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## Active noise cancellation of variable frequency narrow band noise using mixture of RLS and LMS algorithms

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**Abstract:** Due to the good tracking behaviour of the LMS adaptive filter in a noisy environment, the FX-LMS algorithm is proposed in the literature as a method of active noise control, ANC. But each of the LMS and RLS algorithms have their own advantages and disadvantages. In this paper, a new approach based on a mixture of the RLS and LMS algorithms, RLMS, is presented. The optimum weights of the mixture are derived and it is proved that the MMSE of the proposed system is reduced compared to those of the RLS and LMS algorithms. Then, the proposed RLMS algorithm is employed for active noise cancellation to form the FX-RLMS algorithm, in a duct. Experimental results show better performance of the RLMS algorithm compared to both the RLS and LMS algorithms of convergence and tracking behaviour in the system identification problem and noisy chirp tracking. The FX-RLMS algorithm shows better results in active noise cancellation compared to the FX-LMS algorithm.

**Keywords:** LMS; least mean square; RLSs; recursive least squares; FX-LMS algorithm; mixture of adaptive filter; ANC; active noise control.

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## 1 Introduction

In recent years, acoustic noise cancelling by active methods, owing to its numerous applications, has been in the focus of interest of many researches. Contrary to the passive method, it is possible by using the active method to suppress or reduce the noise in a small space particularly in low frequencies (below 500 Hz) (Prandoni and Vetterli, 1998; Sadoghi Yazdi et al., 2004). ANC was introduced for the first time by Lveg (1936) for suppressing the noise in a duct (Elliott and Nelson, 1993). In the active control method by producing a sound with the same amplitude but with opposite phase, the noise is removed. For this purpose, the amplitude and phase of the noise must be detected and inverted. The developed system must have the adaptive noise control capability (Strauch and Mulgrew, 1998). In usual manner, an FIR filter is used in ANC whose weights are updated by a linear algorithm (Das and Panda, 2004; Das et al., 2006). Using the linear algorithm of the LMS is not possible owing to the non-linear environment of the duct and appearing the secondary path transfer function  $H(z)$ . Hence, the FX-LMS algorithm is presented in which the filtered input noise  $x'(n)$  is used as an input to the algorithm (Tan and Jiang, 2001; Sicuranza and Carini, 2004). The notable points in the ANC are as follows:

- The duct length and the distance between the system elements are such that the system becomes causal (Prashanth et al., 2008).
- Regarding the speaker response, no decrease will be obtained in frequencies below 200 Hz (Sadoghi Yazdi et al., 2004). Also, passive techniques for reducing the noise in frequencies below 500 Hz have not been successful (Prandoni and Vetterli, 1998; Sadoghi Yazdi et al., 2004). Therefore, the ANC systems are used in the range of 200–500 Hz and above 500 Hz.

The existence of non-linear effects in the ANC environment complicates the use of the FX-LMS algorithm and similar algorithms. Divergence or slow convergence is among these difficulties. Some of research works in recent years (2008–2009) are reviewed in the next section.

### 1.1 Recent works over ANC

ANC has gained a lot of significance in the recent past because of its potential use in low-frequency ANC applications (Elliott and Nelson, 1993). FX-LMS algorithm is one of the simplest methods for ANC applications. Because of the poor performance (Strauch and Mulgrew, 1998) of the FX-LMS algorithm in the case of non-linear noise processes, two non-linear adaptive algorithms, namely Filtered-S LMS (FSLMS) algorithm (Das and Panda, 2004; Das et al., 2006) and Volterra Filtered-X LMS (VFXLMS) algorithm (Tan and Jiang, 2001; Sicuranza and Carini, 2004), have been proposed. Computationally efficient fast implementation of these two algorithms is presented in Prashanth Reddy et al. (2008). The concept of reutilising a part of the computations performed for the first sample while computing the next sample, for a block length of two samples, is exploited to implement the fast and exact versions of the FSLMS and VFXLMS algorithms. Akhtar et al. (2009) investigate new methods for online Feedback Path Modelling and Neutralisation (FBPMN) in multichannel ANC systems for improving the degraded performance of strong acoustic feedback. The modification is to combine the role of the FBPM and FBPN filters into one FBPMN filter, which results in reduced computational complexity. Improved convergence performances with the ANC structure proposed in Akhtar et al. (2006) were obtained by introducing the delay compensation, and by removing the auxiliary noise from the error signal of the control filter. They used two adaptive filters, one for adapting the control filter and one for modelling the secondary path. Carini and Malatini (2008) introduce two improvements in the feedforward ANC system with online secondary path modelling developed by Akhtar et al. (2006). First, optimal variable step-size parameters are derived for the adaptation algorithms of the secondary path-modelling filter and of the control filter. Second, a self-tuning power scheduling for the auxiliary noise is introduced. A few papers like Flotte-Hernández et al. (2008) have worked on real constructed ANC systems where LMS algorithms were used to achieve noise suppression.

In this paper, the FX-LMS and FBFXLMS algorithms were implemented and tested to cancel noise over a prototype of a duct network using DSP processors.

In this work, a new structure is substituted to the LMS and the FX-LMS algorithm is improved to the FX-RLMS algorithm.

## 1.2 Related work in the adaptive filtering theory

Adaptive filtering is employed in a variety of applications to help modelling of time-variations of system parameters. In lack of a priori knowledge of the statistical model of the input signal, a wide range of algorithms has been developed. Among these, the LMS (Widrow and Stearns, 1984) algorithm is very attractive, as it provides an efficient, robust and low-complexity solution. Also, the simplicity of the LMS algorithm has made it an important benchmark for other algorithms. The ability of the LMS algorithm to operate in a non-stationary environment has been investigated by many authors (Widrow and Walach, 1984; Macchi, 1986; Farhang-Borojeny and Gazor, 1996). However, the slow convergence of the LMS algorithm for inputs with large eigenvalue spreads may lead to the use of the RLS algorithm (Haykin, 1996). On the other hand, the low tracking capability of the RLS algorithm in noisy environments (Haykin, 1996) makes it impractical to be used as a suitable adaptive filtering algorithm in low SNRs.

Historically, several methods have been developed to improve the performance of the LMS and RLS algorithms by combining both (Ysebaert et al., 2003; Yu and Chung Ko, 2003; Huang et al., 2008). Reduction of the complexity by combining RLS and LMS is considered in Ysebaert et al. (2003) wherein, a part of Kalman vector of RLS algorithm is updated with the LMS algorithm. Because of its application for per-tone equalisers, this method works in frequency domain and its implementation is too complicated. Owing to utilisation of Kalman filters, this method is restricted only to autoregressive signals. Yu and Chung Ko (2003) and Huang et al. (2008) are two different configurations of cascaded RLS and LMS. To solve the slow convergence problem of the LMS algorithm, a low-order RLS predictor is cascaded prior to the LMS predictor in Yu and Chung Ko (2003). Although Yu and Chung Ko (2003) is one of good research works on mixture of RLS and LMS for lossless compression but the tracking performance and reliability of the system are not considered. Huang et al. (2008) use cascaded RLS–LMS Predictor in MPEG-4 lossless audio coding. This research is constituted from several cascaded RLS, LMS and DPCM predictors. Therefore, the implementation complexity and amount of computations, owing to applying of high-order LMS and RLS predictors, are too much. In Oikawa and Tetsuya Shimamural (2006), a kind of parallelisation of RLS and LMS is discussed. It used two estimators. The first of which is the LMS and the second one is the RLS. This configuration is not really parallel, because the error signal of the first estimator is used as the desired signal for the

second estimator. Moreover, unlike our method, it does not have any combination of RLS and LMS at the end. Schober and Gerstacker (2001) are not talking about combination of RLS and LMS with each other. It has depicted the efficiency of separate combination of RLS and LMS with NSE against carrier phase variations in receivers. None of the above-mentioned works applied for automatic noise cancellation in a duct. Briefly, in this method, the input is decorrelated by using a suitable transformation before applying the LMS adaptation in the frequency domain (Haykin, 1996) or time domain (Oikawa and Tetsuya Shimamural, 2006; Schober and Gerstacker, 2001; Mboup et al., 1994). In Hansler (1990), the probability density function of the signal and error was utilised in the RLS algorithm. The MAP<sup>1</sup> estimator is cascaded to RLS and reduction of MSE<sup>2</sup> was obtained.

We proposed a new combination of RLS and LMS for ANC. Our three principles here are:

- a When comparing the LMS and RLS algorithms individually, we have the LMS as a better tracker while the RLS has faster convergence speed. We are seeking for a combination that first we can benefit from the primary fast convergence speed of the RLS algorithm, and second, we can have the good tracking performance of the LMS algorithm after convergence.
- b An efficient system is the one that is adjustable for different situations. So, we want to design a system that according to application, one of the RLS and LMS methods plays the dominant role in the ANC. In other words in systems with faster convergence,  $w_{RLS}$ , and for systems with more tracking ability,  $w_{LMS}$ , should be more pronounced.
- c Increasing the reliability of the whole system is our last goal in mixture of the LMS and RLS algorithms. In a real constructed system, it is possible to have failure in one of RLS and LMS blocks, which means one of them is not working. In such a situation, we should have a system that is still workable.

The proposed method in this paper, mixture of RLS and LMS, namely the RLMS<sup>3</sup> algorithm has a better tracking performance and a lower MMSE<sup>4</sup> compared with the RLS and LMS algorithms. The proposed RLMS algorithm is configured for the ANC problem.

Section 2 is devoted to the proposed RLMS algorithm. Section 3 of the paper concerns the investigation of the ANC in a duct. In Section 4, application of the proposed RLMS algorithm is presented in the system identification problem, the noisy sinusoidal chirp tracking and ANC and finally, conclusions are derived in the final.

## 2 RLMS algorithm

We combine the LMS and RLS algorithms in a parallel form. At first, we briefly review the LMS and RLS algorithms.

### 2.1 The LMS algorithm

The LMS algorithm is an important member of the family of gradient algorithms. A significant feature of the LMS algorithm is its simplicity and good tracking properties in identification problem at low SNRs. The LMS is a linear adaptive filtering algorithm that consists of a filtering process and an adaptation process according to the following equations:

Filtering process:

$$y_k = X_k^T W_k. \quad (1)$$

Adaptation process:

$$W_{k+1} = W_k + \mu e_k X_k,$$

where

$$e_k = d_k - y_k. \quad (2)$$

The weight vector of the estimator at time index  $k$  is  $W_k = [W_1, \dots, W_L]^T$  and  $X_k = [x_1, \dots, x_L]^T$  the  $L$  element vector of the samples of a buffered data sequence, which is a stationary random process, and  $L$  is the number of filter taps and  $e_k$  is the estimation error and  $d_k$  represents the desired response and  $\mu$  is the step size.

### 2.2 The RLS algorithm

The RLS filter is an adaptive, time-update version of the Wiener filter. Its goal is to minimise the weighted sum of

the squared error, i.e., the error function in the time domain obtained from equation (3)

$$\epsilon_k = \sum_{i=1}^k \lambda^{k-i} e_i^2, \quad (3)$$

where  $e_k$  is the error signal,  $e_k = d_k - X_k^T W_k$  and  $\lambda$  is the forgetting factor. The filter weights are obtained as,

$$W_{k+1} = W_k + R_k^{-1} X_k e_k, \quad (4)$$

where  $R_k$  is the input autocorrelation matrix and its inverse,  $R_k^{-1}$  is obtained recursively from the following equation (Haykin, 1996),

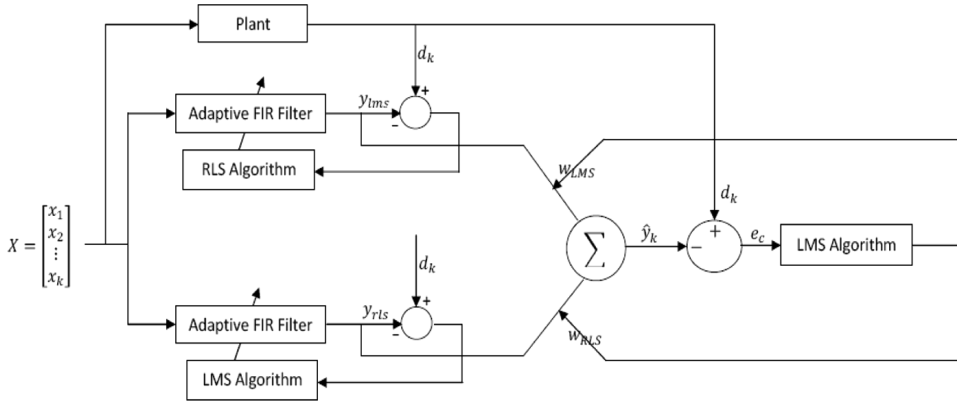
$$R_k^{-1} = \lambda^{-1} R_{k-1}^{-1} - \frac{\lambda^{-2} R_{k-1}^{-1} X_k X_k^T (R_{k-1}^{-1})^T}{1 + \lambda^{-1} X_k^T R_{k-1}^{-1} X_k}. \quad (5)$$

### 2.3 The proposed RLMS algorithm

We combine the LMS and RLS algorithms in a parallel form, as shown in Figure 1. In a system identification configuration, the outputs are fed to an adaptive linear combiner.

As depicted in Figure 1, we have increased the reliability of the whole system with parallelisation of the LMS and RLS. In real constructed systems, with failure in one of the RLS and LMS blocks, the whole system is still workable in the noise cancellation configuration, although with a reduced efficiency.

Figure 1 The configuration for RLMS algorithm, in system identification problem



In the following equations, the optimum weights of the combiner are derived and it is proved thopat the MMSE of the RLMS is decreased compared with those of the RLS and LMS algorithms.

$$e_c = d - (w_{LMS} y_{LMS} + w_{RLS} y_{RLS}) \quad (6)$$

where  $e_c$  is the error output of the proposed system and  $d$  is the desired signal and  $w_{LMS}$  and  $w_{RLS}$  are weights by which the outputs of the LMS and RLS algorithms are weighted, respectively.

$y_{LMS}$  and  $y_{RLS}$  are outputs of the LMS and RLS filters, respectively.

$$y_{LMS} = d - e_{LMS} \quad (7)$$

$$y_{RLS} = d - e_{RLS}. \quad (8)$$

$e_{LMS}$  and  $e_{RLS}$  are the output errors of the LMS and RLS algorithms, respectively. By substitution of equations (7) and (8) into equation (6), we obtain:

$$\begin{aligned} e_c &= d - w_{LMS} (d - e_{LMS}) - w_{RLS} (d - e_{RLS}) \\ &= d(1 - w_{LMS} - w_{RLS}) + w_{LMS} e_{LMS} + w_{RLS} e_{RLS}. \end{aligned} \quad (9)$$

Assuming  $w_{LMS} + w_{RLS} = 1$  and for convenience, we let  $w = w_{LMS}$  then,

$$e_c = we_{LMS} + (1-w)e_{RLS}. \quad (10)$$

Assuming  $E\{e_{LMS}e_{RLS}\} = 0$  and taking expectation from squares of both sides of equation (10), we have,

$$E\{e_c^2\} = w^2 E\{e_{LMS}^2\} + (1-w)^2 E\{e_{RLS}^2\} \quad (11)$$

or equivalently,

$$\xi_c = w^2 \xi_{LMS} + (1-w)^2 \xi_{RLS}. \quad (12)$$

For finding the optimum weights, we take the derivative of the above-mentioned equation,

$$\frac{\partial}{\partial w} \xi_c = 0 \quad (13)$$

$$w_{opt} = \frac{\xi_{RLS}}{\xi_{RLS} + \xi_{LMS}}, \quad \xi_{cmin} = \frac{\xi_{LMS} \xi_{RLS}}{\xi_{RLS} + \xi_{LMS}}.$$

In the above-mentioned formulae, the MMSE of the proposed approach is lower than those of both the LMS and the RLS algorithms. In practice, combination of the RLS and LMS may be done dynamically with LMS method according to the following equation (14):

$$\begin{bmatrix} w_{LMS} \\ w_{RLS} \end{bmatrix}_k = \begin{bmatrix} w_{LMS} \\ w_{RLS} \end{bmatrix}_{k-1} + \mu_c \begin{bmatrix} y_{lms} \\ y_{rls} \end{bmatrix}_k e_{ck}, \quad (14)$$

where  $\mu_c$  is the step size and  $e_{ck}$  is the output error in  $k$ th sample.

Because of automatic tuning of the weighting system according to equation (14), it is possible to design a system where one of the RLS and LMS methods plays the dominant role in the ANC. For systems with faster convergence,  $w_{RLS}$  and for systems with more tracking ability  $w_{LMS}$ , are more pronounced.

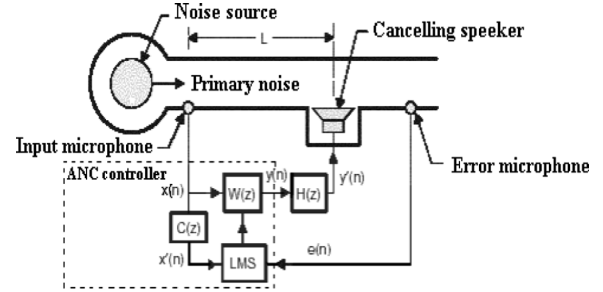
We utilise the proposed RLMS algorithm in the ANC application but, before explaining the proposed FX-RLMS algorithm, the FX-LMS algorithm is introduced in the next section.

### 3 Principle of active noise control in a duct

If we assume that the noise propagates in a one-dimensional form, then it is possible to use a single channel ANC for noise cancellation. For simulation and implementation of this system, a narrow duct is used as in Figure 1. According to Figure 1, the primary noise before reaching to the speaker is picked up by the input microphone. The system uses the input signal for generating the noise cancelling signal  $y(n)$ . The generated sound by the speaker gives rise to a reduction in the primary noise. The error microphone measures the residual signal,  $e(n)$ , which can be minimised using an adaptive filter, which is used for identifying the duct's transfer function. Because of using the input and error microphones, we must take into account some functions, which are known as the secondary path effects. In such a system, usually for cancelling the noise, the FX-LMS algorithm, Figure 2, and equation (1) are considered

(Prandoni and Vetterli, 1998; Elliott and Nelson, 1993; Akhtar et al., 2006, 2009). The vector  $x'(n)$  is a filtered version of the vector  $x(n)$  using LMS adaptive filter (equation (2)).

**Figure 2** Using the FX-LMS algorithm in a single channel ANC system



In the above-mentioned figure,  $C(z)$  is an estimation of  $H(z)$ , which can be obtained by some off-line techniques (Flotte-Hernández et al., 2008). The considerable points in the execution of the FX-LMS are as follows

- cancelling the broadband noise needs a filter of high order, which increases the duct length (Flotte-Hernández et al., 2008)
- to choose the proper step size, we need knowledge of statistical properties of the input data (Kuo and Morgan, 1999; Eriksson et al., 1987)
- to ensure the convergence, the step size is chosen small; hence, the convergence speed will be low and the performance will be weak
- for executing the above-mentioned algorithm, we need to estimate the secondary path
- non-linear behaviour of this system stimulated new researches on developing algorithms in ANC.

Increasing the speed of convergence of the LMS algorithm is the main concern in ANC. So, we increase the convergence speed of the LMS algorithm by mixing it with the RLS algorithm and substitute in filtered input LMS algorithm. We proceed to applying the mixture of the RLS and LMS algorithms in system identification, noisy chirp tracking and ANC in the next section.

### 4 Applications of the RLMS algorithm

We use the RLMS algorithm in identification and in noise reduction from noisy chirp sinusoid problem and, finally, FX-RLMS algorithm is proposed for ANC application.

#### 4.1 Using the RLMS algorithm in identification and noisy chirp tracking

In simulation of RLMS for identification and noise reduction problems, the forgetting factor of the RLS algorithm,  $\lambda$ , is set to 0.5 for increasing the tracking ability

and the step size of LMS algorithm is set to 0.06 and  $\mu_c$  in equation (14) is 0.02.

#### 4.1.1 The system identification problem

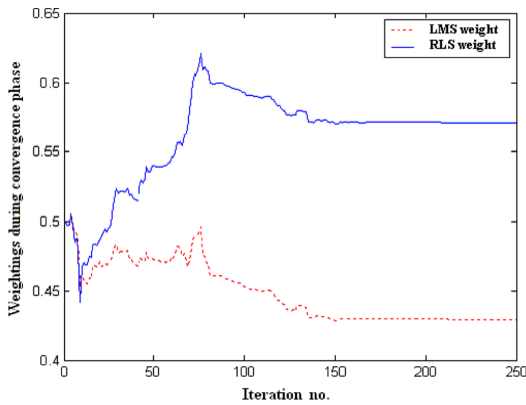
We study the behaviour of the proposed algorithm in two different conditions, stationary and non-stationary environments. For the stationary environment, the error-performance surface is fixed and the essential requirement is to seek the minimum point of that surface. But, in a non-stationary environment, minimum point of the error surface changes with statistical variations of the input. Therefore, adaptive filter must track these variations and optimum weights of filters must be changed in adaptation process as well. It is assumed that optimum weights of the plant change according to a first-order Markov process as (Haykin, 1996),

$$W_{opt(k+1)} = aW_{opt(k)} + \eta \quad (15)$$

where  $a$  is a constant and  $\eta$  is an AWGN.<sup>5</sup>

We expect contribution of RLS algorithm be higher than LMS algorithm in seeking optimum weights at convergence phase in RLMS algorithm, because of slow convergence of the LMS algorithm and fast convergence of the RLS algorithm. Results of the simulation for an identification problem are plotted in Figure 3 at convergence phase. Figure 3 shows the time evolution of the respective weightings for outputs of the LMS (i.e.,  $w_{LMS}$ ) and RLS (i.e.,  $w_{RLS}$ ) filters. From Figure 3, we can conclude that during the convergence phase,  $w_{RLS}$  begins to increase while  $w_{LMS}$  decreases.

**Figure 3** LMS and RLS weightings during the convergence phase (see online version for colours)

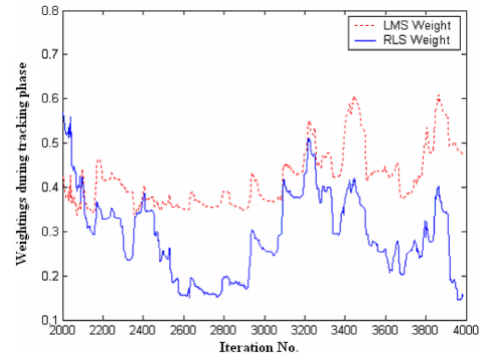


In the tracking phase of the RLMS algorithm, we expect  $w_{RLS} < w_{LMS}$  in finding optimum weights, because of good tracking of LMS algorithm in low SNR (Haykin, 1996). To check the tracking behaviour of the proposed system, optimum weights of the plant are changed according to equation (15). As seen in Figure 4, in tracking phase  $w_{LMS}$  is bigger than  $w_{RLS}$ .

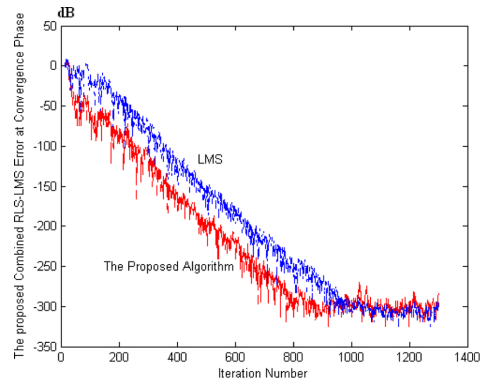
We conclude from the above-mentioned observations that the mixture of the LMS and RLS, i.e., RLMS, according to the proposed scheme has two notable advantages:

- In the convergence phase, the RLS weight ( $w_{RLS}$ ) is greater than the LMS weight ( $w_{LMS}$ ), so we expect that the convergence speed to be higher than that of the LMS algorithm (Figure 5).
- In the tracking phase for the system identification problem,  $w_{LMS}$  is greater than  $w_{RLS}$  (Figure 4). Also in a different SNR, the MSE of RLMS is less than both the LMS and the RLS algorithms (Figure 6).

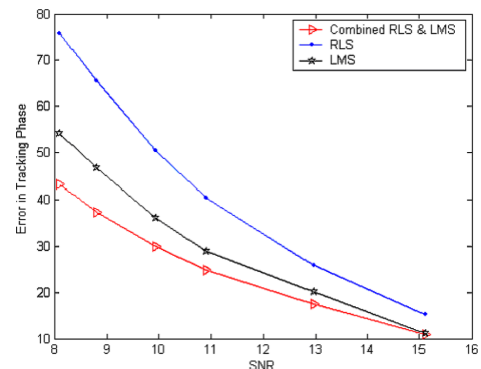
**Figure 4** LMS and RLS weightings during the tracking phase (see online version for colours)



**Figure 5** Learning curve of the proposed and LMS algorithms (see online version for colours)



**Figure 6** MSE vs. SNR in the tracking phase (see online version for colours)



#### 4.1.2 Noisy chirp tracking

We used a dynamic mixture of RLS and LMS in a noise reduction application. Adaptive recovery of a chirp sinusoid buried in noise is a standard method because the chirp sinusoid represents a well-defined form of non-stationarity.

In this experiment, we consider the tracking of a chirped sinusoid. The chirped input signal is given by:

$$S(k) = \sqrt{P_s} \exp(j[(2\pi f_c + \psi k/2)k + \varphi]) \quad (16)$$

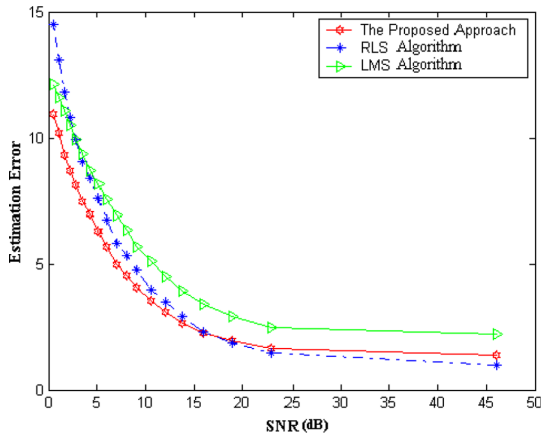
where  $\sqrt{P_s}$  denotes the signal amplitude,  $f_c$  is the centre frequency,  $\psi$  is the chirp rate and  $\varphi$  is an arbitrary phase shift. The signal  $S(k)$  is deterministic but non-stationary because of the chirping.  $S(k)$  is added with noise  $n(k)$ , then tracking of the noisy chirp is a benchmark for testing the RLMS and LMS and RLS algorithms. The SNR is denoted by:

$$\text{SNR} = 10 \log \left( \frac{\sqrt{P_s}}{A_n} \right) \quad (17)$$

where  $A_n$  is the amplitude of the noise.

The estimation error for 1001 samples for 2 s with 1 kHz sample rate of chirp is shown in Figure 7. In low SNR, the proposed method is better than the RLS and LMS algorithms, while in SNR bigger than 20dB, RLS is slightly better than the proposed method. Thus, we propose that for low SNR environment, a dynamic mixture of RLS and LMS is used for noise reduction.

**Figure 7** Estimation error vs. SNR (see online version for colours)



#### 4.2 Using the RLMS algorithm in ANC

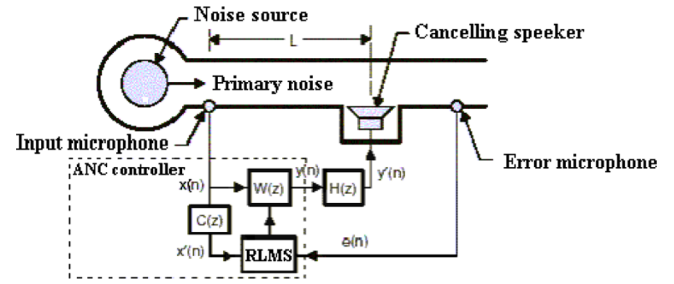
The present network is used to actively cancel the noise as in Figure 8. Two points are interested in the proposed system as

- increasing the convergence speed
- increasing the tracking ability of the RLMS algorithm compared with the RLS and LMS algorithms.

For precise simulation of the proposed algorithm and comparison with the conventional FX-LMS method,

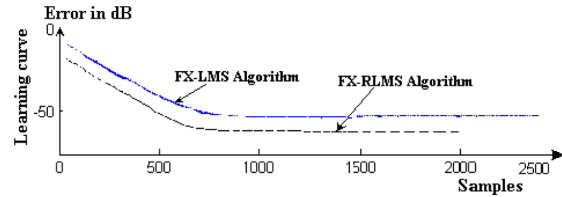
the transfer function of the primary path (the duct transfer function) and the secondary path must be available, which for this purpose, the information given in Lveg (1936), which is obtained practically is utilised.

**Figure 8** A structure for cancelling noise in a duct with the proposed method (see online version for colours)

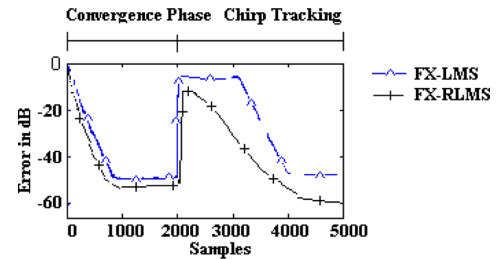


Higher convergence speed and lower error for the proposed algorithm in comparison with the FX-LMS algorithm in Figure 9 is observed. On average, the convergence speed increased 1.6 times and the final MSE minimum error decreased by 16%. Figure 10 shows the convergence and tracking phases of FX-LMS and FX-RLMS proposed algorithms, respectively. This figure shows increasing speed of convergence in the proposed algorithm compared with FX-LMS algorithms and also it shows reducing error in tracking phase for RLMS and FX-LMS algorithms.

**Figure 9** Learning curves to sinusoidal chirp with a variable frequency of 300–305 Hz in ANC in the duct for the FX-LMS (see online version for colours)



**Figure 10** Learning curves for convergence and tracking phases, convergence for frequency of 300 Hz and tracking for sinusoidal chirp with a variable frequency of 300–305 Hz (see online version for colours)



#### 4.3 Active noise cancellation of variable frequency narrow band noise using FX-RLMS algorithm

For best mixing of LMS and RLS algorithms in the FX-RLMS algorithm, suitable  $\mu_c$  in equation (14) is required in wide range frequency 200–500 Hz. We employed a novel approach based on frequency estimation

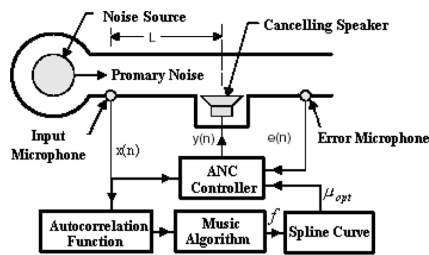
and a look-up table, which is obtained in learning phase. We know which ANC algorithms have different behaviour in variety of frequencies so we find optimum  $\mu_c$  ( $\mu_{opt}$ ) in each frequency. For this purpose, a novel system is

proposed as shown in Figure 11. The proposed system contains three main parts

- a ANC algorithm
- b best step size calculation for fusion of RLS and LMS outputs in ANC
- c frequency estimation.

The FX-RLMS algorithms shown structure in Figure 8 are used for ANC algorithm. Best step size of ANC algorithm,  $\mu_{opt}$ , is obtained in range 200–500 Hz, by changing of step size for reaching to minimum error in Figure 8.  $\mu_{opt}$  is obtained by step 25 Hz in range 200–500 Hz. Table 1 shows  $\mu_{opt}$  vs. frequency in ANC application.

**Figure 11** The proposed system for cancellation of variable narrow band frequency



**Table 1** Obtained  $\mu_{opt}$  vs. frequency in ANC application

$f$	200	237.5	275	300	312	325
$\mu_{opt}$	0.025	0.009	0.025	0.029	0.030	0.03
$f$	350	375	400	425	475	500
$\mu_{opt}$	0.026	0.017	0.011	0.022	0.031	0.03

For selecting of  $\mu_{opt}$ , frequency is required. So, we used MUSIC algorithm for frequency estimation (Eriksson and Allie, 1988; Bouchard and Yu, 2001). Using MUSIC algorithm, frequency of signal is estimated by extracting of Eigenvalues of autocorrelation function.

In the above-mentioned figure, a spline curve has been fitted over the  $\mu_{opt}$  vs. frequency.

## 5 Conclusions

For increasing the convergence speed and decreasing the MSE in the tracking mode, combining of the adaptive filters is a suitable method. A new approach based on a mixture of the RLS and LMS algorithms was presented, namely the RLMS algorithm. We proved that the MMSE of the proposed algorithm is reduced compared with those of the RLS and LMS algorithms. The RLMS algorithm was employed for active noise cancellation to form of the FX-RLMS algorithm, in a duct. Our new approach has several superiorities to the above-mentioned works, namely:

- our work is presenting a new configuration for combining of RLS and LMS methods in a real parallel form, considering convergence speed, tracking performance and error reduction

- applying the final system for ANC in a duct
- increasing the reliability of the whole system, which is not considered in previous research works.

Obtained results showed increasing performance of the RLMS algorithm in system identification, noisy chirp tracking and active noise cancellation.

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## Notes

- <sup>1</sup>Maximum A Posteriori.
- <sup>2</sup>Mean Square Error.
- <sup>3</sup>Recursive Least Mean Square.
- <sup>4</sup>Minimum Mean Square Error.
- <sup>5</sup>Additive White Gaussian Noise.

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