

# An Improved Regula Falsi Method for Finding Simple Zeros of Nonlinear Equations

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## Abstract

An improved regula falsi (IRF) method based on classic regula falsi (RF) method is proposed in this paper. The method is verified on a number of test examples and numerical results confirm that the proposed method is very effective with respect to the classic regula falsi method.

**Keywords:** Regula falsi method, Nonlinear equations, Numerical examples

**Mathematics Subject Classification:** 65B99, 65G40

## 1 Introduction

The goal of this paper is to investigate the problem of finding the zeros of a nonlinear equation

$$f(x) = 0, \tag{1}$$

where  $f$  be a real-valued continuous equation on  $[a, b] \subset R$ . Suppose that  $f(x)$  has at least one zero, say  $r$ , in  $[a, b]$ . One of the oldest method for solving this function is regula falsi method. This method for solving (1) consists of successive application of the following two statements.

- (i) For the initial interval  $[a, b]$  including the zero  $r$ , take  $c$  as an approximation of  $r$  where  $c$  is the zero of the linear interpolation polynomial of  $f$  between  $a$  and  $b$ .

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- (ii) Replace the interval  $[a, b]$  with  $[a, c]$  if  $f(a)f(c) < 0$  or with  $[c, b]$  if  $f(c)f(b) < 0$ .

For a convex or concave  $f$ , the method retains one of the end points of the interval containing the zero and converges to the zero very slowly. For a good review the theory of regula falsi method and its algorithm see [4,5].

Our scheme is a predictor - corrector method that the classic regula falsi would be as a predictor and our algorithm would be as a corrector. Aslam Noor and his co-workers [1,2,3] consider some of predictor - corrector methods like Newton - Raphson or regula falsi or other classic or improved iterative methods as a predictor or corrector.

In the next section, we discuss our proposed method with some of figures that shows the efficiency of our algorithm. In the third section, we present our numerical algorithm and in the fourth section we give some numerical examples and compare it with the classic regula falsi method in the same precision.

## 2 Modified regula falsi method

Suppose that  $[a, b]$  is an initial interval that the nonlinear equation (1) has at least one zero  $r$  in it. We will have two main cases :

case (1) :  $c = \frac{af(b)-bf(a)}{f(b)-f(a)}$  is the first approximation of the root (1) with classic regula falsi method that be obtained from the linear interpolation

$$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a), \quad (2)$$

with setting  $y = 0$  and  $x = c$ . Assume that  $f(c)f(a) < 0$  and this linear interpolation intersect the line

$$y = \frac{-kf(b)}{b - a}(x - a), \quad (3)$$

which is straight line that connect points  $(a, 0)$  to  $(b, -kf(b))$ .

With intersecting (2) and (3) and eliminating  $y$ , we will have

$$x = \frac{(1+k)bf(a) - f(b)}{(1+k)f(a) - f(b)}. \quad (4)$$

If  $f(x)f(a) < 0$  then  $b$  is replaced with  $x$  (see Fig. 1) and if  $f(x)f(a) > 0$  we replace  $b$  with  $c$  and  $a$  with  $x$  (see Fig. 3)

case (2) : consider  $c = \frac{af(b)-bf(a)}{f(b)-f(a)}$  and line (2) again. Now suppose that  $f(c)f(a) > 0$  and the classic regula falsi line (2) intersect with the line

$$y - f(a) = \frac{-kf(a)}{b - a}(x - a), \quad (5)$$

that is the straight line between points  $(b, 0)$  and  $(a, -kf(a))$ . With intersecting (2) and (5) and eliminating  $y$ , we will have

$$x = \frac{(1+k)af(b) - f(a)}{(1+k)f(b) - f(a)}. \quad (6)$$

If  $f(x)f(b) < 0$ , replace  $a$  with  $x$  (see Fig. 2) and if  $f(x)f(b) > 0$ , replace  $a$  with  $c$  and  $b$  with  $x$  (see Fig. 4) and the algorithm is continued.

The parameter  $k \geq 0$  is a weight factor that has direct relation with  $f(c)$  and the inverse relation with  $b - c$  or  $a - c$  when  $f(c)f(a) < 0$  or  $f(c)f(a) > 0$  respectively.

Note that, when  $k = 0$  our method return to the classic regula falsi method. So we will have:

$$k = \begin{cases} \left| \frac{\alpha f(c)}{b-c} \right| & \text{if } f(c)f(a) < 0, \\ \left| \frac{\alpha f(c)}{a-c} \right| & \text{if } f(c)f(a) > 0. \end{cases}$$

parameter  $\alpha$  is an arbitrary number and we set  $\alpha = 1$  in our examples.

### 3 The algorithm

Step 1 : Get  $a, b$  ( initial interval ) and  $\varepsilon$  ( precision).

Step 2 : Compute  $c = \frac{af(b)-bf(a)}{f(b)-f(a)}$ .

Step 3 : If  $f(c)f(a) < 0$  set  $k = \left| \frac{\alpha f(c)}{b-c} \right|$ , compute  $x_i = \frac{(1+k)bf(a)-f(b)}{(1+k)f(a)-f(b)}$  and

if  $f(x_i)f(a) < 0$  set  $b = x_i$

else set  $b = c$  and  $a = x_i$

else if  $f(c)f(a) > 0$  set  $k = \left| \frac{\alpha f(c)}{a-c} \right|$ , compute  $x_i = \frac{(1+k)af(b)-f(a)}{(1+k)f(b)-f(a)}$  and

if  $f(x_i)f(a) > 0$  set  $a = x_i$

else set  $a = c$  and  $b = x_i$ .

Step 4: If  $|x_{i+1} - x_i| < \varepsilon$  or  $|f(x_{i+1})| < \varepsilon$ , then stop.

step 5: Set  $i = i + 1$  and go to step 2.

## 4 Numerical experiments

In all of our examples, the maximum number of iteration is  $n = 200$  and our examples are tested with precision  $\varepsilon = 1 \times 10^{-10}$ . The following stopping criteria is used for our computer programs:

- (i)  $|f(x_{i+1})| < \varepsilon$ .
- (ii)  $|x_{i+1} - x_i| < \varepsilon$ .

Table 1 presents comparison of the iteration number between RF and IRF in given precision.

Equation	Initial interval	Iteration of		$x_n$
		RF	IRF	
$xe^x - 1 = 0$	$[-1,1]$	21	6	0.5671433083
$11x^{11} - 1 = 0$	$[0.1,0.9]$	35	9	0.8041330975
$e^{x^2+7x-30} - 1 = 0$	$[2.8,3.1]$	37	7	3.0000000000
$\frac{1}{x} - \sin(x) + 1 = 0$	$[-1.3,-0.5]$	13	5	-0.6294463362
$x^3 - 2x - 5 = 0$	$[2,3]$	23	5	2.0945515532
$\frac{1}{x} - 1 = 0$	$[0.5,1.5]$	32	6	1.0000000000

Table 1: The number of iteration of RF and IRF methods for specifics precision.

In Table 2, the CPU time ( per second ) of RF and IRF methods are compared. All numerical results show here, are obtained on a pentium IV processor at 3.00 GHz.

Equation	RF	IRF
$xe^x - 1 = 0$	0.40625000	0.09375000
$11x^{11} - 1 = 0$	0.60937500	0.15625000
$e^{x^2+7x-30} - 1 = 0$	0.68750000	0.10937500
$\frac{1}{x} - \sin(x) + 1 = 0$	0.26562500	0.10937500
$x^3 - 2x - 5 = 0$	0.42187500	0.09375000
$\frac{1}{x} - 1 = 0$	0.54687500	0.07812500

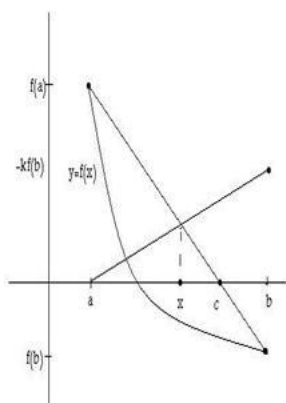
Table 2: The CPU time of RF and IRF methods.

As Tables 1 and 2 show that the CPU time and accuracy of our new regula falsi method is better than classic regula falsi method.

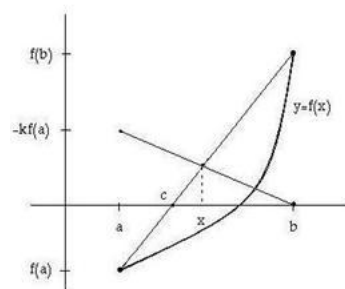
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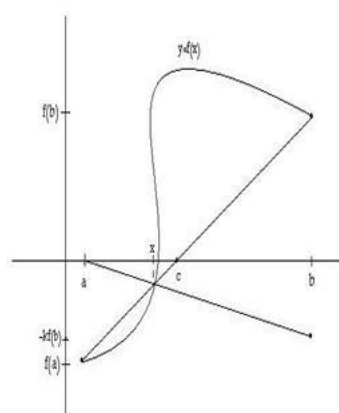
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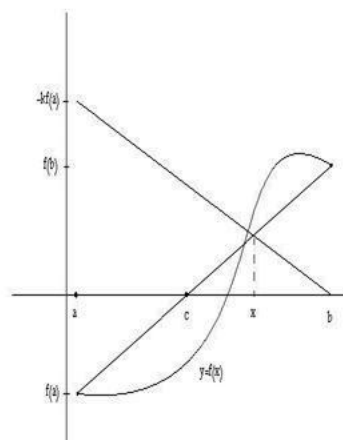
**Fig. 1**  $f(x)f(a) < 0$



**Fig. 2**  $f(x)f(b) < 0$



**Fig. 3**  $f(x)f(a) > 0$



**Fig. 4**  $f(x)f(b) > 0$

Figures for cases (1) and (2)