International Heat Transfer Conference (IHTC-14/2010)
Washington D.C., U.S.A.
August 8-13,2010

Effects of Suction and Blowing on Flow and Heat Transfer between Two Rotating Spheres with Time-Dependent Angular Velocities

Asgahr B. Rahimi

Professor, Faculty of Engineering, Ferdowsi University of Mashhad, P.O. Box No.91775-1111, Mashhad, Iran. E-mail:rahimiab@yahoo.com (Corresponding Author)

Abstract:

Effect of suction and blowing in the study of flow and heat transfer of a viscous incompressible fluid between two vertically eccentric rotating spheres is presented when the spheres are maintained at different temperatures and rotating about a common axis while the angular velocities of the spheres are arbitrary functions of time. The resulting flow pattern, temperature distribution, and heat transfer characteristics are presented for the various cases including exponential and sinusoidal angular velocities. These presentations are for various values of the flow parameters including rotational Reynolds number Re, and the blowing/suction Reynolds number Re_w. The effects of transpiration and eccentricity on viscous torques at the inner and outer spheres are studied, too. As the eccentricity increases and the gap between the spheres decreases the viscous torque at the outer sphere decrease. Results for special case of concentric spheres are obtained by letting eccentricity tend to zero.

1- Introduction

The flow and heat transfer in an annulus between two spheres have been studied in various cases by many researchers. Such studies can be classified into two main groups. In the first group, there is neither suction nor blowing at the spherical walls. Such containers are used in engineering designs like centrifuges and fluid gyroscopes and also are important in geophysics. Available theoretical works concerning such problems are primarily of a boundary-layer or singular-perturbation character considered by Howarth [1], Proudman [2], Lord & Bowden [3], Fox [4], Greenspan [5], Carrier [6] and Stewartson [7]. The first numerical study of time-dependent viscous flow between two rotating spheres has been presented by Pearson [8] in which the cases of one (or both)sphere is given an impulsive change in angular velocity starting from a state of either rest or unifom rotation. Munson and Joseph [9] have considered the case of steady motion of a viscous fluid between concentric rotating spheres using perturbation techniques for small values of Reynolds number and a Legendre polynomial expansion for larger values of Reynolds numbers. Thermal convection in rotating spherical annuli has been considered by Douglass, Munson and Shaughnessy [10]. A study of viscous flow in oscillatory spherical annuli has been done by Munson and Douglass [11] in which a perturbation solution valid for slow oscillation rates is presented and compared with experimental results. Another interesting work is the study of the axially symmetric motion of an incompressible viscous fluid between two concentric rotating spheres done by Gagliardi et al. [12], and also the study by Jen-Kang Yang et al. [13] and the finite element study by Ni and Nigro [14]. These problems include the case where one or both spheres rotate with prescribed constant angular velocities and the case in which one sphere rotates due to the action of an applied constant or impulsive torque. Recently a numerical study of flow and heat transfer between two rotating spheres has been done by Jabari Moghadam and Rahimi [15] in which the fluid contained between two vertically eccentric spheres maintained at different temperature and rotating about a common axis with different angular velocities when the angular velocities are arbitrary functions of time. Jabari Moghadam and Rahimi [16] have also studied the similarity solution for spheres rotating with constant angular velocity.

In the second group, the effects of transpiration on flow in an annulus between two spheres have been investigated. The study of flow in a spherical annulus along with transpiration is used in many practical applications, such as rotary machines and spherical heat exchanger and in the design of spherical fluid storage systems. In these applications transpiration is used to regulate the rate of heat transfer.

Effects of transpiration on free convection in an annulus between two stationary concentric porous spheres have been considered by Gulwadi et al. [17]. Gulwadi et al. [18] studied the laminar

flow in an annulus between rotating porous spheres and with injection and suction at spherical walls. They used a perturbation technique to solve the steady-state Navier-Stokes equations of motion and also used a finite difference method to validate their analytical results. Their results are valid for small values of the rotational Reynolds number and an injection/suction Reynolds number and the heat transfer has not been considered. A review of the literature reveals that there is no studies on the transient motion and the heat transfer between two rotating spheres with uniform transpiration.

In the present study, a numerical solution of unsteady momentum and energy equations is presented for the general case of viscous flow between two vertically eccentric rotating spheres maintained at different temperatures along with suction and blowing at their boundaries, which are rotating with time-dependent angular velocities. Results for some example functions including exponential and sinusoidal angular velocities and various blowing/suction Reynolds number Re_w are presented when the outer sphere initially starts rotating with a constant angular velocity and the inner sphere starts rotating with a prescribed time-dependent function. Results for the special case of concentric spheres are obtained by letting eccentricity tend to zero.

2 -Problem Formulation

The geometry of the spherical annulus considered is indicated in Fig. 1. The vertical eccentricity of the outer sphere is measured by the distance *e*. If the outer sphere is placed above the central position, *e* has a positive value, otherwise it is negative. The origin of the spherical coordinate system is the inner sphere center and the characteristic radius of the outer sphere, R_o' , is a function of θ. A Newtonian, viscous, incompressible fluid fills the gap between the inner and outer spheres, which are of radii R_i and R_o and with constant surface temperatures T_i and T_o and rotate about a common axis with angular velocities Ω_i and Ω_o, respectively. The components of velocity in r, θ and ϕ directions are v_r, v_ϕ and v_ϕ, respectively. These velocity components for incompressible flow and in meridian plane satisfy the continuity equation and are related to stream function ψ and angular momentum function Ω in the following manner:

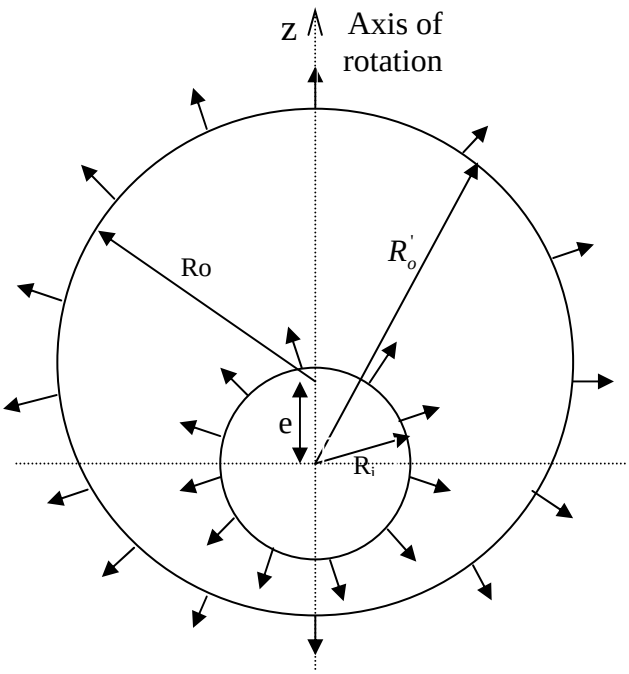

Fig.1. Geometry of eccentric rotating spheres

$$v_r = \frac{\psi_\theta}{r^2 \sin\theta}, \qquad v_\theta = \frac{-\psi_r}{r \ \sin\theta}, \qquad v_\phi = \frac{\Omega}{r \ \sin\theta} \tag{1}$$

The blowing/suction Reynolds number is defined as:

$$\mathrm{Re}_w = \frac{v_{r_o} r_o}{\nu} \tag{2}$$

in which v_{r_o} and r_o are radial velocity and radius reference values, respectively. The blowing/suction Reynolds number Re_w is positive for blowing at inner sphere and negative for suction. Since the flow is assumed to be independent of the longitude, ϕ, the non-dimensional Navier-Stokes equations and energy equation can be written in terms of the stream function and the angular velocity function as follows:

$$\frac{\partial\Omega}{\partial t} + \frac{\psi_\theta \Omega_r - \psi_r \Omega_\theta}{r^2 \sin\theta} = \frac{1}{(\mathrm{Re})} D^2\Omega \tag{3}$$

$$\frac{\partial}{\partial t}(D^2\psi) + \frac{2\Omega}{r^3 \sin^2\theta}[\Omega_r r\cos\theta - \Omega_\theta \sin\theta] - \frac{1}{r^2 \sin\theta}[\psi_r (D^2\psi)_\theta - \psi_\theta (D^2\psi)_r] +$$

$$+ \frac{2D^2\psi}{r^3 \sin^2\theta}[\psi_r r\cos\theta - \psi_\theta \sin\theta] = \frac{1}{(\mathrm{Re})} D^4\psi \tag{4}$$

$$\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} = \frac{1}{(Pe)}[\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial T}{\partial \theta}] + (Ek)\{2[(\frac{\partial v_r}{\partial r})^2 +$$

$$+ (\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r})^2 + (\frac{v_r}{r} + \frac{v_\theta}{r}\cot\theta)^2] + [r\frac{\partial}{\partial r}(\frac{v_\theta}{r}) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}]^2 + [\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}(\frac{v_\phi}{\sin\theta})]^2 +$$

$$+ [r\frac{\partial}{\partial r}(\frac{v_\phi}{r})]^2\} \tag{5}$$

in which the non-dimensional quantities Reynolds number (Re), Prandtl number (Pr), Peclet number (Pe) are defined as:

$$\mathrm{Re} = \frac{\omega_o r_o^2}{\nu}, \quad \mathrm{Pr} = \nu/\alpha, \quad Pe = \mathrm{Re}.\mathrm{Pr} = \frac{\omega_o r_o^2}{\alpha}, \quad Ek = \frac{\nu\omega_o}{c_p(T_o - T_i)} \tag{6}$$

The following non-dimensional parameters have been used in the above equations and then the asterisks have been omitted:

$$t^* = t\omega_o, \qquad r^* = \frac{r}{r_o}, \qquad \psi^* = \frac{\psi}{r_o^3 \omega_o},$$

$$\Omega^* = \frac{\Omega}{r_o^2 \omega_o}, \qquad T^* = \frac{T - T_i}{T_o - T_i} \tag{7}$$

In which ω_o is reference value. The non-dimensional boundary and initial conditions for the above governing equations are:

For $t \prec 0$:

$$\begin{cases} \psi = 0 \\ \Omega = 0 \\ T = 0 \end{cases} \quad \text{, every where}$$

For $t \geq 0$:

$$\theta = 0 \rightarrow \{\psi_r = 0, \psi_\theta = 0, \Omega = 0\}, \frac{\partial T}{\partial \theta} = 0$$

$$\theta = \pi \rightarrow \{\psi_r = 0, \psi_\theta = 0, \Omega = 0\}, \frac{\partial T}{\partial \theta} = 0$$

$$r = R_i / R_o \rightarrow \left\{ \psi_\theta = \frac{\mathrm{Re}_w}{\mathrm{Re}} \sin\theta, \psi_\mathrm{r} = 0, \Omega = \frac{\Omega_\mathrm{i} R_i^2}{\omega_o R_o^2} \sin^2\theta, \mathrm{T} = 0 \right.$$

$$r = R_o' / R_o = e\cos\theta + \sqrt{(1 - e^2 \sin^2\theta)}$$

$$\rightarrow \left\{ \psi_\theta = \frac{\mathrm{Re}_w . R_o'^2}{\mathrm{Re}. R_o^2} \sin\theta, \psi_\mathrm{r} = 0, \ \Omega = \frac{\Omega_o R_o'^2}{\omega_o R_o^2} \sin^2\theta, \ \mathrm{T} = 1 \right. \tag{8}$$

where

$$D^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} - \frac{\cot\theta}{r^2}\frac{\partial}{\partial \theta}$$

These governing equations along with the related boundary and initial conditions are solved numerically in the next section.

3-Computational Procedure

The two equations governing the fluid motion show that each is describing the behavior of one of the dependent variables Ω and ψ. On the other hand, these two equations are coupled only through nonlinear terms. To solve the problem, the momentum equations were discretized by the finite-difference method and implicit scheme. Because of the known velocity field, the energy equation is linear and is solved keeping all its terms. In each time step (n+1), the value of the dependent variables are guessed from their values at previous time steps (n), (n-1), and (n-2) and after using them in difference equations and repeating it until the desired convergence, will lead to the corrected values at this time step. This procedure is applied for the next time step.

4 Presentation of Results

The contours of ψ and T for Re = 1000, $\mathrm{Re}_w = 5$, Pr = 1, $\Omega_{io} = -\exp(1-t)$, $e = 0.1$, and Ek=0 are shown in Figs. 2 and 3, respectively. As can be seen from the Fig. 2 (a), at the beginning the eddies are created in upper hemisphere and near the pole $(\theta = 0)$, so that two stagnation points exist on the pole. Note that due to eccentricity the flow field is asymmetric with respect to the equator plane and on the other hand because of more Coriolis force in the lower hemisphere the eddies are eliminated near the pole $(\theta = 180)$. Also it is seen from this figure that by decreasing the eccentricity, the size of the eddies in upper hemisphere decrease while as the eccentricity tends to zero, the eddies will be formed in lower sphere, too. Then in concentric case, four stagnation points exist on the poles. Also, the effects of blowing on vortices can be obtained in comparison with Ref. [15]. It is observed that the blowing removes all of the vortices in fourth quadrant and also the vortices near the outer sphere in first quadrant. As time advances, the streamlines in vicinity of equator become irregular while in vicinity of the poles the streamlines are smoother, Fig. 2 (b). At this time, there is no eddy in vicinity of the poles. Now, considering the contours of (ψ), the distribution of temperature (T) can be described better. From Fig. 3 (a), it is observed that at the beginning the distribution of temperature in the annulus space is nearly uniform and eddies in upper hemisphere don't affect the temperature field. As time passes, the blowing effect covers the entire temperature field so that it grows less than the case $\mathrm{Re}_w = 0$, but because of smoother streamlines in the vicinity of the poles the cold flow of the inner sphere toward the outer sphere transfers more than the regions far from the poles.

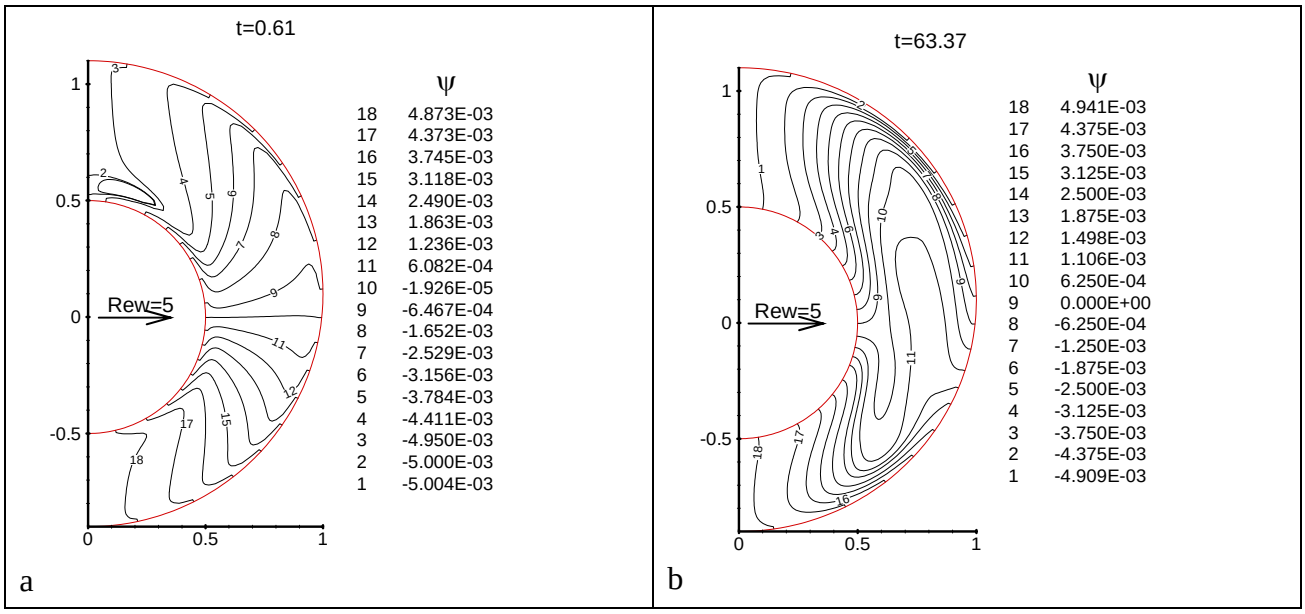

Fig.2. Contours of ψ for Re = 1000, $\mathrm{Re}_w = 5$, $\Omega_{io} = -\exp(1-t)$, $e = 0.1$

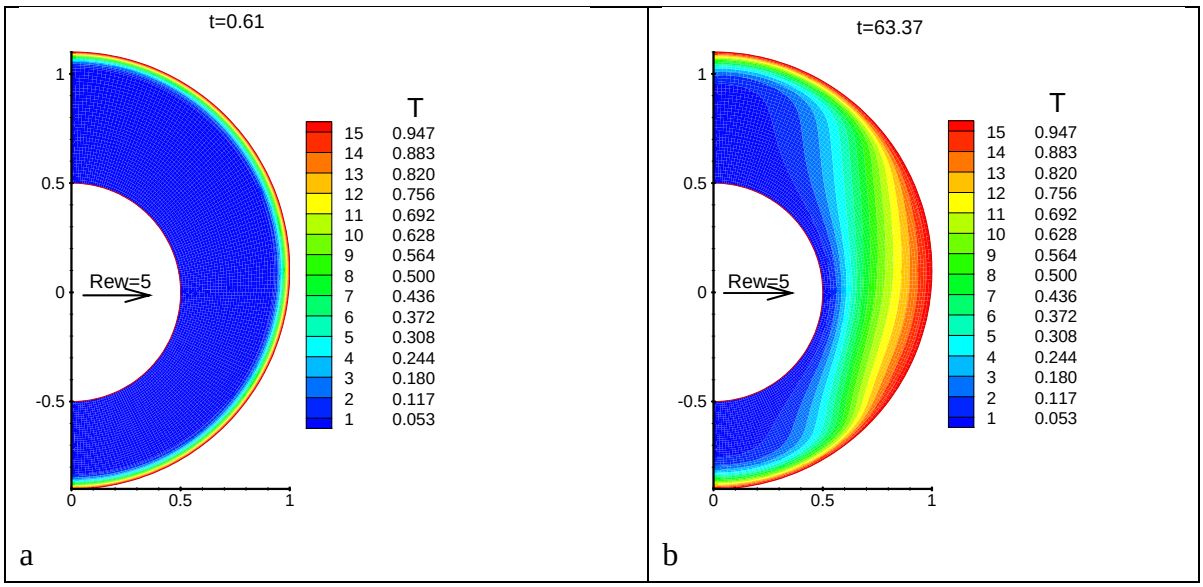

Fig.3. Contours of T for Re = 1000, $\mathrm{Re}_w = 5$, Pr = 1, $\Omega_{io} = -\exp(1-t)$, $e = 0.1$

With increase in suction, it is observed that at the beginning eddies are eliminated as is seen in Fig.4 (a) compared to Fig.3 (a). Also Fig.4 (b) shows that the streamlines penetrate with time from the first quadrant into fourth quadrant. As can be seen in Fig.5 (b), the effect of suction on the diffusion of heat from the outer sphere into the field is considerable. Note that the factors such as Prandtl number (Pr) and blowing/suction Reynolds number have important role in the diffusion of heat, so that an increase in Prandtl number or blowing/suction Reynolds number decreases the heat diffusion of outer sphere into field.

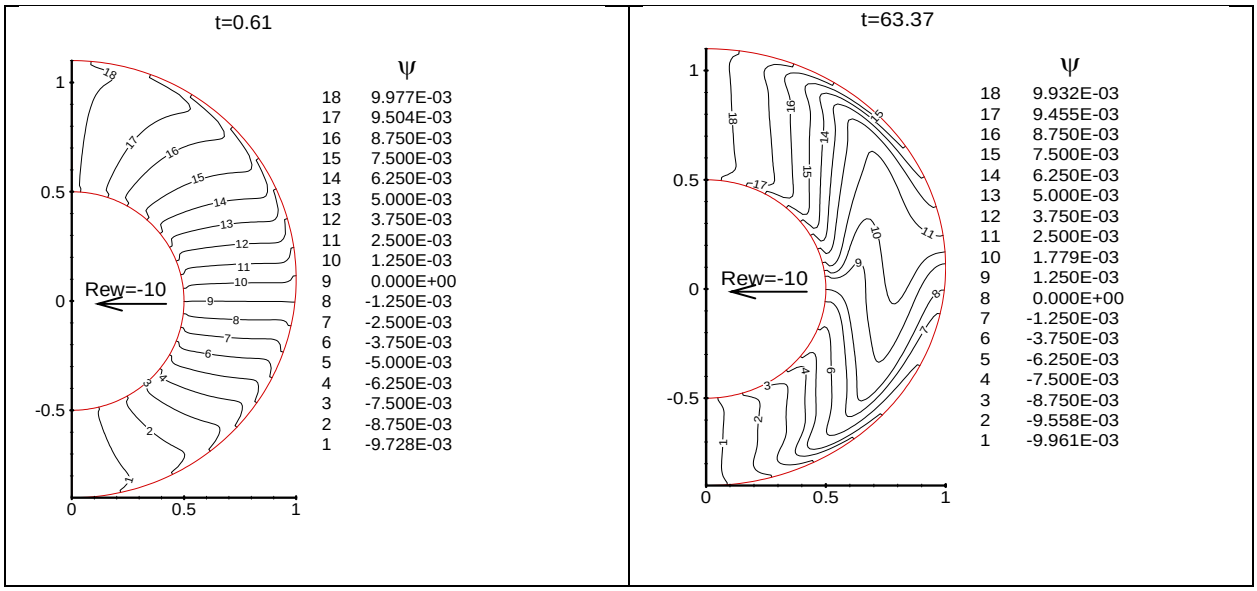

Fig.4. Contours of ψ for Re = 1000, $\mathrm{Re}_w = -10$, $\Omega_{io} = -\exp(1-t)$, $e = 0.1$

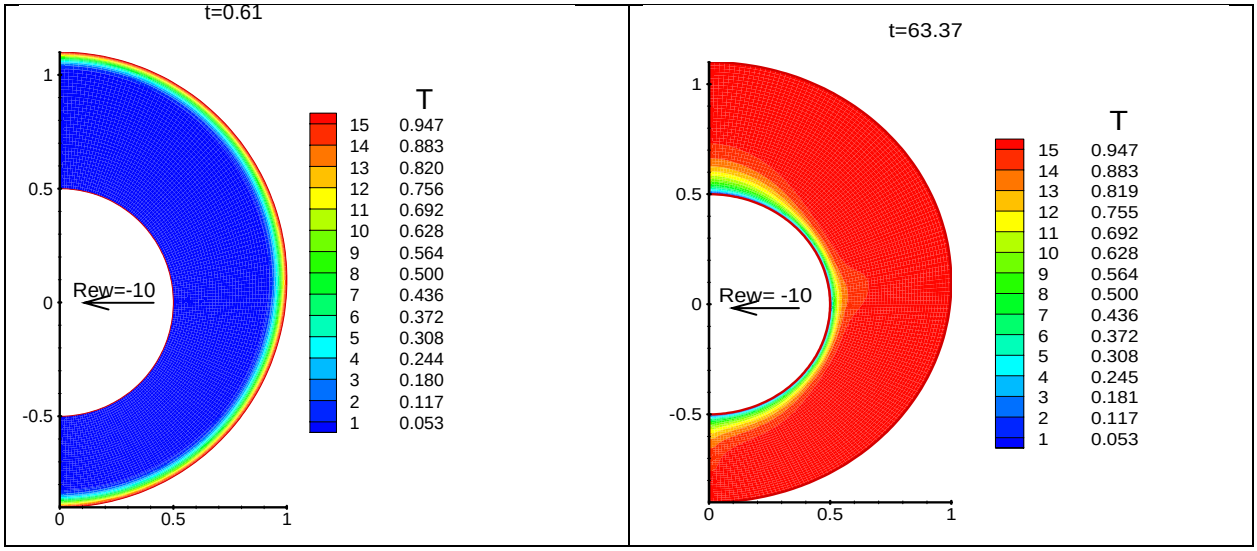

Fig.5. Contours of T for Re = 1000, $\mathrm{Re}_w = -10$, Pr = 1, $\Omega_{io} = -\exp(1-t)$, $e = 0.1$

In the case $\mathrm{Re}_w = 10$ and t = 4.01, eddies approximately are eliminated in the entire flow field. In this case, the blowing helps the Coriolis forces to remove eddies in flow field. On the contrary, at t =11.01, Fig.6 (b), because the inner sphere rotates counter to the outer sphere (inner sphere angular velocity is Ω_{io}= -1.998) the effect of Coriolis forces are against the blowing effect. In comparison with Fig.7 (a) the eddies have not been eliminated although in compared to Fig.7 (b) eddies are smaller (because of larger value of Re_w). As a result, a change in the value and direction of rotation of the spheres and or rate of blowing/suction can be used to regulate the flow field and therefore the rate of the heat transfer. The contours of T is shown in Fig.7.

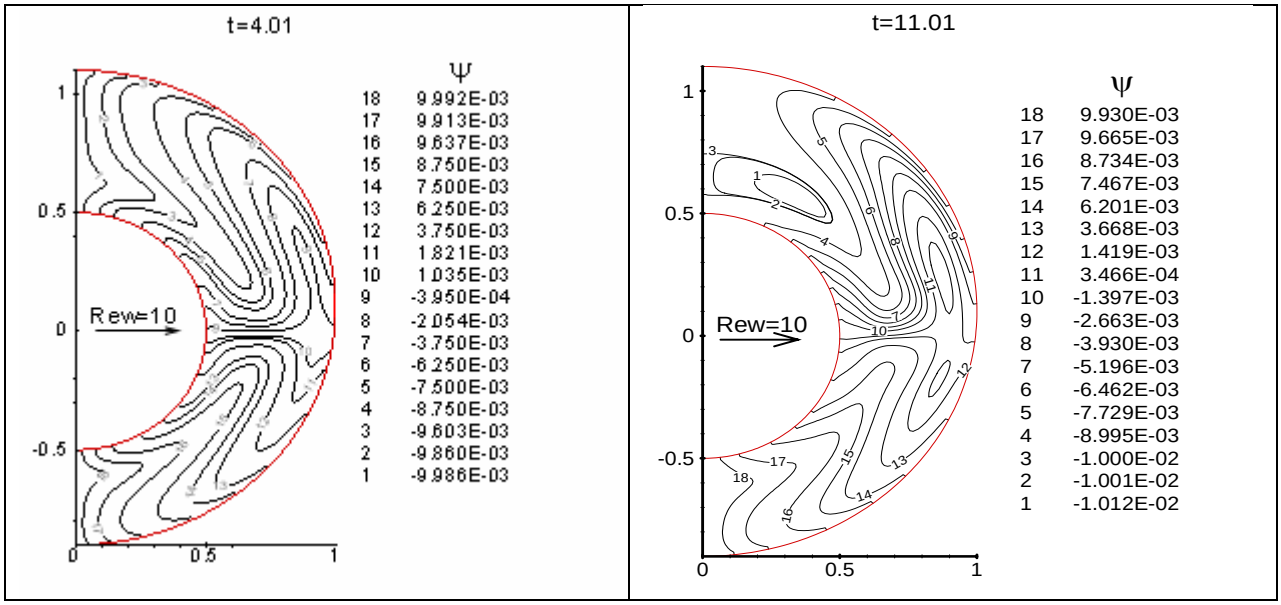

Fig.6 Contours of ψ for Re = 1000, $\mathrm{Re}_w = 10$, $\Omega_{io} = 2\sin(\pi t/2)$, $e = 0.1$

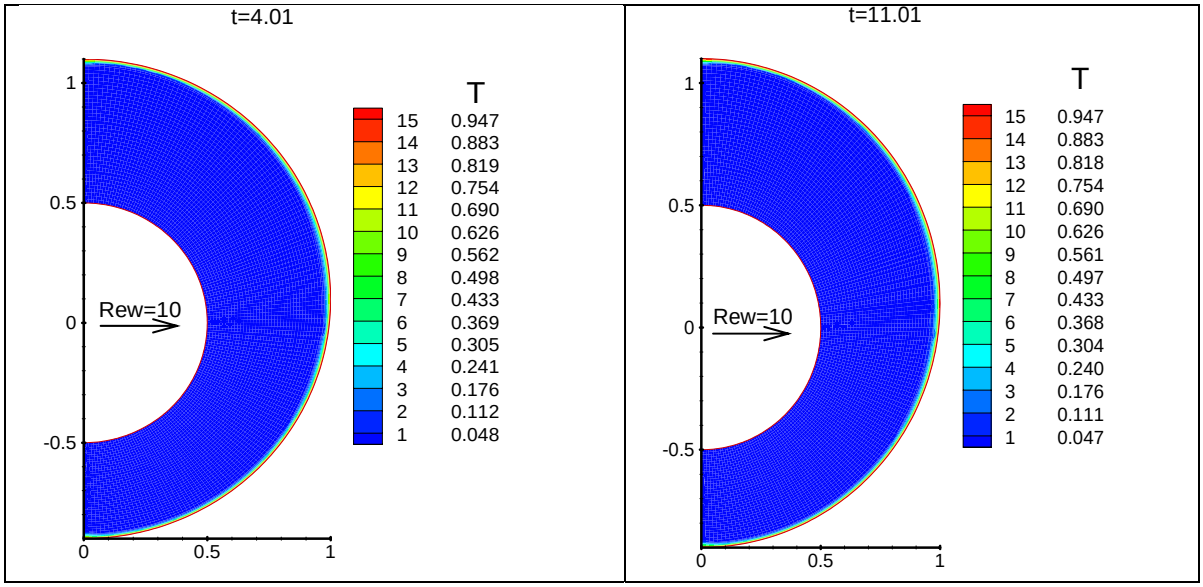

Fig.7. Contours of T for Re = 1000, $\mathrm{Re}_w = 10$, Pr = 10, $\Omega_{io} = 2\sin(\pi t/2)$, $e = 0.1$

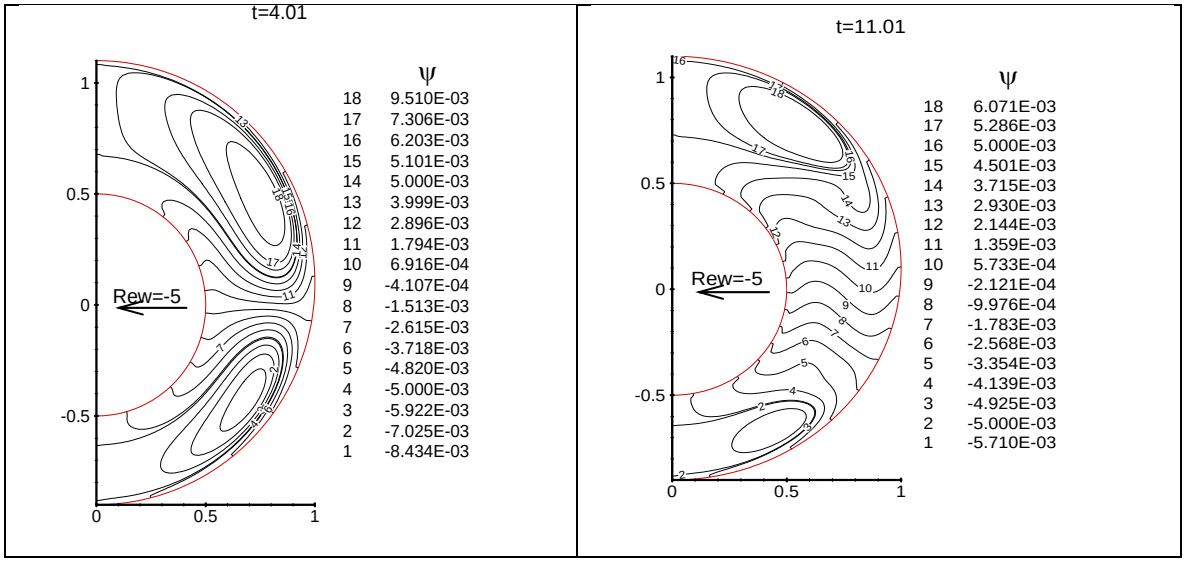

Fig.8. Contours of ψ for Re = 1000, $\mathrm{Re}_w = -5$, $\Omega_{io} = 2\sin(\pi t/2)$, $e = 0.1$

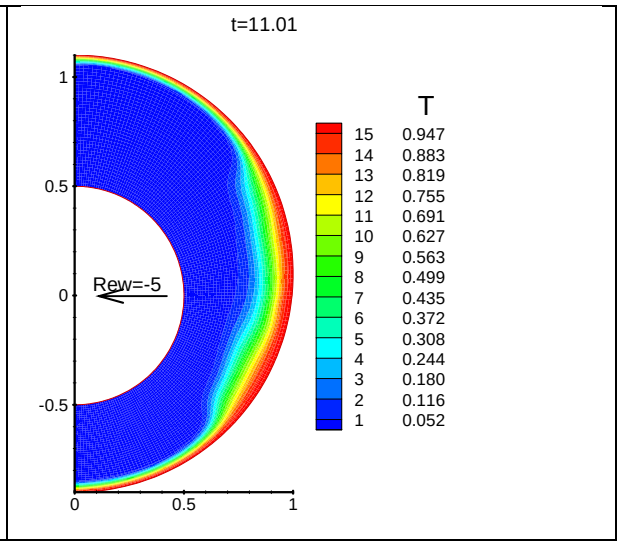

Fig.9. Contours of T for Re = 1000, $\mathrm{Re}_w = -5$, Pr = 10, $\Omega_{io} = 2\sin(\pi t/2)$, $e = 0.1$

Figures 8 and 9 present the flow field and heat transfer results for sinusoidal and exponential inner angular velocities for the case of concentric spheres (e = 0). As it is observed from Fig.21 (a) in this case ($\mathrm{Re}_w = 5$) two stagnation points exist on the streamlines at the poles and also two stagnation points are at the equator. Streamlines for $\mathrm{Re}_w = 10$ have been drawn in Fig.22 (a) and it can be seen that there are no eddies in the flow field in this case.

Conclusions

In this paper, the effects of transpiration on flow and heat transfer in an annulus between two rotating spheres (concentric and eccentric) have been studied when the spheres have time-dependent prescribed values of angular velocities. The results have been presented for various values of blowing / suction Reynolds number which indicates the strength of transpiration. Results show that increasing values of blowing or suction can be used to remove the eddies created in flow field which are as preventive means for heat transfer. It is observed, as the eddies create in upper and lower poles in eccentricity case, with decreasing the eccentricity the size of the eddies in upper pole (where the distance between two spheres is bigger) and therefore the distance between two stagnation points on upper pole decreases while in lower pole the effect is opposite so that in concentric case the size of the eddies will be equal. With increasing the blowing and suction the eddies are removed faster in lower hemisphere due to more Coriolis forces.

Temperature field results show how the blowing and suction can be used to regulate the rate of heat transfer. In eccentricity case, the diffusion of heat is more where the distance between two spheres is less. Results show that viscous dissipation effects appear near the equator because of higher velocities gradients and causes more heat diffusion in this region. Finally, the effects of blowing and suction and also the eccentricity on viscous torques are studied. It is seen that the effects of the eccentricity on viscous torque aren't considerable while the blowing and suction have considerable effects on viscous torques.

REFERENCES

[1] Howarth, L., Philosophy Magazine Series, 7, 42, p: 1308, 1951.
[2] Proudman, I., Journal of Fluid Mechanics, 1, p: 505, 1956.
[3] Lord, R.G. & Bowden, F. P., Preceding of Royal Society A 271, p: 143, 1963.
[4] Fox, J., NASA TN D-2491, 1964.
[5] Greenspan, H.P., Journal of Fluid Mechanics, 21, p: 673, 1964.
[6] Carrier, G.F., Journal of Fluid Mechanics, 24, p: 641, 1966.

[7] Stewartson, K., Journal of Fluid Mechanics, Vol. 26, part 1, p: 131-144, 1966.
[8] Pearson, C., *A numerical study of the time-dependent viscous flow between two rotating spheres*, Journal of Fluid Mechanics, Vol. 28, part 2, p: 323-336, 1967.
[9] Munson, B.R., Joseph, D.D., *Viscous incompressible flow between concentric rotating spheres*, Part I: Basic flow, Journal of Fluid Mechanics 49, p: 289-303, 1971.
[10] Douglass, R.W., Munson, B.R., and Shaughnessy, E.J., *Thermal convection in rotating spherical annuli-1. Forced convection*, Int. J. Heat and Mass Transfer, Vol. 21, p: 1543-1553, 1978.
[11] Munson, B.R., Douglass, R.W., *Viscous flow in oscillatory spherical annuli*, Journal of Physics of fluids, 22 (2), p: 205-208, Feb. 1979.
[12] Gagliardi, J.C., Nigro, N.J., Elkouh, A.F., and Yang, J.K., *Study of the axially symmetric motion of an incompressible viscous fluid between two concentric rotating spheres*, Journal of Engineering Mathematics 24, p:1-23, 1990.
[13] Jen-Kang Yang, N. J. Nigro, and A. F. Elkouh, *Numerical study of the axially symmetric motion of an incompressible viscous fluid in an annulus between two concentric rotating spheres*, I n ternational Journal for numerical methods in fluids, Vol. 9, p: 689-712, 1989.
[14] W. Ni, N. J. Nigro, *Finite element analysis of the axially symmetric motion of an incompressible viscous fluid in a spherical annulus*, International Journal for numerical methods in fluids, Vol. 19, p:207-236, 1994.
[15] Jabari Moghadam, A. and Rahimi, A.B., *A numerical study of flow and heat transfer between two rotating spheres with time-dependent angular velocities*, Journal of Heat transfer, 2008
[16] Jabari Moghadam, A. and Rahimi, A.B.,
[17] Gulwadi, S.D., Elkouh , A.F., *Effects of transpiration on free convection in an annulus between concentric porous spheres*, Journal of Engineering Mathematics, Vol. 28, p: 483-499, 1994
[18]Gulwadi, S.D., Elkouh, A.F., and Jan, T.C., *Laminar flow in an annulus between two concentric rotating porous spheres*, ACTA MECHANICA, Vol. 97, p:215-228 ,1993
[19] Press, W.H., Flannery, B.P., Teukolsky, S.A., and Vetterling, W.T., 1997, Numerical Recipes, The Art of Scientific Computing, Cambridge University Press, Cambridge.