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Design, Construction and Dynamic Modeling of a Snake Robot

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Abstract:
In this paper, dynamic equation of n-link snake robot is derived using Lagrange's method for two friction model: simple viscous and coulomb. A simplified form for the final dynamic equation in matrix format is presented.In order to verify the derived dynamic equation, snake robot model prepared in SimMechanics toolbox of MATLAB package is used. Results obtained by the derived dynamic equation closely agree with those of SimMechaics. In order to experimentally evaluate the results, a 5-link snake an undulatory robotic prototype has been developed using off-the-shelf components and conventional fabrication techniques. Experimental verification of the derived dynamic equation has also be perfomed.

Introduction :
Snake robots are a class of hyper-redundant mechanisms that locomote through internal shape change. Snake robots are serially connected, multilink articulated mechanisms, which propel themselves by body shape undulations. Despite having challenges in the area of control and inefficiency in locomotion due to high friction, snake robots have attracted the attention of researchers for applications not suitable for wheeled and legged robots. Applications such as ruins of collapsed buildings or narrow passages in search and rescue operations are good examples where snake robot may be used. two other challenges of snake robots over wheeled mechanisms are difficulty in analyzing and synthesizing snakelike locomotion mechanisms as well as its control. To overcome the first problem, we drive dynamic equations of an n-link snake robot using Lagrange's method that results in a simplified final dynamic equation. Verification of the derived dynamic equation has been performed both theoretically and experimentally.

Snake robot model :
A planar snake robot consisting of n links connected through n-1 joints is depicted in Fig. 1. Each link is rigid with uniformly distributed mass and is equipped with a torque actuator (motor). Each link is of mass m_i, length l_i and moment of inertia J_i. Let (x_{ci},y_{ci}) and θ_i define the center of gravity and the angle between the link and the x-axis,

respectively. Values of d_i represent location of mass center of i-th link. (x_b,y_b) is coordinate of the end of tail link. Free-body diagram of the robot is depicted in Fig. 2, where T_i are the joint torques from the actuators and f_{ni} and f_{ti} are the force due to the friction between the links and the horizontal surface. As illustrated in Fig. 2, φ_i, $(i=1,\ldots,n-1)$ are relative angles of two adjacent links.

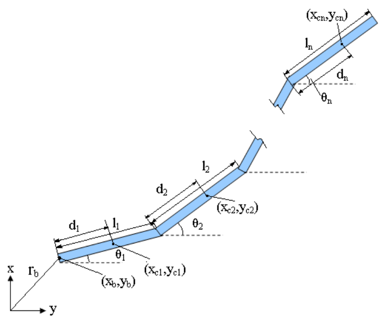

Figure 1.*n*-link snake robot

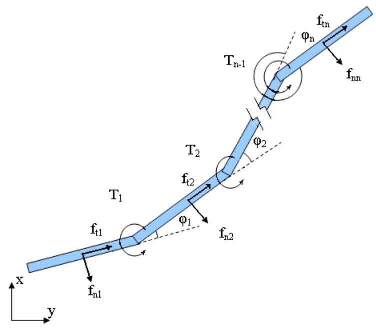

Figure 2. Free body diagram of *n*-link snake robot.

In order to generate significantly different friction coefficients in normal and tangential directions a fixed wheel axis or a blade may be attached to each link. In our experimental model we used fixed wheels. We consider a simple coulomb friction model. Friction force is modeled by the following equations

$$f_{ei} = -m_i g \mu_e sign(v_i^e) \tag{1}$$

$$f_{ei} = -m_i C_e v_i^e$$

Where $e = t, n$ (t and n represents tangential and normal directions). g is the gravity constant. μ_t and μ_n are normal and tangential coulomb friction coefficients. C_t and C_n are normal and tangential viscus friction coefficients. Subscript i corresponds to the i-th link, f_{ti} and f_{ni} are friction forces in tangential and normal directions, respectively. v_{it} and v_{in} are velocities of the center of mass of i-th link. The signum function is denoted by $sign(x)$; i.e., $sign(x)$ is 1 if $x > 0$, 0 if $x = 0$, and -1 if $x < 0$.
According to Fig. 1, the coordinates of the mass center of i-th link are

$$x_{ci} = x_b + \sum_{j=1}^{i-1} l_j \cos\theta_j + d_i \cos\theta_i \tag{3}$$

$$y_{ci} = y_b + \sum_{j=1}^{i-1} l_j \sin\theta_j + d_i \sin\theta_i \tag{4}$$

Velocities of mass center of each link are

$$\dot{x}_{ci} = \dot{x}_b - \sum_{j=1}^{i-1} l_j \dot{\theta}_j \sin\theta_j - d_i \dot{\theta}_j \sin\theta_i \tag{5}$$

$$\dot{y}_{ci} = \dot{y}_b + \sum_{j=1}^{i-1} l_j \dot{\theta}_j \cos\theta_j + d_i \dot{\theta}_j \cos\theta_i \tag{6}$$

Thus the kinetic energy of the *n*-link snake robot can be defined as

$$K=\sum_{i=1}^{n}[\frac{1}{2}I_i\dot{\theta}_i^2+\frac{1}{2}m_i(\dot{x}_c^2+\dot{y}_c^2)] \quad (7)$$

The instantaneous system configuration will be known upon having (x_b,y_b) and θ_i *($1 \le i \le n$).* Therefore, the generalized coordinates are selected as follows

$$q_j=[\theta_1,\theta_2,...,\theta_n,x_b,y_b] \quad (9)$$

As there is no variation in potential energy, the equations of motion can be written as

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right)+\frac{\partial K}{\partial q_i}-Q_i=0 \ \ (i=1,2,....n+2) \quad (10)$$

Non conservative forces that do work when generalized coordinates are given virtual displacements are actuators torques (T_i *i=1,2,…,n-1*) and friction forces (f_{xi} *and* f_{yi} *i=1,2,…,n*). Thus generalized forces are defined as

$$Q_{\theta j}=d_j(f_{yi}\cos\theta_j-f_{xi}\sin\theta_j)+l_j[\cos\theta_j\sum_{i=j+1}^{n}(f_{yi})-\sin\theta_j\sum_{i=j+1}^{n}(f_{xi})]+T_{j-1}-T_j \quad (11)$$

$$Q_{xb}=\sum_{i=1}^{n}(f_{xi}) \quad (12)$$

$$Q_{yb}=\sum_{i=1}^{n}(f_{yi}) \quad (13)$$

Where $Q_{\theta j}$ are generalized forces related to generalized coordinate θ_j. Q_{xb} and Q_{yb} are generalized forces related to x_b and y_b, respectively. By substituting Eqs. (8), (11), (12) and (13) into Lagrangian formulation, Eq. (10), the dynamic model for the *n*-link snake robot can be derived as

$$BT=M(\theta)\ddot{q}+H(\theta,\dot{\theta})+F(\theta) \quad (14)$$

where $M(\theta)$ is the *(n+2)×(n+2)* positive definite and symmetric inertia matrix, $H(\theta, \dot{\theta})$ is the *(n+2)×1* matrix related to centrifugal and Coriolis terms with *i*-th member of H_i, $F(\theta)$ is an *(n+2)×1* matrix related to friction forces, *B* is an *(n+2)×(n-1)* constant matrix. *T* is *(n-1)×1* matrix of input torques and $q,\dot{q},\ddot{q}$ are *(n+2)×1* matrix of generalized coordinates and their derivatives. $\theta,\dot{\theta},\ddot{\theta}$ are *n×1* matrix of links absolute angles and their derivatives. The final dynamic equation, Eq. (14), has a simplified matrix format and can easily be expanded for any number of links.

Shape to motion

In this section we attempt to drive at the motion of the snake robot given the relative angles of the adjacent links. Another words, given instantaneous relative angles and their derivatives ($\varphi,\dot{\varphi},\ddot{\varphi}$), Eq. (14) can be solved in order to find applied torques and coordinates of the tail of the robot which will determine the motion of the robot. Relation between absolute value and relative value of joint angles is:

$$\varphi_i=\theta_{i+1}-\theta_i \quad (15)$$

Where *i=1,2,…,n-1*. We can rewrite Eq. (15) in matrix form as

$$\theta=E\varphi+e\theta_1 \quad (16)$$

Where φ is an n-dimensional vector of $[\varphi_1,\varphi_2,\ldots,\varphi_{n-1}]$, θ_1 is the absolute angle of the tail link, E_{ij} and e are defined as

$$E_{ij}=\begin{cases}1 & i>j\\ 0 & others\end{cases} \qquad e=[1,1,\ldots 1]^T \tag{17}$$

In order to obtain motion based on given shape, we first decouple the dynamic Eq. (14) into two parts

$${}^pM(\theta)\ddot{\theta}+{}^pN(\theta)\ddot{r}_b+{}^pH(\theta,\dot{\theta})+{}^pf(\theta)=DT \tag{18}$$

$${}^qM(\theta)\ddot{\theta}+{}^qN\ddot{r}_b+{}^qH(\theta,\dot{\theta})+{}^qf(\theta)=0 \tag{19}$$

where

$$M=\begin{bmatrix}{}^pM_{n\times n} & {}^pN_{n\times 2}\\ {}^qM_{2\times n} & {}^qN_{2\times 2}\end{bmatrix},\quad H=\begin{bmatrix}{}^pH_{n\times 1}\\ {}^qH_{2\times 1}\end{bmatrix},\quad r_b=\begin{bmatrix}x_b\\ y_b\end{bmatrix} \tag{20}$$

And matrix ${}^pf$, ${}^qf$ and D are defined in appendix. By substituting second derivative of Eq. (16) into Eq. (19), we arrive at

$$\ddot{r}_b=-{}^qN^{-1}({}^qM\ddot{\theta}+{}^qH\dot{\theta}+{}^qf)=-{}^qN^{-1}\,{}^qM(E\ddot{\varphi}+e\ddot{\theta}_1)-{}^qN^{-1}({}^qH\dot{\theta}+{}^qf) \tag{21}$$

Substituting Eq. (21) into Eq. (18), we obtain

$$\begin{aligned}&DT+({}^pN\,{}^qN^{-1}\,{}^qM-{}^pM)e\ddot{\theta}_1=({}^pM-{}^pN\,{}^qN^{-1}\,{}^qM)E\ddot{\varphi}\\ &-{}^pN\,{}^qN^{-1}({}^qH\dot{\theta}+{}^qf)+{}^pH\dot{\theta}+{}^pf\end{aligned} \tag{22}$$

Equation (22) is an n-dimensional linear equation representing the dynamic of the n-link snake robot. In the direct dynamic formulation, inputs are the n-dimensional vector of joint angles $[\varphi_1,\varphi_2,\ldots,\varphi_{n-1}]$ and the outputs are n unknown variables $\ddot{\theta}_1\in\mathrm{R}$ and torque, $T\in\mathrm{R}^{n-1}$. Therefore, by solving Eq. (22), we can obtain the joint torques, T_i, and tail link rotation acceleration, $\ddot{\theta}_1$. Substituting these values back into Eq. (21), will obtain acceleration of the tail end, $\ddot{r}_b$. The tail link joint angle (θ_1) and its angular velocity ($\dot{\theta}_1$) as well as velocity of the tail end ($\dot{r}_b$) and its moving distance (r_b) are all obtained through integration. The complete parameters defining robot motion are derived for the case when changes in body shape, φ, are known. Therefore, upon specifying changes in body shape we can drive at necessary joint torques to generate the desired robot motion.

SimMechanics Model

A 5-link snake robot with identical links is modeled using SimMechanics. As illustrated in Fig. 3, there is a revolute joint between each link block of the model except for the first link. A planar joint is used to connect first link block and ground block. In order to calculate friction forces of Eq.(1) or Eq.(2), it is necessary to measure velocity of center of mass of each link compare to tangential and normal directions. As illustrated in Fig. 3 and Fig .4, velocity measurement for each link is carried out using sensor block in SimMechanics toolbox.

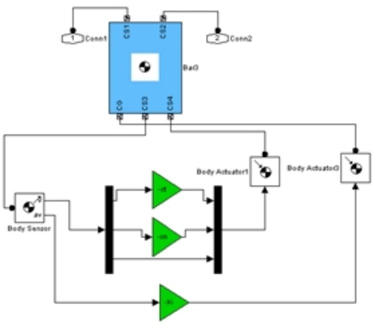

Figure 3. Viscus friction model of each link.

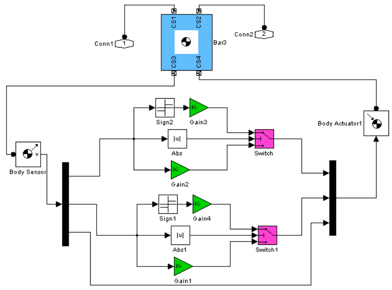

Figure 4. Coulomb friction model of each link.

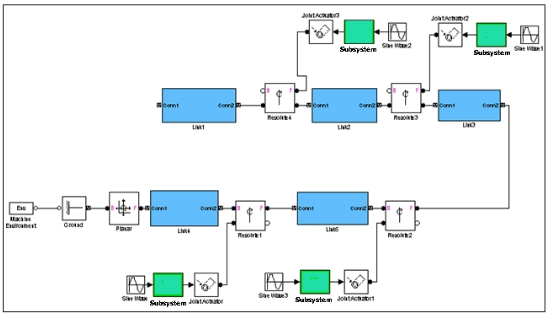

Figure 5. 5-link snake robot model in SimMechanics.

Comparing Lagrange's Method with SimMechanics

In order to verify the derived dynamic equation we consider a case study and model it with SimMechanics. Next we solve derived dynamic equation, Eq. (15). we consider a 5-link snake robot with identical links with l_i=2m,d_i=1m, m_i=1 Kg, I_i=0.33, C_t=1 and C_n=30. we apply sinusoidal relative angle with constant phase difference as shown in Fig. 6 to each joint. Simulation time is 20 seconds.

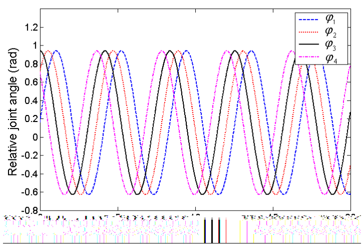

Figure 6. Relative joint angles applied to each joint.

Design and construction snake robot

In order to experimentally evaluate the results obtained in previous sections, an undulatory robotic prototype has been developed, using off-the-shelf components and conventional fabrication techniques. The prototype used in the present study, shown in Fig. 7, is composed of five Plexiglas links (each link: weight 80g, length 110mm, width 40mm and height 30mm) with the special rotary joints allows vertical motion of links which makes its motion easier when encountering uneven terrains.

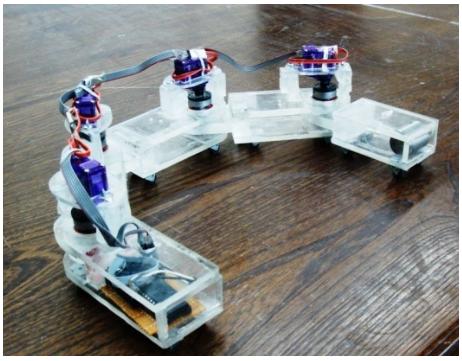

Figure 7. Snake robot experimental prototype.

Our approach in hardware design is to find an efficient model to test and implement our dynamic motion equation. The important task here is to find a reasonable joint design since the joints should have a one-dimensional rotation in vertical axis with a small movement in the plane in the direction parallel to the axis. Otherwise robot cannot move in the jagged surfaces even with a small one. Since some of the links would not have full contact with the beneath surface. This decreases the friction force, which is the principal of our locomotion system. In order to build a joint with the mentioned characteristics, we design a specific hinge with rectangular cross section. As is presented in Fig. 8. this joint is composed of a groove and a rode with a rectangular cross section at the end of the first link which both of them have an approximately same width but with different lengths. These would cause small freedom in the movement in the plane across the groove but inhibit rotation of the rod. (Fig. 10.)

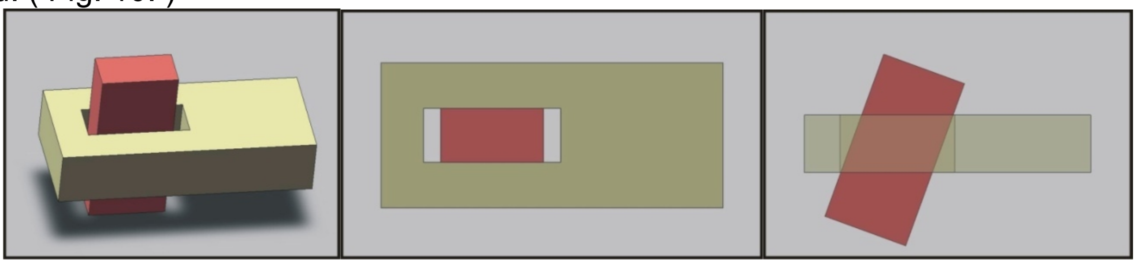

Figure 8. Groove and Rod

To provide the rotational movement of one link relative to the other link, the rod is connected to the second link using two ball bearings. Fig. 12.

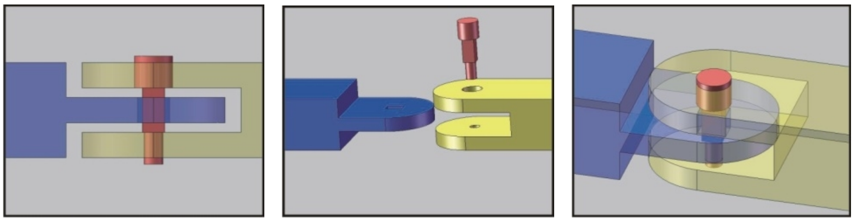

Figure 9. A final joint between two links

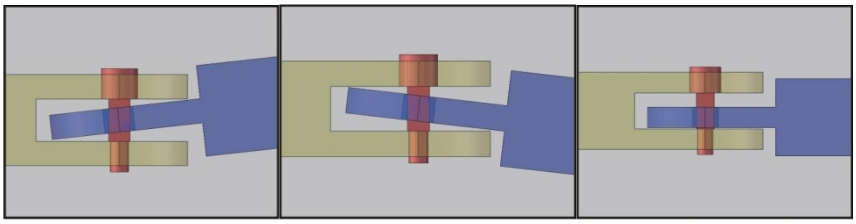

Figure 10. small movement in joint

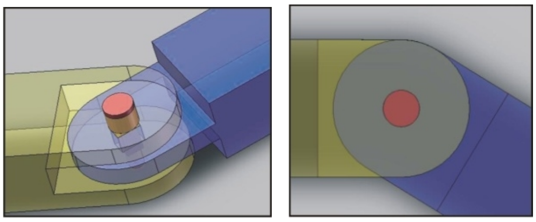

Figure 12. rotational movement in joint

The motor which is used in this robot is a 4 volts servo DC motor with a 3.6 Kg.cm torque, and 0.15 degree precision. Motors are equipped with an absolute encoder which satisfies the need of finding the initial position of the joints when powered on. In order to supply sufficient torque to rotate the joint, we use direct connection of motor shaft to the rod. Reducing destructive looses and mechanical losses are some of the advantages of direct connection (without gear, belt and …). From one side the output shaft of the motor with direct coupling is connected to the upper sides of the rod and on the other side the body of the motor is connected to the second link. As the motor turns, first link rotates relative to the second one in the direction to the vertical axis.

To construct the body of the robot we utilize Plexiglas sheets due to low weight, easily cutting and suitable soldering. As mentioned before we need minimum friction in the direction perpendicular to the links and maximum friction in the orthogonal direction. As the result we utilize four passive wheels for each links .To provide electrical energy we used distributed electric power cells on each links instead of one power supply. Since this arrangement would equally set the weight of the battery to all of the robot links. Otherwise we would face to the problem of unsatisfactory locomotion. In order to implement our equations for robot locomotion as following we use microprocessor core to produce essential signals for each motor.

Realization of Serpentine Gait

All simulation results obtained thus far have used the derived dynamic equations. In order to validate dynamic equation of the robot, we adjust geometrical parameters (length, mass, link inertia) to represent the physical model. The coefficients of tangential and normal frictions for the actual surface are physically measured and are found to be $\mu_t=0.05$ and $\mu_n=0.56$. We apply relative angles equation . to both simulation and physical model (with values of $\alpha=\pi/4$, $\beta=\pi/4$) so that the snake robot moves with serpentine gait. Position of center of the mid link is measured by analyzing pictures taken during robot motion.
We compare path followed by center of the mid link of the physical robot with simulated model (Fig. 13). Differences between these paths are mainly due to inaccurate friction coefficients and incomplete dynamic equations for ignoring effects such as, joint friction, gear box and small differences between links. Another source of discrepancy between the two results may be due to difficulty in recording the actual path followed by the experimental model. Considering limitations discussed above, it can be concluded that using the dynamic equation a good approximation of the actual motion of the robot can be obtained

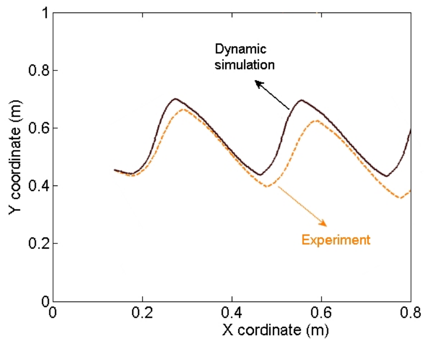

Fig. 13.Comparison of motion predicted by simulation and experimental model

Reference

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[2] Crespi, A., Badertscher, A., Guignard, A. and Ijspeert, A. J. " Amphibot I: an amphibious snake-like robot" Robotics and Autonomous Systems 50 (2005) 163-175.