Robust Control of dc/dc PWM Converters : A Comparison of $\mathbf{H}_\infty$, μ , and Fuzzy Logic Based Approaches

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Abstract- ***In this paper small and large signal responses and robust control of dc/dc PWM converters are investigated. The system under study is a CUK converter as a nonlinear and variable structure plant with very complex and chaotic behavior. Results obtained from application of the H_∞ and μ , and fuzzy controllers have been presented and discussed . They show successful application of the H_∞ and μ controllers in providing an excellent robust stability and performance against the changes in the input dc voltage of the converter where the nominal averaged and linearized model of the converter is used and only small signal study is the goal. The results showed that the μ controller can provide a response which is lower conservative and more robust (in the sense of stability and/or performance). It has been shown that the under study system can be unstable and chaotic at large signal conditions where the above mentioned robust but linear controllers are used. A robust fuzzy logic based controller with a nonlinear and time varying feed forward controller has been proposed and it has been shown that it can provide an excellent response and robust performance against the changes in the converter input voltage and /or load resistance while the startup transient response of the converter is studied.***

Index Terms - dc/dc Converter, Instability, Chaos, Robust Control , Fuzzy Controller, $\mathbf{H}_\infty$ Controller , μ Controller, State Space Averaging.

1. INTRODUCTION

Dc/dc pulse width modulated (PWM) power converters have wide applications in industry . From control point of view , operation of these systems can be considered as a tracking problem so that the output quantity should follow the reference command with low transient and steady state error. The topic of control of this family of systems is very attractive and important [1-11]. Many of These systems are inherently, time varying (Variable structure), and Nonlinear systems with some complex behavior. Also the state space averaging method the well known approach for modeling dc/dc converters results in an approximate dynamical model for the system. There are some important sources of uncertainty in the modeling of these converters . Some uncertainties arise by the modeling approach (state space averaging) which provide an approximate model for low frequencies[3]. Another source of uncertainty is due to the linearization approach used. The linearized model is a function of the operating point of the converter. Any change in the reference command or input dc voltage will change the operating point of the system which will change the parameters of the linearized model. So, the robust performance and robust stability of a dc/dc converter is very important . In some cases, the non linear power converter may show a very complex and chaotic behavior. In this paper robust control of a dc/dc CUK converter which is a well-known bad behavior system has been investigated using the $\mathbf{H}_\infty$ [4], μ [5], and Fuzzy Logic Based controllers. Results show that $\mathbf{H}_\infty$ and μ controllers can provide robust performance/stability ability for a large range of input voltage while the best robust performance can be achieved by μ controller. Those controllers are linear and are designed based on the averaged and linearized model of the converter. Simulation results showed that the resulting system can represent a complex and chaotic behavior at large signal operation . Hence, a fuzzy logic based controller has been designed and its excellent performance in chaos rejection and robustness against the large changes in load resistance and/or input dc voltage has been shown.

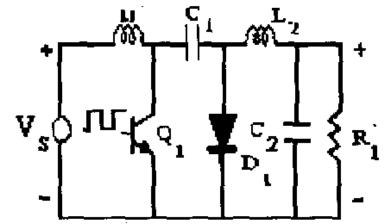


Fig. 1 : Cuk converter circuit under study.

2. SYSTEM UNDER STUDY

Fig. 1 shows the circuit of the system under study which is a dc/dc PWM CUK converter as [1]. The converter nominal input and out put

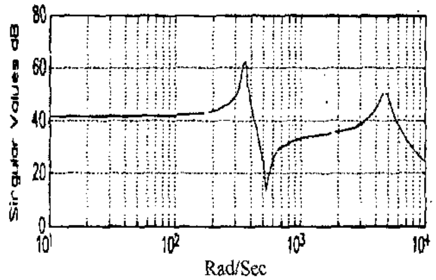


Fig. 2 : Frequency response of the converter LTI model.

voltages are $V_s = 25$ v and $V_o = -30$. Considering relation between input, output, and steady state duty cylce D as shown in Eq. (1) the resulted nominal value of the duty cycle is as $D_0 = 0.5455$.

$$\frac{V_o}{V_s} = \frac{-D_0}{1-D_0} \Rightarrow D_0 = \frac{-V_o}{-V_o + V_s} \quad (1)$$

The circuit other nominal parameters are as : Load Resistance $R = 30\,\Omega$, and Switching Frequency $f_s = 10\,kHz$. The system is non-minimum phase and with very bad behavior. Fig. 2 shows the frequency response of the converter model resulted from state space averaging and linearising techniques[3] which result an linear and time invariant (LTI) model for the system. The control input "d " and output "y" are the deviations around the nominal values of the duty cycle and output voltage respectively. The averaged model state vector X, source voltage V, duty cycle D, and output voltage Y can be decomposed into dc and ac components around the nominal values as follow where the lower case variables show the ac signals:

$$X = X_0 + x \quad (2)$$

$$V = V_s + v \quad (3)$$

$$D = D_0 + d \quad (4)$$

$$Y = V_0 + y \quad (5)$$

Transfer function of the LTI model is as follows and as seen it has two complex zeros with positive real parts, two low damped low frequency poles, and two medium damped high frequency poles. Superscript * shows the complex conjugate of the quantity.

$$G(s) = k\frac{(s-z_1)(s-z_1^*)}{(s-p_1)(s-p_1^*)(s-p_2)(s-p_2^*)} \quad (6)$$

Where:
$k = 1.219\times10^9$
$z_1 = 65.7 + j518.2$
$p_1 = -5.8 + j354.77$
$p_2 = -454.9 + j4739.1$

3. DESIGN OF $\mathbf{H}_\infty$ CONTROLLER

Weighting functions W_1 and W_2 related to the desired transfer function T(s) and sensitivity function S(s) respectively have been selected to provide good performance that is fast transient response and excellent tracking ability as well as good robustness against the external disturbance "v" and parameter variations due to changes in the operating point of the converter. A well known source of parameter variation is the changes in the value of the source voltage V_s. Where V_s changes, the closed loop system should change amount of D to keep the output voltage near its nominal value. Weighting functions W_1 and W_2 have been selected via some trail and error as shown in (7) and (8):

$$W_3^{-1} = 1.3\frac{1+s/3000}{1+s/500} \quad (7)$$

$$W_1^{-1} = 0.02\frac{1+s}{1+s/11000} \quad (8)$$

Fig. 3 shows the resulting transfer function T(s) and sensitivity function S(s) while Fig. 4 shows the unit step response of the close loop system applied to the reference input , disturbance input(v) at nominal point of operation. This figure also shows the response of the system where the source voltage changes from 25 to 15 and 30 volts. The controller has been placed in cascade with the converter LTI model and the closed loop system is with unity gain in the feedback path. The resulting system has robust stability within a wide range of input voltage(15-30 volts) while the nominal input and output voltages are 20and -30 volts respectively. Simulation results show that the system is stable within this range of the source voltage and is unstable for other values. Since the converter is non minimum phase, any increase in the bandwidth of the system may result in some peak in the S(s) which causes instability. On the other hand, very low bandwidth results in a conservative design and provides a poor tracking ability. The authors' experience showed that designing a stabilizing controller with more wide stable range for the source voltage is very difficult.

4. DESIGN OF μ CONTROLLERS

$\mathbf{H}_\infty$ design is very conservative. Hence for achieving a controller to be lower conservative we have used μ controllers. Weighting functions for performance and stability have been selected via trail and error. Results of two different designs have been reported. In the first, the robust performance while in the next, the robust stability are the goal. In the first system, the closed loop system shows a robust performance where the input voltage changes between 10 and 35 volts. In the second , the system shows robust stability within a wide range of input voltage(5-100 volts) .

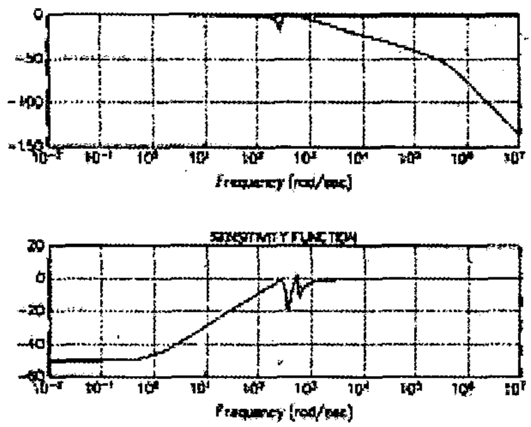
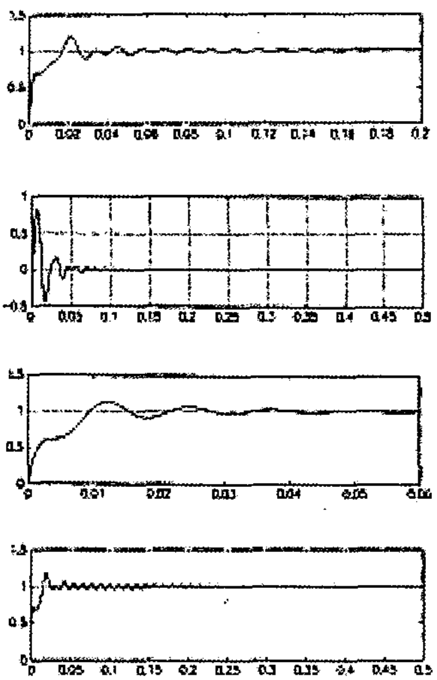


Fig. 3 : up :T(s) and down : S(s) of the $\mathbf{H}_\infty$ controlled system.
Vertical axes: dB



Time(Seconds)

Fig. 4 : Response of the $\mathbf{H}_\infty$ controlled system. Up to Down : step signal applied to the reference(Vs=25) , step signal applied to the disturbance input(Vs=25), step signal applied to reference(Vs=15), and step signal applied to the reference(Vs=30).Vertical axes: volts

A) Robust performance Design

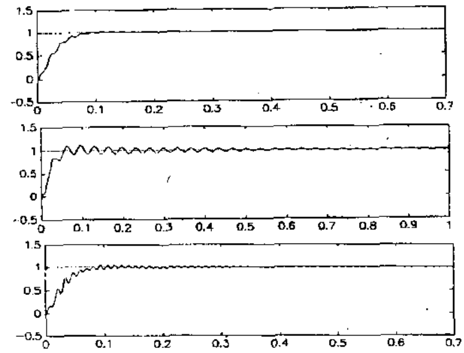
In this design, the weighting functions W_T , W_s , and W_I related to transfer function, sensitivity function , and uncertainty model respectively have been selected as follow :

$$W_T = 25\frac{s+200}{s+100000} \tag{9}$$

$$W_S = 1/6\frac{s+300}{s+0.5} \tag{10}$$

$$W_I = 100\frac{s+200}{s+50} \tag{11}$$

D-K iterative approach has been used [6] and the resulting step response of the system has been shown in Fig. 5. As seen the system presents a slower but robust performance with good tracking ability where the source voltage changes from 10 to 35 volts.

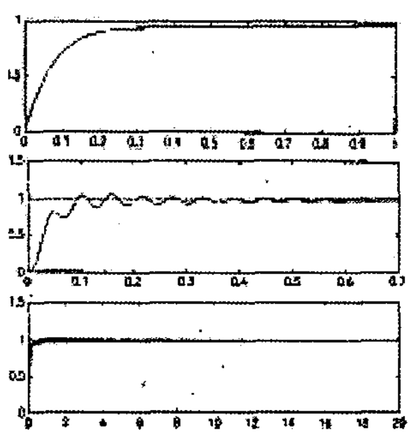


Time(Seconds)

Fig. 5 : Response of the μ controlled system (Design #1). Up to Down : step signal applied to the reference(Vs=25, 10, and 35 volts)
Vertical axes: volts

B) Robust Stable Design

The weighting functions have been selected as (9)-(11) but the uncertainty model W_I introduced by (11) has been multiplied by 10 to provide a more robust response. The resulting step response of the system has been shown in Fig. 6. As seen the system presents a slow response but it is stable for a very wide range of source voltage from 5 to 100 volts. Fig. 7 shows the resulting T(s) and S(s) of the system with the open loop step response which shows the slow and oscillatory response of the system under control. The results show the superiority of the μ controller .



Time(Seconds)

Fig. 6 : Response of the μ controlled system (Design #2). Up to Down : step signal applied to thereference(Vs=25, 5, and 100 volts.
Vertical axes: volts

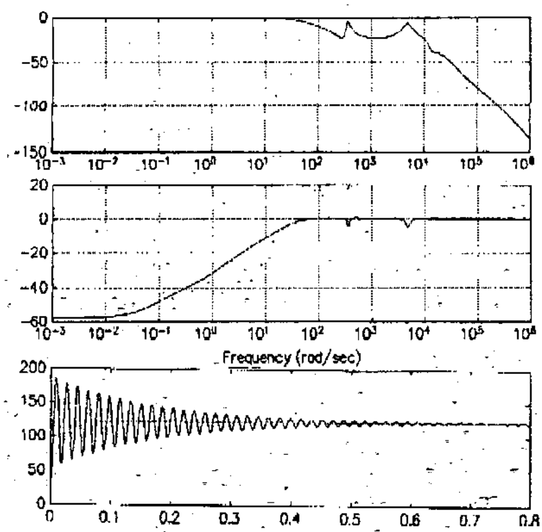


Fig. 7 : up to down: T(s) , S(s) responses of the μ controlled system.(design #2), and step response of the open loop system. Vertical axes: dB, Horizontal Axes: red/sec or seconds.

5.LARGE SIGNAL BEHAVIOR AND CHAOS

Fig. 8 shows the block diagram of the converter system under study with a conventional PI controller which is the most popular control scheme for control of dc/dc converters in practice. Simulation results show unstable and chaotic response of the system in many cases where the exact mathematical model of the converter is used instead of the approximate LTI model and large signal behavior of the closed loop system has been considered.

Fig. 9 shows the startup response of the open loop system together with the response of the closed loop system with the following parameters for the PI controller: $K_I = 20$ K_p=1. It also shows the state trajectory of two state variables of the system (Voltages of capacitors V_{c_1} and V_{c2}).

Simulation Results show that some of the above investigated linear controllers provide very poor and chaotic response in large signal conditions where the converter is being started up[7,8]. Fig. 10 shows the large signal startup of the system with the $\mathbf{H}_\infty$ controller presented in section 3 and as seen the system is unstable and chaotic. The results show that linear controllers may not be a good system for controlling dc/dc converter systems.

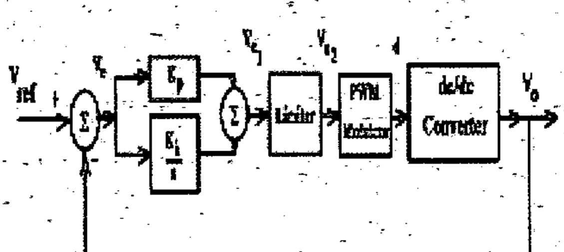


Fig. 8 : Block diagram of the PI control system

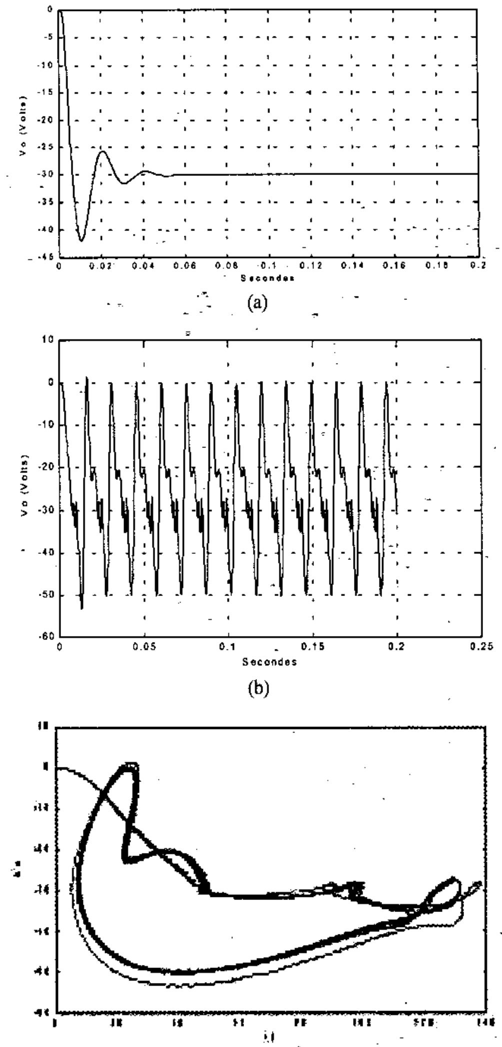


Fig. 9: a) startup response of the open loop system., b) startup response of the closed loop system with PI controller, c) State trajectory of system , V_{c_2} versus V_1

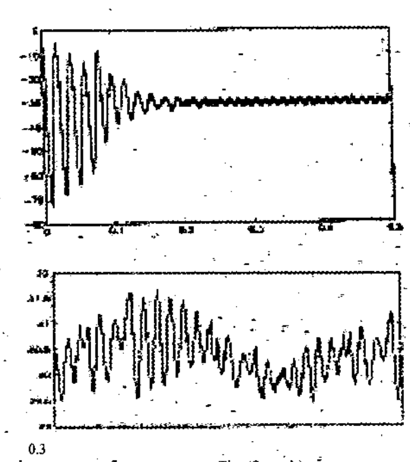


Fig. 10: up) startup response of the open loop system., b) enlarged figure from .3 to 0..31 seconds. Vertical axes: volts

6.DESIGN OF FUZZY LOGIC BASED CONTROLLER

For such applications where the under control system is very nonlinear and may represent chaotic behavior , it seems that a nonlinear controller such as fuzzy controller[9-11] may work more proper. As shown in Fig. 11, a Proportional-Derivative (PD_ Fuzzy controller with conventional PI controllers for achieving zero error at steady state and a nonlinear feed forward controller has been used. A single edge PWM strategy has been used for controlling the switching of the converter.

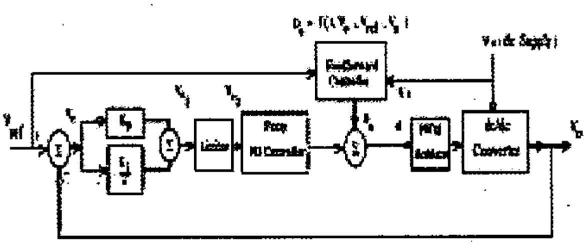


Fig. 11 : Block diagram of the fuzzy logic based control system of the Cuk converter.

The fuzzy logic based controller receives the signal (x)and its derivative(x') from output of the PI controller. There are the following parameters can be tuned for achieving a good response:

-PI controller parameters K_I and K_p

-Input membership functions of x and x'.

-Output membership functions.

-Rule Base parameters.

-Nonlinear feed forward controller to reduce the effect of V_s .

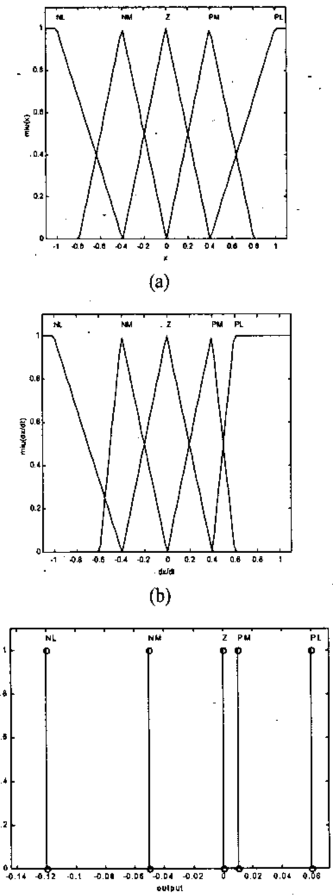


Fig. 12: Fuzzy controller membership functions. a) input x, b) input x', c) output d.

A. Fuzzy controller design

Based on the conceptual information and after some try and error, the rule base has been formed as shown in table 1 while the table elements meanings are:

NL=Negative Large

NM=Negative Medium

Z=Zero

PL=Positive Large

PM= Positive Medium

Table1: Rule base of Fuzzy Controller

$\downarrow x\ x' \rightarrow$	NL	NM	Z	PM	PL
NL	NL	NL	NL	NM	NM
NM	NL	NL	*NL*	NM	Z
Z	NM	Z	Z	Z	PM
PM	Z	*Z*	PM	PM	PL
PL	*NL*	*PM*	PL	PL	PL

Also the inputs and output membership functions have been selected via try and error as shown in Fig 12 to achieve a fast transient response at startup.

B. Feed forward Controller design

For improving the transient response of the system a feed forward controller has been proposed to tune the amount of D_0 at input of the modulator. As addressed earlier, this parameter depends on the source voltage V_s and the desired output voltage V_{ref}. Authors' experiences show that a time varying controller which increases D_0 to the final value continuously provides a better response. The following equation shows the mechanism applied to the feed forward controller where $D_0(k)$ is the nominal duty cycle at k -th switching period.

$$D_0(k+1)=\begin{cases} D_0(k)+\frac{k}{200}*(\frac{-V_{ref}}{-V_{ref}+V_s}) & if\ D_0(k+1)\le(\frac{-V_{ref}}{-V_{ref}+V_s}) \\ (\frac{-V_{ref}}{-V_{ref}+V_s}) & if\ D_0(k+1)\ge(\frac{-V_{ref}}{-V_{ref}+V_s}) \end{cases}$$

Finally, amounts of PI controller parameters are as K_I=1 and K_p=100. Fig. 13 shows the control system response for the nominal V_s and R(Load resistance) together with the responses of the system for different values of R and V_s. The results obtained show the robust behavior of the system against the changes in R and V_s.

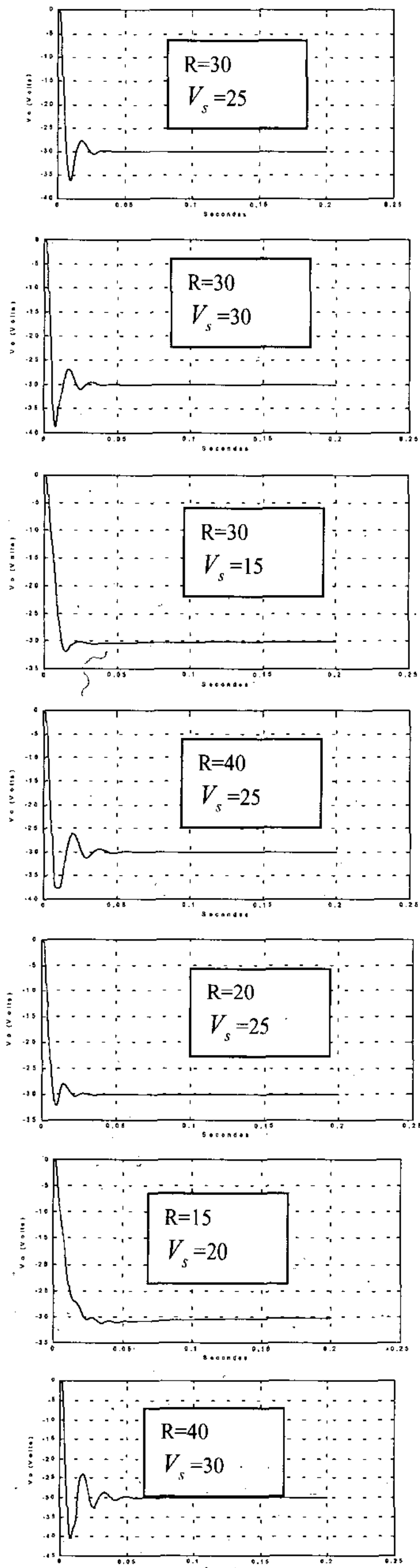


Fig. 13: Large signal response of the fuzzy controlled system at startup for different values of the load resistance and source voltage.

The results show a stable large signal behavior with good transient and steady state response . Also the system shows a wide range of robustness against the changes in the load resistance and input voltage. The results show the superiority of the Fuzzy controllers over linear controllers.

7.CONCLUSION

In this paper robust control of dc/dc converters in small and large signal conditions were investigated. The system under study was a CUK converter as a nonlinear and variable structure plant with very complex and chaotic behavior . Comparative study of three important types of robust controllers that is $\mathbf{H}_\infty$, μ , and fuzzy controllers showed the excellent robust stability and performance of $\mathbf{H}_\infty$ and μ controllers where the small signal response of the converter is considered while the μ controller can provide more robust response. It was shown that the under study system can be unstable and chaotic at large signal conditions where the above mentioned robust but linear controllers are used . A robust fuzzy logic based controller with a nonlinear and time varying feed forward controller was proposed and it was shown that it can provide an excellent response and good robust performance while the startup transient response of the converter is studied. The results obtained show the superiority of the fuzzy controller to control of nonlinear power converters.

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