to a class of control algorithm in which a dynamic process model was used to predict and to optimize process performance. The first MPC techniques were developed in the 1970s because conventional single-loop controllers were unable to satisfy increasingly stringent performance requirements [1]. Linear model were successfully employed to solve control problems. However, many processes were sufficiently nonlinear. This led to the development of Adaptive Model Predictive Control (AMPC) and Nonlinear Model Predictive Control (NMPC) in which were more accurate for process prediction and optimization. Models are a decisive factor in MPC algorithm. Auto-Regressive Moving Average (ARMA), Controlled Auto-Regressive Integrated Moving Average (CARIMA) and Laguerre functional model are the more important linear models.

Laguerre series is one of the most elegant techniques [2]. It can date back to Lee [3] and Wiener [4]. They found that the Laplace transforms of the classical orthonormal Laguerre function are very useful for approximating linear dynamic systems. Zervos and Dumont in 1988 proposed a novel linear MPC algorithm based on Laguerre series in which the control horizon equal one [5]. From 2000 to 2004, Zhang presented a lot of successful industrial application of Laguerre functional series based control algorithm on high temperature semiconductor diffusion furnace, double water tank and distillation columns [6-8]. However, for high nonlinear process, it has better performance to use adaptive Laguerre model for approximation the behavior of the system.

Another approach for modeling the nonlinear process is to use the nonlinear models. The behavior of many systems could be approximated by a static nonlinearity cascaded with a linear part in particular form. These models are known as Hammerstein and wiener block cascade models. These model structures have been successfully ustilized to represent nonlinear system in a number of practical applications in the area of chemical process, biological process, signal processing and control [9]. From an identification point of view, pH process has often been considered in the literature as having a wiener structure .Distillation process have been modeled using both Hammerstein and wiener models [10].

In this paper, we have considered a temperature control problem of CSTR with first order exothermic reaction. Two controllers are design for this purpose. The controllers are constructed through a Laguerre function based adaptive linear model predictive control and a Laguerre-wiener model based nonlinear model predictive control. The rest of the paper is organized as follows:

In section 1, Laguerre function is introduced. In section 2, Laguerre-wiener model is presented. In section 3, model predictive control based on two types of model is introduced. Effectiveness of the proposed scheme is demonstrated in via simulation in section 5. Finally conclusion is drawn in the last section.

II. LAGUERRE FUNCTION

Laguerre function is defined as a functional series [11]:

$$\varphi_i(t)=\sqrt{2p}\,\frac{e^{-pt}}{(i-1)!}.\frac{d^{i-1}}{dt^{i-1}}\left[t^{i-1}.e^{-2pt}\right]\quad i=1,2,...,\infty \tag{1}$$

where p is a constant called time scaling factor and $t\in[0,\infty]$ is a time variable.

The Laplace transformation of Laguerre function is:

$$\varphi_i(s)=L\left[\varphi_i(t)\right]=\sqrt{2p}\,\frac{(s-p)^{i-1}}{(s+p)^{i}}\quad i=1,2,...,\infty \tag{2}$$

Open loop stable system can be approximated by N order Laguerre series.

$$Y_m(s)=\sum_{i=1}^{N}c_i\varphi_i(s)U(s)=\sum_{i=1}^{N}c_il_i(s) \tag{3}$$

The state space expression of Incremental mode Laguerre functional model after discretization is:

$$\Delta L(k+1)=A\Delta L(k)+b\Delta u(k) \tag{4}$$
$$\Delta y_m(k)=C^T\Delta L(k)$$

where $\Delta L(k)=L(k)-L(k-1)=\left[\Delta l_1(k)\ \Delta l_2(k)...\Delta l_3(k)\right]^T$ is the state vector of the incremental mode Laguerre functional model; $\Delta y_m=y_m(k)-y_m(k-1),\Delta u(k)=u(k)-u(k-1)$ are the input and output of this model in kth sampling period, respectively; $C^T=[c_1,...,c_N]$ is the Laguerre coefficients vector. Matrices, A and b, are calculated as follows:

$$A=\begin{bmatrix}\tau_1 & 0 & ... & 0\\ \frac{-\tau_1\tau_2-\tau_3}{T} & \tau_1 & ... & 0\\ . & . & & .\\ . & . & & .\\ . & . & & .\\ \frac{(-1)^{N-1}\tau_2^{N-2}(\tau_1\tau_2+\tau_3)}{T} & ... & \frac{-(\tau_1\tau_2+\tau_3)}{T} & \tau_1\end{bmatrix} \tag{5}$$

$$b^T=\left[\tau_4,(-\tau_2/T)\tau_4,...,(-\tau_2/T)^{N-1}\tau_4\right]$$

where

$$\tau_1=e^{-pT},\tau_2=T+\frac{2}{p}\left(e^{-pT}-1\right),\tau_3=-Te^{-pT}-\frac{2}{p}\left(e^{-pT}-1\right),$$
$$\tau_4=\sqrt{2p}\,\frac{(1-\tau_1)}{p}$$

and T the sampling period.

In the above equation Δu were calculated as a replacement for u in the controller. Owing to the fact that this method could import integral mechanism, which in terms could guarantee zero steady-state error in the closed-loop system [12].

III.LAGUERRE-WIENER MODEL

In this section, Laguerre-wiener model is introduced. The Laguerre-wiener model of a nonlinear system is constructed by a nonlinear gain cascaded after Laguerre functional model as linear part.

The input-output relationship using this model could be presented as follows:

$$L(k+1)=AL(k)+bu(k) \tag{6}$$
$$y(k)=\Omega\left(L(k)\right)$$

In this model, linear and nonlinear parts in Laguerre function could be represented by various models such as polynomial, NARMA and neural network. In this paper, a polynomial model was considered as the nonlinear part.

A. Laguerre-wiener model using second order polynomial

The nonlinear gain of many processes can be approximated by second order polynomial. Khaksar and ect.al used second order polynomial for Hammerstein model in controlling the unstable reactor by MPC algorithm [13].After using this polynomial for Laguerre- wiener model, the relationship between input-output can be shown as following equations:

$$L(k)=c_0+\sum_{i=1}^{N}c_il_i(k)$$
$$y(k)=\Omega\left(L(k)\right)=\sum_{i=0}^{2}\gamma_i\left(L(k)\right)^i \tag{7}$$

where $C=[c_1,...,c_N]$ are coefficients of Laguerre functional model and $\gamma=[\gamma_0,\gamma_1,\gamma_2]$ are coefficients of second order polynomial. Offline least square optimization technique were utilized for parameter identification in this model.

IV. MODEL PREDICTIVE CONTROLLER

MPC is an optimization-based control strategy which is well suited for constrained, multivariable process. A sequence of control move was computed to minimize an objective function which includes predicted future values of the controlled output. The predictions are obtained from a process model. The various MPC algorithms proposed different cost function for obtaining the control law. A general expression for such an objective function is shown below:

$$\min_{\Delta u(k|k),\Delta u(k+1|k),...,\Delta u(k+Nu-1|k)} J=\sum_{j=0}^{H-1}\left\|y(k+j|k)-r(k+j|k)\right\|_{Q_j}+\sum_{j=0}^{Nu}\left\|\Delta u(k+j|k)\right\|_{R_j} \tag{8}$$

where $\Delta u(k+j|k)=u(k+j|k)-u(k+j-1|k)$, Nu is the control horizon, H is the prediction horizon, Q is a symmetric positive semi definite penalty matrix on the output, R is symmetric definite penalty matrix on the rate of input and y (k+j|k) the prediction output. An important characteristic of process control problem is the presence of constrain on input, state and output variable. In this work, only input constrain was considered which could be represented as:

$$U^L\le u(k+j|k)\le U^U\quad 0\le j\le Nu-1 \tag{9}$$

The superscript L and U represents the admissible lower and upper bounds for the input variable, respectively. To compensate for the mismatch between the process and the model and to consider unmeasured disturbance in the process, a term such as one shown below must be added to predicted output of the plant:

$$d(k) = y(k) - y_m(k) \tag{10}$$

where y (k) is the output of the real process and y_m (k) the model output. The modified predicted output could be represented as:

$$y_{pred}(k+i) = y_m(k+i) + d(k) \quad i = 1,...,H \tag{11}$$

To employ the MPC strategy, it was necessary to obtain vector of future output from model. For Laguerre functional model, the prediction output could be obtained using the following equations which yield from Equation 4:

$$\begin{aligned}
&\Delta L(k+2) = A^2 \Delta L(k) + Ab\Delta u(k) + b\Delta u(k+1) \\
&\quad . \\
&\quad . \\
&\quad . \\
&\Delta L(k+H) = A^H \Delta L(k) + \sum_{i=0}^{Nu} A^{H-1-i} b\Delta u(k+i)
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
&\Delta y_m(k+1) = C^T \Delta L(k) + C^T b\Delta u(k) \\
&\quad . \\
&\quad . \\
&\quad . \\
&\Delta y_m(k+H) = C^T A^H \Delta L(k) + \sum_{i=0}^{Nu-1} C^T A^{H-1-i} b\Delta u(k+i)
\end{aligned} \tag{13}$$

Considering that:

$$\begin{aligned}
&y_m(k+1) = y_m(k) + \Delta y_m(k+2) \\
&y_m(k+2) = y_m(k) + \Delta y_m(k+1) + \Delta y_m(k+2) \\
&\quad . \\
&\quad . \\
&\quad . \\
&y_m(k+H) = y_m(k) + \Delta y_m(k+1) + ... + \Delta y_m(k+H)
\end{aligned} \tag{14}$$

Thus:

$$Y_m(k+1) = SH_l \Delta L(k) + SH_u \Delta U_{Nu}(k) + Qy_m(k) \tag{15}$$

where

$$Y_m(k+1) = \left[y_m(k+1),...,y_m(k+H) \right]^T$$

$$\Delta U_{Nu}(k) = \left[\Delta u(k),...,\Delta u(k+Nu-1) \right]^T$$

$$S = \begin{bmatrix} 1\,0\,...\,0 \\ 1\,1 \\ . \\ . \\ . \\ 1\,1\,...\,1 \end{bmatrix}_{H\times H} \quad Q = \begin{bmatrix} 1 \\ 1 \\ . \\ . \\ . \\ 1 \end{bmatrix}_{H\times 1} \quad H_l = \begin{bmatrix} C^T A \\ C^T A^2 \\ . \\ . \\ . \\ C^T A^H \end{bmatrix}_{H\times Nu}$$

$$H_u = \begin{bmatrix} C^T b & 0 & ... & 0 \\ C^T Ab & C^T b & & \\ & & & \\ C^T A^{Nu-1} b & ... & ... & C^T b \\ . & & & \\ . & & & \\ . & & & \\ C^T A^{H-1} b & ... & ... & C^T A^{H-Nu} b \end{bmatrix}_{H\times Nu}$$

Coefficients of Laguerre model were identified on-line by RLS (Recursive Least Square) algorithm with forgetting factor [14]. For Laguerre-wiener model, the signal L (k) was defined as follows:

$$L(k) = \left[L(k+1),...,L(k+H) \right]^T$$

Therefore, Laguerre-wiener model based output prediction could be computed as:

$$Y_m(k+1) = \begin{bmatrix} \Omega(L(k+1)) \\ . \\ . \\ . \\ \Omega(L(k+H)) \end{bmatrix} \tag{16}$$

Finally, adaptive Laguerre based model predictive control and Laguerre –wiener models were transferred to sequential quadratic programming problem.

V. SIMULATION RESULTS

Reactors are the heart of many chemical processes and dynamic simulation of these critical units is absolutely essential for the safe and profitable operation of the entire plant [15]. In exothermic reactors where irreversible reactions were taken place, the most challenging criteria are the potential for temperature runaways. The case study which was considered in this paper is high nonlinear CSTR reactors which is the most common type of reactor used in industry.

Consider a reactor in which the following exothermic reaction takes place:

$$A \rightarrow B$$

The reaction rate is given by:

$$-r_A = kc_A \tag{17}$$

where k is the reaction constant dependant on temperature and is defined as:

$$k = k_o \exp(-E/RT) \tag{18}$$

Using the mass and energy balance fundamentals, the reactor could be modeled as:

$$\begin{aligned}
&\frac{dc_A}{dt} = \frac{q}{V}(c_{Af} - c_A) - k_o c_A \exp(-E/RT) \\
&\frac{dT}{dt} = \frac{q}{V}(T_f - T) + \frac{(-\Delta H) k_o c_A}{\rho c_p} \exp(-E/RT) \\
&+ \frac{\rho_c c_{pc}}{\rho c_p V} q_c \left[1 - \exp\left(\frac{-hA}{q_c \rho c_{pc}} \right) \right] (T_{cf} - T)
\end{aligned} \tag{19}$$

where the variable c_A is concentration of A and T is reactor temperature. T_{cf} is the coolant temperature which was considered as a manipulated variable and V is the reactor volume considered constant. The main objective of this work was to control the reactor temperature. The reactor parameters adopted in this study are given in Table1 [16].

Moreover, the feed flow rate q of this work was considered as unmeasured disturbance of the process. Figure 1 demonstrates the open-loop response of the process for $\pm$20 % step change in the coolant temperature.

To categorize the process, a uniform random signal was generated in MATLAB as an excitation signal. The switching time between different levels were selected for 16 samples.

This signal was applied as the input signal to the process. Input and output data are gathered with sampling time of 0.06 min and with 2000 samples for identification purpose. Figure 2 shows the input and output which were collected for the identification of the process.

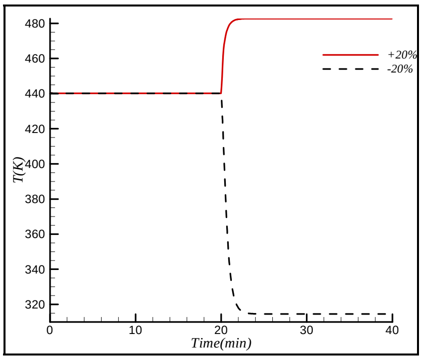
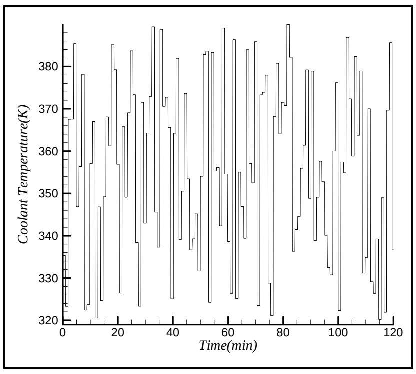
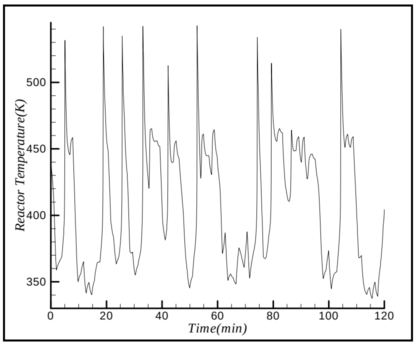


Figure1. Open-loop step-response of the CSTR reactor for change in the coolant temperature.



(a)



(b)

Figure2. Signals for identification of CSTR reactor a) input and b)output

Table1. Nominal CSTR operating condition and parameters

$q = 100\,\mathrm{min}^{-1}$	$\rho, \rho_c = 1000 gl^{-1}$
$c_{Af} = 1 moll^{-1}$	$c_p, c_{pc} = 1 ca\lg^{-1} K^{-1}$
$T_f = 350\,K$	$q_c = 103.41\, l\,\mathrm{min}^{-1}$
$T_{cf} = 350\,K$	$T = 440.2K$
$V = 100\, l$	$c_A = 8.16 \times 10^{-2}\, moll^{-1}$
$hA = 7 \times 10^5\, cal\,\mathrm{min}^{-1} K^{-1}$	$E / R = 9.95 \times 10^3\, K$
$k_o = 7.2 \times 10^{10}\,\mathrm{min}^{-1}$	$-\Delta H = 2 \times 10^5 calmol^{-1}$

Subsequent to identifying the process, initial parameters for Laguerre functional model and parameters for Laguerre-wiener model were obtained. These parameters are shown in Table 2:

Table2. Model's Parameter.

Model	Parameter
Laguerre Functional Model	$c_1 = 1.0444, c_2 = -0.1125, c_3 = -0.2575, c_4 = -0.3192$ $N = 4, p = 1.9608, \tau_1 = 0.889, \tau_2 = -0.0532, \tau_3 = 0.0599$ $\tau_4 = 0.1121$
Laguerre-Weiner Model	$c_1 = 1.0444, c_2 = -0.1125, c_3 = -0.2575, c_4 = -0.3192$ $N = 4, p = 1.9608, \tau_1 = 0.889, \tau_2 = -0.0532, \tau_3 = 0.0599$ $\tau_4 = 0.1121, \gamma_2 = -0.4819, \gamma_1 = 1.469, \gamma_0 = -0.06401$

By applying the Equation 20, the forgetting factor which has been utilized in RLS algorithm was updated.

$$\lambda = 1 - \frac{e^2(k)}{1 + e^2(k)} \tag{20}$$

where e is error between process output and model output.

Figures 3 and 4 illustrate the performance of temperature tracking for the proposed controllers as well as the control action. The transient response of the system for load rejection was also studied in this work. The temperature transient response for the controllers and their corresponding control actions are shown in Figure 7 and 8. The control horizon and prediction horizons were tuned by trial and error 5 and 10, respectively. The weighting matrices were selected as Q = 100 I and R = 0.3 I. To imposing saturation constraints in manipulated variable, a lower limit of 297 K and an upper limit of 372K were chosen. These figures demonstrate that AMPC based on Laguerre function has a good performance in set-point tracking and load rejection. In this work, the robustness of AMPC model mismatch was also examined and the deviation in the heat of reaction was considered as the model uncertainty. The performance of the proposed controller and control action in presence of model mismatch are shown in Figures 7 and 8, respectively.

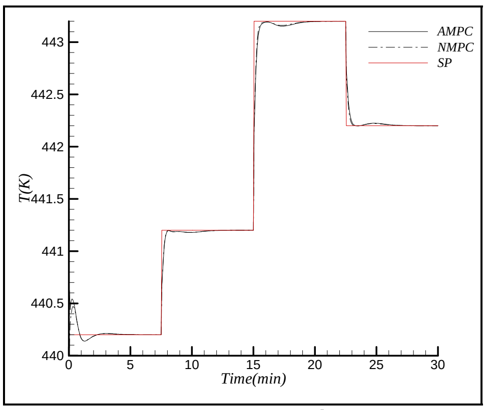


Figure 3. Set-point tracking.

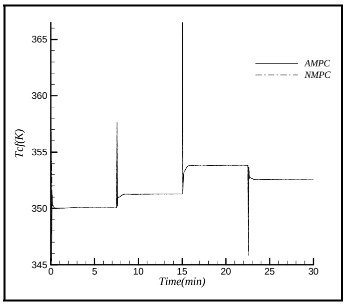


Figure 4. control action for set-point tracking.

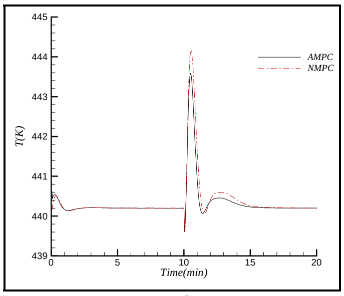


Figure5. Load rejection.

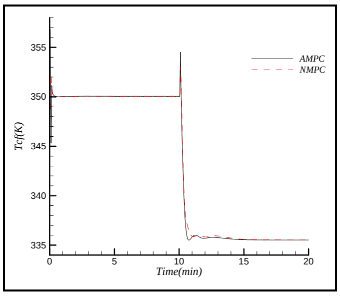


Figure 6: Control action for load rejection.

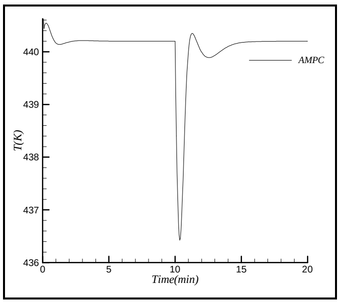


Figure 7. Performance of AMPC for deviation in the heat of reaction.

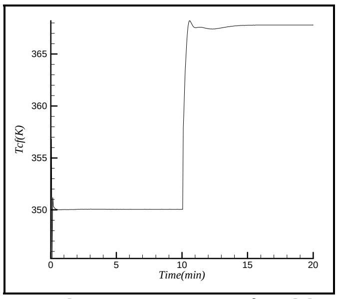


Figure8. Control action in presence of model mismatch.

VI. CONCLUSION

In this paper, an adaptive model predictive control using Laguerre functional model was presented. This controller for the control of CSTR reactor process was applied and simulated. The simulation result reveals that AMPC has a good performance in set-point tracking and load rejection. The results have also been compared with NMPC controller. Simulation results demonstrate that two purposed controller almost have the same performance.

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