



## A New Approach for Solving of Linear Time Varying Control Systems

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**Abstract**—This paper is concerned with the solution of Linear Time Varying [LTV] control systems. The concept of a solution for LTV systems is defined on the basis of finding the fundamental matrix corresponding to LTV control systems. There are some numerical methods such as Euler method, Taylor method and Runge-Kutta method for obtaining approximate solution of LTV system [LTVs], each of them has some limitations. In the recent years, other kinds of constructive approaches for the solution of LTVs are presented limited to the particular cases of it. In this paper, we introduced a new approach that we call it Extended AVK [EAVK] approach to obtain a global optimal approximation of the fundamental matrix of LTVs, by introducing a problem in calculus of variations corresponding to our LTVs problem. A global optimal approximate solution (general solution of LTV systems) by using Linear Programming [LP] is considered. EAVK approach exhibits some interesting advantages (simplicity, high performance, flexibility, etc.) comparable with some existing methods. The results of simulations to solve some numerical examples showed the efficiency of our method.

**Index Terms**—Fundamental Matrix, Linear Time Varying Control Systems, Linear Programming.

### I. INTRODUCTION

Linear time varying systems are of great importance because they are very frequently used to represent the dynamical behavior of the physical systems encountered in engineering practice. We are going to make use of a new approach to obtain an approximation of the fundamental matrix of LTVs to obtain a general solution to the initial-value problem for LTV control systems. Until now, a number of constructive theories for the solution of LTV systems have been available [1], [2], [3], [4], [5]. Some limitations of previous methods are: Euler method [6], [7] has not acceptable accuracy; Taylor method [6], [7] involves computation of coefficients high order derivatives so it cannot be used for LTVs with non-smooth coefficients; in Runge-Kutta method [6], [7], [8] error control is possible when system is smooth, and otherwise it does not guarantee solution to be accurate and

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