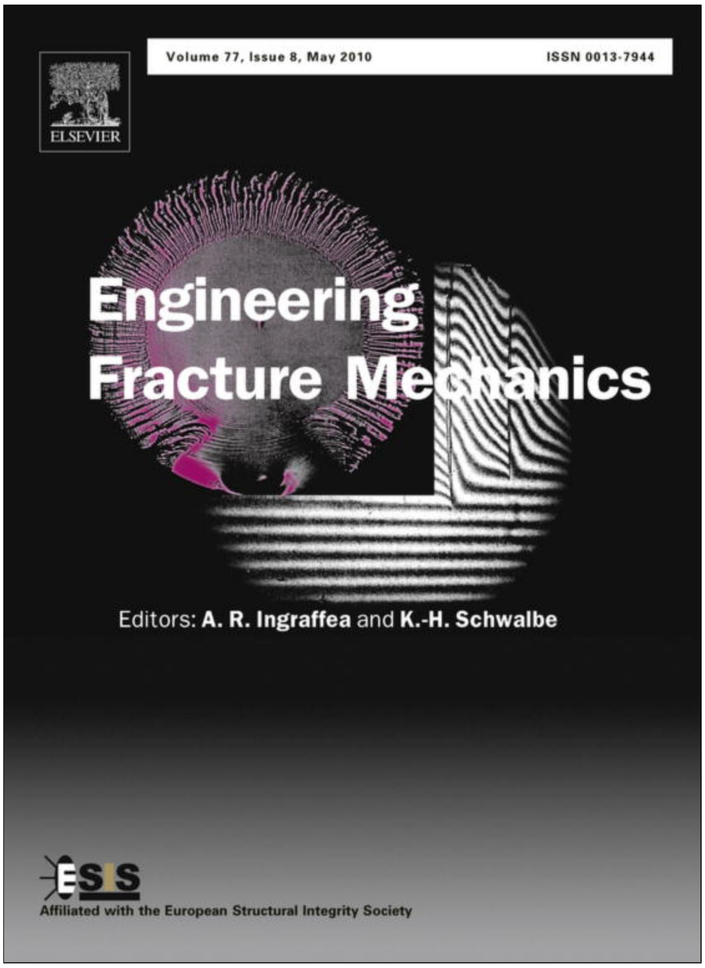
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Letter to the Editor

Comments on "Detection of multiple cracks using frequency measurements" and suggestion for improvement in solution for damage parameters

M. Raghebi *, A. Farshidianfar

Department of Mechanical Engineering, Ferdowsi University of Mashhad, PO Box 91775-1111, Iran

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1. Introduction

Recently, Patil and Maiti [1] have presented interesting work describing a method of analysis for detection of multiple open cracks in slender Euler–Bernoulli beams using frequency measurements. Their method is based on transverse vibration modeling through transfer matrix method and representation of a crack by rotational spring, proposed by Hu and Liang [2]. The beam is virtually divided into a number of segments, which the number can be decided by the analyst, and each segment is considered to be associated with a damage parameter s. The procedure gives a linear relationship between the changes in natural frequencies of the beam and the damage parameters:

$$\left\{\frac{\Delta\omega}{\omega}\right\}_{q\times 1} = 2[H]_{q\times m}\{S\}_{m\times 1}, \tag{1}$$

where for a simply-supported beam $h_{nj} = \int_{L_j} \frac{1}{2}\sin^2(n\pi\beta)L\,d\beta,\ n = 1,\ldots,q,\ j = 1,\ldots,m$ are the elements of $[H]$ matrix, where $\beta = x/L$ is the position of the crack.

Relation (1) can be solved by pseudo-inverse technique [3], now we have:

$$\{S\}_{m\times 1} = 0.5[H^T H]^{-1}_{m\times m}[H]^T_{m\times q}\left\{\frac{\Delta\omega}{\omega}\right\}_{q\times 1}. \tag{2}$$

Liang et al. [4] said in page 1480 that:

> "It is noted that the pseudo-inverse technique will only give the best estimate of S in the least-squares sense. Therefore, it is possible to obtain values of $S_i < 0$. ...Consequently when the pseudo-inverse technique gives values of $S_i < 0$, we set the section modulus change at those locations (elements) equal to zero and repeat the calculation for the remaining S_is. In this way, the number of S_is to be solved can be gradually reduced to the one equal to the number of known (or measured) eigenfrequency changes, thus, effectively reducing the inconsistent linear system to a consistent system, where normal matrix inversion procedure applies".

* Corresponding author. Tel.: +98 9155187824.
E-mail addresses: raghebi@yahoo.com (M. Raghebi), Farshid@um.ac.ir (A. Farshidianfar).

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Nomenclature

a	depth of the crack
b	width of the beam
E	modulus of elasticity
h	depth of the beam
L	length of the beam
m	number of elements in the beam
q	number of measured frequencies
x	position of the crack
β	non dimensional position of the crack
ρ	mass density
$\{S\}$	vector of damage parameters
ω	natural frequency of the beam

Table 1
Natural frequencies of simply-supported beam with and without cracks.

Crack location and size				Solution method	Natural frequencies (rad/s)				
β_1	a_1/h	β_2	a_2/h		ω_1	ω_2	ω_3	ω_4	ω_5
Uncracked beam				Analytical	59.007	236.029	531.065	944.116	1475.182
0.25	0.07971	0.45	0.0986	Ref. [1]	58.625	235.142	528.096	942.515	1469.103

$E = 2.8 \times 10^{10}$ N/m^2, $\rho = 2350$ kg/m^3, $L = 10$ m, $h = 0.6$ m, $b = 0.2$ m.

Table 2
Convergence of identification process for simply-supported beam.

		S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
Iteration 1	Patil's method [1]	0.00066	0	0	0	0	0.05317	−0.00699	0.03901	−0.00343	0
	Liang's method [4]	0.3829	0.0599	0.1191	0.4329	0.1616	−0.3442	−0.4946	−0.0016	−0.0329	−0.6834
	Proposed method	0.3829	0.0599	0.1191	0.4329	0.1616	−0.3442	−0.4946	−0.0016	−0.0329	−0.6834
Iteration 2	Patil's method [1]	0.00066	−0.00343	0	−0.00699	0	0.05317	0	0.03901	0	0
	Liang's method [4]	0.00066	−0.00343	0.03901	−0.00699	0.05317	0	0	0	0	0
	Proposed method	0.00066	−0.00343	0.03901	−0.00699	0.05317	0	0	0	0	0
Iteration 3	Patil's method [1]	1238.76	0	782.02	0	0	0.06	0	−781.98	0	−1238.8
	Liang's method [4]	0.009414	0	0.031318	0	0.05040	0	0	0	0	0
	Proposed method	−0.00713	0	0.03462	0	0.05055	0	0	0	0	0
Iteration 4	Patil's method [1]	0.01	0	0.03	0	−5374.4	5374.47	0	0	0	0
	Liang's method [4]										
	Proposed method	0	0	0.033486	0	0.04896	0	0	0	0	0
Iteration 5	Patil's method [1]	0.009414	0	0.031318	0	0	0.05040	0	0	0	0
	Liang's method [4]										
	Proposed method										

The authors have a simple comment about the paragraph mentioned above. Notice that in some cases after setting negative S_i's equal to zero, the number of damage indices can be less than the number of measured frequency changes. Therefore, we have an over determined system of equations. In this case (1) should be solved repeatedly until all indices are positive. It should be noted that for solving (1) all of its equations are preserved. In the method that has been described in Refs. [1,4], the equations corresponding to higher frequencies have been ignored, thus their results are inaccurate and imprecise.

For comparison purposes, an example that is mentioned in Ref. [1] is going to be solved in the next section.

2. Example: crack detection in a simply-supported beam

Patil and Maiti [1] have supposed a simply-supported beam containing two cracks and divided it into 10 equal segments for detection of these two cracks; therefore in (1) $m = 10$. Details of the geometric, material properties and the first five natural frequencies of the beam with and without cracks have been mentioned in Table 1.

Since, five natural frequencies are considered, in (1) $q = 5$ is selected. Patil and Maiti have solved (1) and the results are shown in Table 2. In this table the results of the Liang's and the proposed method have been shown.

From Table 2 it can be stated that:

1. In Patil's method after five iterations three damage parameters are positive or equivalently three cracks in segments 1, 3 and 6 have been predicted; but in the actual model only two cracks exist.
2. In Liang's method after three iterations the same results have been obtained, in other words three cracks in segments 1, 3 and 5 have been predicted; but in the actual model only two cracks exist. It is worth to mention that because of symmetry of boundary conditions in the simply-supported beam, elements 5 and 6, 4 and 7, 3 and 8, 2 and 9 or 1 and 10 are the same and therefore the results of Liang and Patil are equivalent.
3. In iterations 1 and 2 the results of Liang's method and the proposed method are similar because they use pseudo-inverse technique for solving (1). However, the results of Patil's method is different because he uses a method based on the fewest possible nonzero components. In other words, Patil uses $x = A/b$ command of MATLAB for solving $Ax = b$: however in the proposed method, $x = \mathrm{pinv}(A) * \mathrm{b}$ or $x = \mathrm{inv}(A' * A) * A' * b$ command has been used. To get the results employing $x = \mathrm{pinv}(A) * b$, there is a need to specify tolerance level. The tolerance level prescribed by the default of MATLAB software was used by the authors. (For detailed discussion on this subject, the interested reader may refer to help of MATLAB, version 7.0.0.19920 (R14) and search for pinv.)
4. In the proposed method after four iterations, the results are positive and the method predicts that the beam contains two cracks in segments 3 and 5 which the actual model also contains the same number of cracks. It should be mentioned that in iteration 2, the values of S_2 and S_4 are negative thus they have been set to zero. Therefore, for iteration 3 the number of damage parameters that should be estimated is equal to 3 which is less than the number of equations and we have an over determined system of equations. Liang and Patil in this manner have ignored the corresponding high frequency data; i.e., 4th and 5th frequencies. Then they have solved a linear consistent system of equations using normal matrix inversion. Hence the results are inaccurate and imprecise. However, in the proposed method all the equations have been preserved and solving (1) repeatedly gives positive damage parameters after four iterations which in result two cracks in segments 3 and 5 have been predicted.

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