

A Psychological Model Applied to Mathematical Problem Solving

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(Received March 28, 2009. Accepted July 14, 2009)

Students' approaches to mathematical problem solving vary greatly with each other. The main objective of the current study was to compare students' performance with different thinking styles (divergent vs. convergent) and working memory capacity upon mathematical problem solving. A sample of 150 high school girls, ages 15 to 16, was studied based on Hudson's test and Digit Span Backwards test as well as a math exam. The results indicated that the effect of thinking styles and working memory on students' performance in problem solving was significant. Moreover, students with divergent thinking style and high working memory capacity showed higher performance than ones with convergent thinking style. The implications of these results on math teaching and problem solving emphasizes that cognitive predictor variable (Convergent/Divergent) and working memory, in particular could be challenging and a rather distinctive factor for students.

Keywords: psychological model, convergent/divergent (Con/Div) thinking style, working memory capacity (WMC), mathematical problem solving

ZDM Classification: C34

MSC2000 Classification: 97C30

WORKING MEMORY CAPACITY

Most current psychological models of learning have been developed based on the fact that individual's brain has a space of limited capacity, in which conscious thought takes

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place (Miller, 1956; Baddeley, 1986; Johnstone & El-Banna, 1986; Johnstone & Al-Naeme, 1991; Holmes & Adams, 2006; Alamolhodaei, 2009). The working memory (WM) is a limited capacity system being responsible for the manipulation and storage of information during the performance of cognitive tasks such as comprehension learning, problem solving, and reasoning (Baddeley, 1986). The multi-component model of WM proposed by Baddeley & Hitch (1974), is arguably the most widely cited model which comprises four sub-components. The phonological loop (Baddeley, 1986) and the visuo-spatial sketchpad (Logie, 1995) are two sub-components that are assumed to be responsible for storing and manipulating verbal or visuospatial information. They are coordinated by a domain-general limited capacity system, the control executive (the third sub-component), which commands a number of functions including planning, inhibition, switching attention, and monitoring the process of temporarily held information (Baddeley, 1986; 1996; Baddeley & Logie, 1999). The fourth part, the episodic buffer (Baddeley, 2000), is considered to be responsible for the integration of information from the subcomponents of WM and long term memory (LTM).

Psychological Model

The present study introduces a psychological model developed based on working memory capacity (WMC) and convergent/divergent (Con/Div) thinking style to study the performance of mathematical problem solving in students.

In recent years various studies have shown that achievements in science and mathematical problem solving depend on cognitive variables such as mental capacity and thinking styles (Talbi, 1990; Johnstone & Al-Naeme, 1991; Johnstone, Hogg & Zian, 1993; Niaz, 1996; Ashcraft & Kirk, 2001; Holmes & Adams, 2006; Alamolhodaei, 2001, 2002; 2009).

Working Memory and Mathematical Problem Solving

There are some considerable evidences suggesting that WM may be important for mathematics learning and problem solving. For example, Adams & Hitch (1998) suggested that mental arithmetic performance relies on working memory. Significant associations have been found between the phonological loop and mental arithmetic performance in children (e.g., Adams & Hitch, 1997; 1998; Javris & Gathercole, 2003; Holmes & Adams, 2006). More specifically, Holmes & Adams (2006) reported a significant association between children's WM ability and their mathematics attainment. They found that WM could predict national curriculum-based mathematical skills. In addition, the result of their study confirmed that the central executive is an important predictor of children's mathematical performance. The phonological loop showed a

stronger association with the older children's mental arithmetic performance.

Moreover, the Alamolhodaei (2009) and others (e.g., Ekbia, 2000) have found that the students with high WMC (HWMC), regardless the gender, are more capable of solving mathematics word problems compared to those with low WMC (LWMC).

Con/Div Thinking Style and Mathematical Problem Solving

A widely used dimension of thinking style in education is the Convergence/Divergence style, which specifies an individual's mode of perceiving, thinking, problem solving and visualizing.

Convergent style is characterized by the generation of one accepted correct answer from the available information. In contrast, divergent style is a propensity to produce a number of potentially acceptable solutions to the problem (Hudson, 1966; Kolb, 1984; Guilford, 1959; 1978; Reese *et al.*, 2001; Cassidy, 2004). According to Hudson (1966), divergent students are very similar to intuitive students. Both tend to be creative and score better on open-end tests than those in which only a single answer is demanded.

Guilford *et al.* (1965) suggested a positive correlation between divergent thinking and learning mathematics. Besides, it was found that learning various aspects of calculus tasks demands different dimensions of thinking style in learners. For instance, divergent thinkers show favour towards pictorial thinking, curve interpretation and calculus word problems (Alamolhodaei, 1996; 2001). Ghorbani (2006) reported that divergent students significantly achieved higher results in calculus open-end questions than convergent students. These findings all confirm that divergents show higher performances than convergents. In other words, divergents have a strong bias towards visual thinking and visualization than convergents. This may be reasonable to suggest that the nature of mathematical tasks indicates that students should cope well with convergent and divergent thinking in the problem solving. In fact, at the beginning of solving a mathematical problem they need to think openly and critically and converge step by step towards the desired solution (Alamolhodaei, 2001). Accordingly, it can therefore be concluded that in general divergent students show higher performances compared to convergent students in mathematical problem solving (Alamolhodaei, 1996; Ekbia, 2000; Akbarian, 2009).

Con/Div Thinking style and WMC

Bahar & Hansell (2000) investigated the interaction between convergent/divergent thinking style and working memory capacity (WMC). The result of their study showed a significant positive correlation between convergence/divergence test results and the results of the WMC test. According to this, divergent students tend to have a high WMC

while convergent ones tend to have a low WMC. An important relationship can be found between WMC and thinking style in the literatures on dyslexia. Dyslexic students have very poor performances in inferential questions due to WM deficiency that can be attributed to a cognitive cause (Simmons & Singleton, 2000). Moreover, children with mathematical difficulties are less likely able to use direct memory retrieval to solve arithmetic questions, and count more slowly and inaccurately than children with normal abilities (Holmes & Adams, 2006).

Other studies show that children with arithmetic learning disabilities also suffer from deficiencies in their WMC (e. g., Shafir & Siegel, 1994; Siegel & Ryan, 1989).

The main aim of the present study is to identify students' difficulties in mathematical problem solving. The focus of this research was to provide profile of students' performance with different WMC and different thinking styles (Con/Div). Thus, the main question addressed here is:

"Is there any interactions between students' WMC and Con/Div styles and their performance in mathematical problem solving?"

In an attempt to answer this question the following objectives were sought in order to find out whether

- There would be a relationship between students' WMC and Con/Div thinking style.
 - Divergent learners show higher WMC than convergent learners.
 - Students with divergent style show higher performance in mathematical problem solving than those with convergent style.
 - Students of IWMC exhibit better performance in mathematical problem solving than those of LWMC.
- i) Divergent students with LWMC (Div + LWMC) demonstrate higher achievements in mathematical problem solving than convergent ones with LWMC (Con + LWMC).
 - ii) Students with divergent style and HWMC (Div + HWMC) could attain higher performance in math exam than convergent ones with HWMC (Con + IWMC).

METHIOD

Participants

150 school girls ages 15 to 16 were selected from high schools across Khorasan Province using random multistage stratified sampling design.

Procedures

The participants were required to take the following tests:

- 1- Con/Div Test,
- 2- Digit Span Backwards Test (DBT), and
- 3- Math Exam.

1- Con/Div Test:

A special version of Con/Div test designed by Johnstone & Al-Naeme (1991) and Alamolhodaci (1996; 2001) was used. It was comprised of six short tests which were each assigned a limited amount of time to complete. The students were required to write as many answers as possible for every question they were given. One mark was given for every single correct response (Hudson, 1966). The highest possible score that could be gained in these six tests was 130. Under this situation, a normal distribution of performance was obtained.

A slice of one quarter of a standard deviation (SD) on either side of the mean scores was classified as "Intermediate" and excluded from the hypotheses test in this study to obtain two contrasting groups (Con/Div). The quantity of the mean score ± 0.25 SD was regarded as a crucial point between moving from convergent thinking style towards divergent thinking one or vice versa. Therefore, moving up from the mean score $+0.25$ SD of sample population was classified as divergents, while moving down from mean score -0.25 SD was grouped as convergents.

Table 1 represents statistical information of the tests carried out to evaluate the (Con/Div) scores obtained by students, while Table 2 shows the number of students in each group of the styles (convergent/intermediate/divergent).

Table 1. Statistical information of Con/Div Test

<i>Group</i>	<i>Mean Score</i>	<i>SD</i>	<i>Maximum Score</i>	<i>Minimum Score</i>
<i>N=150</i>	52.03	12.75	88	23

Table 2. The distribution of thinking styles over the sample

<i>Group</i>	<i>Convergent</i>	<i>Intermediate</i>	<i>Divergent</i>
<i>N=150</i>	<i>N=62</i>	<i>N=27</i>	<i>N=61</i>
Total	41.3%	18.0%	40.7%

Reliability coefficient (Cronbach's α) for Con/Div test was estimated to be 0.68.

2- Digit Span Backwards Test (DBT):

For measuring the students' WMC, the DBT was used (Case, 1974; Scardamalia, 1977; Johnstone, 1988; Talbi, 1990; Johnstone *et. al.*, 1993). The digits were read by an expert and the students were required to listen carefully, then turn the number over in their mind and write it down from left to right on their answering sheets. Students took the DBT test and a retest within a one month period. A copy of the test is available in Appendix I. The Pearson correlation between the test and the retest was significant (p -value < 0.001, $r = 0.55$). Students who scored above the sample mean were labeled as having high WMC, and those who scored less than the sample mean were labeled as having low WMC. Table 3 shows this distribution.

Table 3. The students' WMC distribution over the sample

<i>Group</i>	<i>Low</i>	<i>High</i>
<i>N</i> = 150	<i>N</i> = 88	<i>N</i> = 62
Total	58.7%	41.3%

3- Math Exam:

The effectiveness of these psychological factors (thinking style and WMC) were investigated by the students' performance in mathematical problem solving. Thus a mathematics exam with ten questions as dependent variables was designed (Appendix II). The maximum score for this task was 20. Normality assumption for the exam was considered.

Referring to the first objective of this study, a relationship was found between students, with Con/Div thinking style and WMC. The Pearson's correlation between these psychological variables was significant ($p = 0.001$). The results of correlation analysis between independent and dependent variables are given in Table 4.

Table 4. Pearson's correlations between students' WMC, Con/ Div and Math

<i>Group</i>	<i>WMC</i>	<i>Con/Div</i>	<i>Math</i>
WMC	1	0.31**	0.36*
Con/Div	0.31**	1	0.30**
Math	0.36*	0.30*	1

** Correlation is significant at $p = 0.001$.

With respect to the second objective, which was to explore whether divergent students would show higher WMC than convergent students, WMC scores obtained by Con and Div students in DBT test were compared. The mean scores and standard deviations (SD) in DBT test related to Con/ Div thinking styles are set out in Table 5.

Table 5. Mean scores and SD of WMC in different groups of Con/Div

Group	Con(N=62)		Div(N=61)	
	Mean	SD	Mean	SD
Math Exam	4.24	0.84	4.60	0.95

According to Table 5, the divergent students obtained a higher mean compared to convergent ones in DBT test. To maximize the effect of thinking style, the results of convergent and divergent group were compared, with the intermediate group being disregarded. According to *t*-test, it was found that there was a significant difference between the mean for WMCs obtained for Con and Div students at $p < 0.001$.

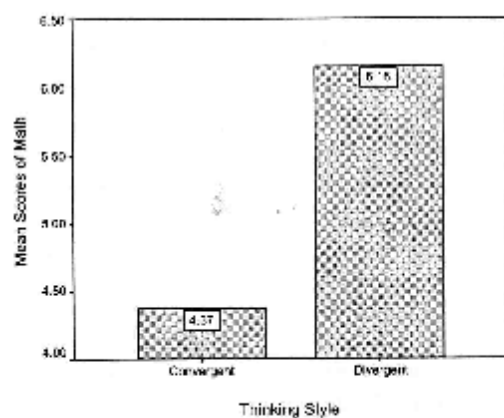
To find out whether Div students would exhibit higher performance than Con students in mathematical problem solving, the mean and SD for scores on math exam were related to Con/Div thinking style as set out in Table 6.

Based upon the *t*-test results for the mean scores on math exam, a significant difference was found between two groups of students at $P = 0.03$. It should be noted that normality assumption for math data was strongly confirmed at $P = 0.03$, which is in agreement with one- sample Kolomgorov test.

Table 6. Mean scores of Con/Div students in Math exam

Group	Con(N=62)		Div(N=61)	
	Mean	SD	Mean	SD
Math Exam	4.37	1.95	6.16	2.65

Figure 1 represents the superiority of students with different styles, (Con vs. Div) based on their mean scores on math exam.

**Figure 1.** The students' achievement with different styles on math exam.

The comparison between performance of the students with low and high WMC in math exam was the fourth objective of this research. According to *t*-test results, the two independent groups of low and high WMC were found to be significantly different ($p < 0.001$) in terms of the mean scores they obtained on math exam (Table 7). This shows the superiority performance of the students with HWMC in mathematical problem solving compared to those with LWMC.

Table 7. Mean & SD of low/ high WMC in math exam

	<i>WMC</i>	<i>Mean</i>	<i>SD</i>
Math	Low ($N=88$)	4.64	2.10
Exam	High ($N=62$)	6.11	2.70

As for the fifth objective of the study, the effects of WMC and Con/Div variables on math exam were examined by performing two-way analysis of variance. According to Table 8, the interaction between WMC and Con/Div was non-significant ($p = 0.82$). In addition, WMC and Con/Div had significant effects on math exam scores at $p = 0.003$ and 0.008, respectively.

Table 8. The effects of WMC and Con/Div on math exam

<i>Independent variable</i>	<i>Dependent variable</i>	<i>F</i>	<i>P-value</i>
WMC	Math	6.19	0.003
Con/Div	Math	7.15	0.008
WMC*Con/Dev	Math	0.19	0.82

Moreover, according to the students' mean scores in Table 9, the Con students with high WMC achieved higher scores than Con students with low WMC in mathematical problem solving.

The same result was found for Div students. The difference between each group was significant as confirmed by test at $p = 0.030$ and 0.035, respectively for Con/Div styles in math exam

Table 9. Mean & SD of Con/ Div students with different WMC on math exam

<i>Group</i>	<i>WMC</i>	<i>Mean</i>	<i>SD</i>
Con	low ($N=43$)	4.0	1.85
	high ($N=19$)	5.21	2.0
Div	low ($N=30$)	5.43	2.35
	high ($N=31$)	6.85	2.80

The last objective of the study was to explore whether Div students with LWMC/HWMC would achieve higher result compared to Con students with LWMC/HWMC in math exam. According to test, the significant superiority of Div + HWMC students was confirmed in mathematical problem solving at $p = 0.007$ and 0.019 , respectively (Tables 10 and 11).

Table 10. Mean & SD of (Con+LWMC) & (Div+LWMC) in math exam

Group	<i>Con+LWMC (N=43)</i>		<i>Div+LWMC (N=30)</i>	
	Mean	SD	Mean	SD
Math Exam	4.00	1.85	5.43	2.34

Table 11. Mean & SD of (Div + HWMC) & (Con + HWMC) in math exam

Group	<i>Div+HWMC (N=31)</i>		<i>Con+HWMC (N=19)</i>	
	Mean	SD	Mean	SD
Math Exam	6.85	2.80	5.21	1.97

DISCUSSION

This study reveals a positive relationship between thinking style (Con/Div) and working memory capacity (WMC) confirming that the Div students achieve higher performance on working memory test (DBT) than Con ones. It was also observed that students with Con styles tend to show weak performance in mathematical problem solving compared to Div students. Moreover, these findings exhibited that the students with HWMC achieved higher performance than those with LWMC in math exam. These findings support previous claims that WMC and thinking styles (Con/Div) could predict mathematical performance (e.g., Talbi, 1990; Askcraft & Kirk, 2001; Alamolhodaei, 2001, 2002, 2009; Akbarian, 2009).

As can be inferred from the results of the present research, the WMC is a more distinctive and challenging variable to the learners with the same thinking styles.

Therefore, a math teacher should encourage learners to use strategies that reduce their mental overloading, and to effectively use their working memories. One main objective of the present study was to discover the effect of WMC and Con/Div style on mathematical problem solving. The findings confirmed that the effect of the two psychological factors (WMC and Con/Div) on the dependent variable (i.e., math exam) are additive. In other words, the effect of one factor (e.g., Con/Div) on the dependent variable (math exam score) is independent from the level of the second factor (e.g., WM).

As the results for the last objective were confirmed, the group with Div + HWMC has

superiority over the other groups. It seems that, (Div + IIWMC) learners would show higher performances than those of other styles in mathematical problem solving. On the other hand, Div learners tend to show higher WMC as compared to Con learners.

Therefore, it would be safe to suggest that, to reach the same level of mathematical performance, Con students would need more mental space to compensate for their thinking style characteristic. It seems to be important from a teaching point of view to filter out signal (related material) from noise (unrelated material) to allow the students to use the potential mental space fully for effective processing. Additionally, to avoid loss of information during working memory processes, larger units of math information in a problem must be chunked into smaller units, or conceptual entities (Alamolhodaei, 2009).

The balance between analytical and pictorial way of thinking is not often valued in textbooks and math teaching methods. Therefore, it is fair to suggest that teaching styles and mathematical tasks should be planned to benefit both thinking styles (Con/Div) of learners. Classroom instructions should incorporate certain skills, thinking styles, and mental capacity whenever and as often as possible. Students need external organizers such as cognitive maps, open-ended questions, graphs, images, and pictures to promote their meaningful learning. They need time to brainstorm, share ideas, and have opportunities to express themselves creatively.

The findings of the present study are based upon the investigation carried out on female high school samples. Consequently, further works need perhaps under more specific conditions for finding more results in particular, for male students. It would also be valuable to extend this study in more advanced mathematical areas in high school and at the undergraduate level.

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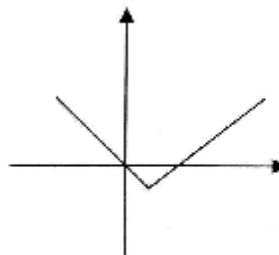
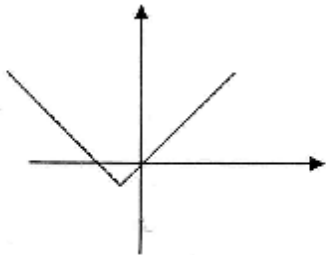
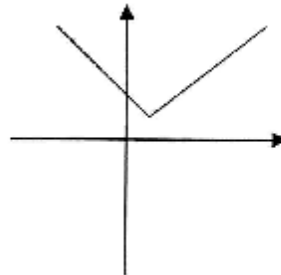
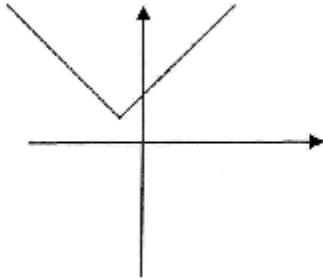
APPENDIX I: MATH EXAM

1- Find domain of the variable of the following equations and then solve them.

A. $\frac{x-2}{x+1} + \frac{x+2}{x-1} = -3$

B. $\sqrt{x+1} - x = -5$

2- Which of the following graphs is the graph of the function $f(x) = |x-1| + 1$? Describe your answer.



3- Let A' be a mirror image of $A(3, 2)$ in the x-axis and A'' be a mirror image of A' in the y axis, then which of the following coordinates is the coordinate of midpoint of the line segment AA'' ? Describe your answer.

1. $\left(\frac{3}{2}, 1\right)$

2. $\left(1, \frac{3}{2}\right)$

3. $(1, 1)$

4. $(0, 0)$

4- Find the perimeter of isosceles triangle which height is 4m and its area is equal with the perimeter of circle with area

$$\frac{36}{\pi m^2}$$

APPENDIX II: DIGITS BACKWARD TEST (DBT)

Directions- Start by saying:

Now I'm going to give another set of number, but this time there's a complication. When I have finished saying each set of number, I want you write them down in reverse order. For example, if I say "719", you would write down "917".

Now, no cheating. Do not write from right to left. You listen carefully, turn the number over in your mind and write from left to right. Have you got that? Then let's begin.

Series:

2	2	4						
	5	8						
3	6	2	9					
	4	1	5					
4	3	2	7	9				
	4	9	6	8				
5	1	5	2	8	6			
	6	1	8	4	3			
6	5	3	9	4	1	8		
	7	2	4	8	5	6		
7	8	1	2	9	3	6	5	
	4	7	3	9	1	2	8	
8	9	4	3	7	6	2	5	8