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OPTIMAL FORCE AND MOMENT BALANCE OF A FOUR-BAR LINKAGE VIA GENETIC ALGORITHM

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Abstract
The present article describes the application of Genetic Algorithm to force- and moment-balance of a four bar linkage. This technique permit competing design objectives to be considered through the investigation of trade-offs between those objectives. The objective functions of the design parameters are determined and their values are minimized by adjusting the independent variables of the designer. The technique permits both partial force and partial moment balance to be accomplished simultaneously. Genetic algorithm is a powerful and widely used stochastic optimization technique which relies on analogies to natural selections and natural genetics.

Keywords: Genetic - Algorithm - Force - Moment - Balance

Introduction
Genetic Algorithms are stochastic search methods that mimic the metaphor of natural biological evolution. Genetic algorithms operate on a population of potential solutions applying the principle of survival of the fittest to produce better and better approximations to a solution. At each generation, a new set of approximations is created by the process of selecting individuals according to their level of fitness in the problem domain and breeding them together using operators borrowed from natural genetics. This process leads to the evolution of populations of individuals that are better suited to their environment than the individuals that they were created from, just as in natural adoption. [1]

A Genetic Algorithm starts with a random creation of a population of strings and thereafter generates successive populations of strings that improve over time. Genetic Algorithm mostly used binary strings. A simple Genetic algorithm is composed of three operators:

1. Reproduction: Reproduction is a process in which individual strings are copied according to their objective function values, f (fitness function). [1]

2. Crossover: Crossover produces new individuals in combining the information contained in the parents. Members of newly reproduced strings in the mating pool are mated at random. The single point crossover chooses a random cut-off point in each of the two strings to form two substrings one to the left of the point, and one to the right. Then the left part of the string of one parent will spliced with the right part of the string of the other parent. [2]

3. Mutation: Mutation is a random alteration of the value of a string position. In binary coding, this means changing a 1 to 0 and vice versa. Mutation is needed because, even though reproduction and crossover effectively search and recombine extant notion, occasionally they may become overzealous and lose some potentially useful genetic material. Mutation introduces some extra variability into the population in order to avoid local minima. [1, 2]

In general, design optimization is the process of achieving the best solution of a given objective or objectives while satisfying certain restrictions. If a single objective function is to be minimized, then

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the problem is of single criterion optimization nature. Quite often, however, there exist several conflicting objectives. Then the problem is formulated as multi criteria, also called multiple objective or vector optimization problem, in which the goal is to minimize and/or maximize several objective functions simultaneously.

Formulation of an optimization problem consists of constructing a mathematical model, which describes the behaviour of a physical system encompassing the problem area. This model must closely approximate the actual behaviour of the system in order for the solution obtained to be adequate and useful.

Mechanism design cannot be complete without focusing attention on the interface between that mechanism and its mounting frame. Berkof and Lowen have addressed this problem in depth. Two methods, complementing each other, have been developed, permitting elimination of both shaking forces and shaking moments transmitted to ground.

Force balance is achieved by developing a set of linearly independent time-dependent vectors. These vectors define distribution of mass locations of the centres of mass such that the centre of mass for the entire system remains fixed. Thus the vector sum of the forces transmitted to ground from a force-balanced linkage is zero.

However, force balance does not eliminate shaking moments transmitted to the frame. To achieve total balance, moment-of-momentum equations are written for the system. When the vector sum of the moments of momentum becomes zero, the shaking moments are eliminated. This may be accomplished through the addition of inertia counterweights and restrictions on link configuration (when these changes are possible given space constraints of a specific application).

As a linkage moves it transmits forces to its surroundings. Unless it is balanced, these forces result in vibration, noise, wear, and cause fatigue problems. When completely force balanced, the vector sum of the forces acting on the frame is zero. This is accomplished by making the total centre of mass for the mechanism stationary. One method to achieve this result is the method of linearly independent vectors introduced by Berkof and Lowen, which redistribute link masses so that time-dependent terms of the equations of motion for the centre of mass become zero. This becomes possible if one can obtain a position equation which provides time-dependent vectors that linearly independent. Force balancing provides a zero vector sum of the inertia forces acting on the frame supports but does not provide zero forces at individual supports. The resultant of these forces will in general be a pure time-varying couple: the shaking moment. If the shaking moment also can be reduced to zero, the mechanism can be completely balanced, avoiding the unpleasant problems of vibrations, noise, wear and fatigue. Optimization techniques will permit minimization of shaking moments when complete balance is impossible because of conflicting requirements for force and moment balancing.

Formulation of the problem

Figure.1 is a four-bar linkage, $O_A ABO_B$, containing three moving links of arbitrary mass distribution. Consider an xoy system associated with the linkage system with O at O_A. Since the centre of mass of entire mechanism is kept stationary, full force balance is maintained regardless of variation in input speed. According to Berkof and Lowen equations when both fixed pivot links (2 and 4) are chosen to receive counterweights we write:

$$W_i^* r_i^* = [(W_i r_i)^2 + (W_i^o r_i^o)^2 - 2W_i r_i W_i^o r_i^o \cos(y_i - y_i^o)] \qquad (1)$$

$$y_i^* = \arg\{(W_i r_i \cos y_i - W_i^o r_i^o \cos y_i^o) + j(W_i r_i \sin y_i - W_i^o r_i^o \sin y_i^o)\} \qquad (2)$$

And if time-dependent terms vanish: [1]

$$W_2 r_2 = \frac{W_3 r_3' a_2}{a_3} \quad and \quad y_2 = y_3' \qquad (3)$$

$$W_4 r_4 = \frac{W_3 r_3 a_4}{a_3} \quad and \quad y_4 = y_3 + p \qquad (4)$$

Where:

$i = 2,4$

W_i^o, r_i^o, y_i^o : parameters for the unbalanced linkage

W_i^*, r_i^*, y_i^* : parameters for counterweights

W_i, r_i, y_i : parameters obtained from equations (3) and (4) (N)

W_i : Weight of link i (N)

y_i : Angle between line from pin to pin and line from pin to centre of mass of link i (radian)

r_3' : Distance from pin 3 to centre of mass of link 3 (m)

y_3' : Angle between line connecting pins 2 and 3 and centre of mass of link 3 (measuring from pin 3) (radian)

Utilization of the concepts of inertia counterweights and the physical pendulum permits complete balance of all mass effects (both linear and rotary, but excluding external loads), independent of input angular velocity. Inertia counterweights permit any unbalanced planar moment, which is proportional to angular acceleration, to be balanced. Since no net inertia forces are introduced by this addition, the shaking force balance is unaffected. Unfortunately the driving torque must increase substantially to drive the now force- and moment-balanced system with the added counterweights. [3]
Because we have changed the centre of mass of link 2 and 4, we must calculate the new radius of gyration for those links. Because we have chosen circular counterweights, the contribution from the counterweights is given by:

$$K_i^* = \frac{r_i^*}{\sqrt{2}} \tag{5}$$

The total weight is:

$$W_i' = W_i^{o} + W_i^* \tag{6}$$

And the new position of the centre of mass is determined by vector addition:

$$\begin{aligned} W_i' r_i' \exp(j y_i') &= W_i^{o} r_i^{o} \exp(j y_i^{o}) \\ &+ W_i^* r_i^* \exp(j y_i^*) \end{aligned} \tag{7}$$

And :

$$\begin{aligned} K_i^2 &= \left(\frac{W_i^{o}}{W_i'}\right)[K_i^{o2} + (r_i^{o} + r_i')^2] \\ &+ \left(\frac{W_i^*}{W_i'}\right)[K_i^{*2} + (r_i^* + r_i')^2] \end{aligned} \tag{8}$$

The weight moments of inertia of the inertia counterweights using 1:1 gearing as in Figure.3 are given by:

$$I_i^{**} = W_i' (K_i^2 + r_i'^2 + a_i r_i') \tag{9}$$

$$I_i^{**} = \frac{p}{2}(g r_i^2 h_i^{**}) r_i^{*2} \tag{10}$$

$$W_i^* = pg\, h_i^* r_i^{*2} \tag{11}$$

Where:

a_i : Length of link i (m)

r_i : Distance from pin to centre of mass (m)

h_i : Thickness of link i (m)

K_i : Radius of gyration of i-th link (m)

$W_i^* r_i^*$: Weight $\times$ Radius for force balance counterweights (r_i^* measured from same pivot as the centre of mass of the i-th link) (N.m)

g : Density of moment balance counterweights (kg/m3)

r_i : The radius of the disk (m)

h_i^* : The thickness of counterweights (m)

h_i^{**} : The thickness of the disk (gear) (m)

In a design optimization task the numerical quantities for which values are to be chosen will be called decision or design variables, or simply variables. There are some restrictions dictated by environment and process and/or resources, which must be satisfied in order to produce an acceptable solution. These restrictions are collectively called constraint functions or constraints. In the process of selection a good solution, which satisfy the constraints, there must be a criterion or some criteria, which allow these solutions to be compared.[4]
In order to formulate the optimization problem mathematically we need to introduce the number of independent variables. We will see W*i, Ψ*i and ρi can be regarded as design variables of both force- and moment-balancing problems.
f_1, f_2, f_3 are three appropriate objective functions that can be defined in the form:

$$f_1 = W_i^* r_i^* - [(W_i r_i)^2 + (W_i^o r_i^o)^2 - 2 W_i r_i W_i^o r_i^o \cos(y_i - y_i^o)]^{1/2} \qquad (12)$$

$$f_2 = y_i^* - \arg\{(W_i r_i \cos y_i - W_i^o r_i^o \cos y_i^o) + j(W_i r_i \sin y_i - W_i^o r_i^o \sin y_i^o)\} \qquad (13)$$

$$f_3 = K_i^2 - \left(\frac{W_i^o}{W_i'}\right)[K_i^{o2} + (r_i^o + r_i')^2] + \left(\frac{W_i^*}{W_i'}\right)[K_i^{*2} + (r_i^* - r_i')^2] \qquad (14)$$

It is obvious that the objective functions defined by equations (12), (13), (14) represent error functions which will be minimized. When these functions are minimized simultaneously, both force and moment balance conditions are satisfied. And W_i^*, y_i^* and r_i indicate the best circular counterweights, angle of counterweights and radius of gears values which added to achieve force and moment balance.

Evolutionary algorithms for multi criteria design optimization

The general formulation of this problem is:

$$x = [x_1, x_2, x_3] \qquad (15)$$

Which for link 2 :

$$x_1 = W_2^*, \quad x_2 = y_2^*, \quad x_3 = r_2 \qquad (16)$$

And for link 4 :

$$x_1 = W_4^*, \quad x_2 = y_4^*, \quad x_3 = r_4 \qquad (17)$$

Where:

$$0.445N < W_2^* < 2.225N, \quad \frac{p}{6} < y_2^* < 8\frac{p}{6}, \quad 0.020m < r_2 < 0.033m \qquad (18)$$

$$1.335N < W_4^* < 2.640N, \quad \frac{p}{6} < y_4^* < 8\frac{p}{6}, \quad 0.036m < r_4 < 0.051m \qquad (19)$$

$x = [x_1, x_2, x_3]$: The vector of decision variables

$f = [f_1, f_2, f_3]$: The vector of objective functions

Parameters of completely unbalanced linkage ,shown in Figure.2 ,are given in Table.1.The links are steel of density $g = 293.782 Kg/m^3$

And:

$$h_2^* = h_4^* = 0.013m \qquad (20)$$

$$h_2^{**} = h_4^{**} = 0.025m \qquad (21)$$

Genetic Algorithm

Genetic and Evolutionary algorithm based techniques can generate the whole set of Pareto optimal solution with single running on a computer program. This advantage provides the decision maker with a full picture of all possible compromise solutions and thus makes the decision process easier. For multicriteria optimization problems, each objective function achieves its minimum at different points. Thus, a Pareto optimality concept is introduced to solve the problem.[5]

Pareto optimum: A point $x^* \in X$ is called Pareto optimal if and only if there exists no $x \in X$ such that $f_i(x) \le f_i(x^*)$, for i = 1, 2, ..., I with $f_j(x) < f_j(x^*)$ for at last one j. [2]

The method of selecting a set of Pareto optimal solutions which is used in this paper is based on the contact theorem [2], which is one of the main theorems in multi criterion optimization. This method was proposed by Osyczka, 1984 and extensively described in [2]. Selection method is based on Pareto set distribution method was proposed by Osyczka & Tamura, 1996.

Let $x^{*r} = [x_1^{*r}, x_2^{*r}, ..., x_N^{*r}]^T$, be a vector of the r-th Pareto optimal solution and $f(x^{*r})$ be a vector of objective functions for the r-th Pareto solution where $r = 1,2,...,R$ and R is the number of existing Pareto solutions. For each new solution x^j that is generated by any method the vector of the vector of the objective functions $f(x^j)$ is evaluated and compared with the existing set of Pareto solutions. The new solution can fall in any of three categories:

1-It is a new Pareto solution which dominates some or at least one solution in the set of Pareto solution found so far. In this case dominated solutions are removed from the set and the new Pareto solution is added to the set.

2-It is a new Pareto solution, but it does not dominate any of the existing Pareto solutions. In this case the new solution is added to the set.

3-It is not a new Pareto solution, and there is no change in the set of Pareto solutions.

This idea is used in the method that used in this paper.

Initial population is typically generated at random. Crossover operates on two chromosomes at a time and generates offspring by combining some features of both chromosomes. Mutation is an operator, which produces spontaneous random changes in various chromosomes and thus it introduces some extra variability into the population in order to avoid local minima.

Results

Initialization of a population to provide the program a starting point is done by generating random strings within the search space and this is the default behavior of the Genetic Algorithms.

The results of running the program for both links (2 and 4) are presented in Table.2 and Table.3 and the set of Pareto optimal solutions are illustrated in Fig.4 and Fig.5 for link 2 and Fig.6 and Fig.7 for link 4.

The values of W_i^*, r_i^*, y_i^* permit both force and moment balance with best solutions in the search space.

Fig.3 shows the configuration for the force- and moment balanced linkage, shaded area is material added to achieve (a) force- and (b) moment-balance. The analytic results for defined for bar are[3]:

$$W_2^* = 1.0312N$$

$$y_2^* = p$$

$$r_2 = 0.0263m$$

$$W_4^* = 2.2134N$$

$$y_4^* = p$$

$$r_4 = 0.0436m$$

These results make it obvious that genetic algorithm optimization, which is used in this paper, generated acceptable results. Genetic algorithms have the ability to solve the other nonlinear problems in the same field as force and moment balance of six bars and synthesis.

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Table 1- Parameters of unbalanced four-bar

Parameters	Link 2	Link 3	Link 4	Link 1
$a_i(m)$	0.025	0.102	0.076	0.07
$d_i(m)$	0.013	0.013	0.013	----
$h_i(m)$	0.005	0.005	0.005	----
$W_i^o(N)$	0.449	0.774	0.650	----
$r_i^o(m)$	0.013	0.051	0.038	----
$y_i^o(rad)$	0	0	0	----
$K_i^o(m)$	0.015	0.040	0.032	----

Table 2 – The set of Pareto optimal solutions (Link 2)

f_1	W_2^* (N)	f_2	y_2^* (rad)	f_3 $\times 10^{-3}$	r_2 (m)
0.0001	1.033	0.0009	3.142	0.087	0.027
0.0018	1.098	0.0009	3.142	0.005	0.027
0.0002	1.038	0.0009	3.142	0.081	0.027
0.0001	1.034	0.0009	3.142	0.086	0.027
0.0000	1.032	3.1416	2.895	0.448	0.022
0.0098	1.365	3.1416	1.051	0.000	0.027
0.0005	1.049	0.0009	3.142	0.067	0.027
0.0016	1.088	0.0009	3.142	0.017	0.027
0.0006	1.052	0.0009	3.142	0.064	0.027
0.0002	1.038	0.0009	3.142	0.082	0.027
0.0000	1.032	0.0009	3.142	0.830	0.031
0.0010	1.068	0.0009	3.142	0.042	0.027
0.0008	1.062	3.1416	1.497	0.016	0.026
0.0014	1.082	0.0009	3.142	0.025	0.027
0.0000	1.032	0.0009	3.142	0.088	0.027
0.0003	1.042	0.0009	3.142	0.076	0.027
0.0017	1.093	0.0009	3.142	0.011	0.027
0.0003	1.040	0.0161	3.157	0.078	0.027
0.0008	1.061	3.1416	3.140	0.051	0.027
0.0008	1.059	0.0009	3.142	0.054	0.027

Table 3 – The set of Pareto optimal solutions (Link 4)

f_1	W_4^* (N)	f_2	y_4^* (rad)	f_3	r_4 (m)
0.0631	2.085	0.9926	4.134	0.0000	0.04
0.0033	2.201	0.0009	3.142	0.0001	0.04
0.0026	2.186	0.0009	3.142	0.0001	0.04
0.0038	2.210	0.0009	3.142	0.0001	0.04
0.0002	2.130	0.0009	3.142	0.0003	0.04
0.0004	2.135	3.1416	3.139	0.0002	0.04
0.0034	2.202	0.0009	3.142	0.0001	0.04
0.0003	2.133	0.0009	3.142	0.0003	0.04
0.0001	2.128	0.0009	3.142	0.0003	0.04
0.0044	2.224	0.0009	3.142	0.0000	0.04
0.0028	2.188	0.0009	3.142	0.0001	0.04
0.0023	2.177	0.0009	3.142	0.0001	0.04
0.0022	2.176	0.0009	3.142	0.0002	0.04
0.0039	2.214	0.0009	3.142	0.0001	0.04
0.0024	2.180	0.0009	3.142	0.0001	0.04
0.0038	2.212	0.0009	3.142	0.0001	0.04
0.0014	2.159	3.1416	3.129	0.0002	0.04
0.0005	2.139	0.0009	3.142	0.0002	0.04
0.0004	2.136	0.0009	3.142	0.0002	0.04

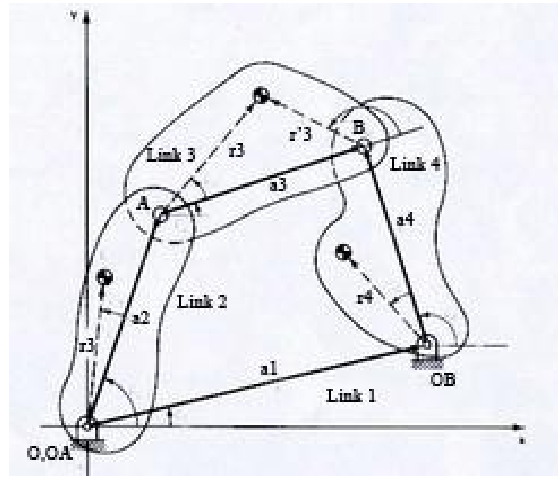

Fig.1 - Four-bar linkage with the arbitrary distribution of link masses.

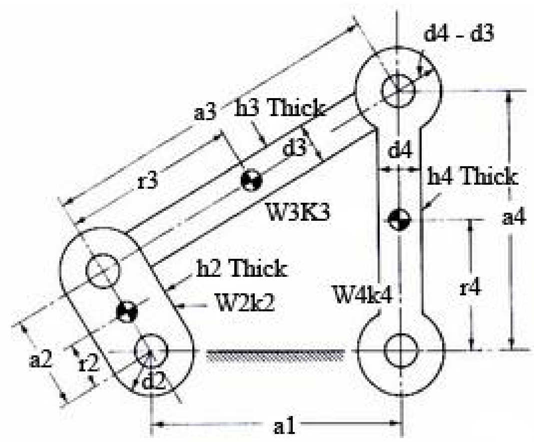

Fig.2 - Unbalanced four-bar linkage

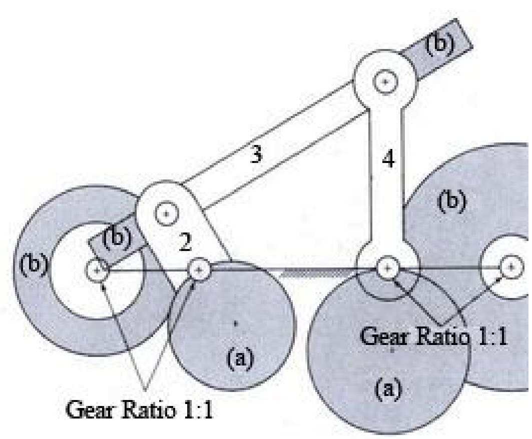

Fig. 3 - Fully force- and moment-balanced in-line four-bar

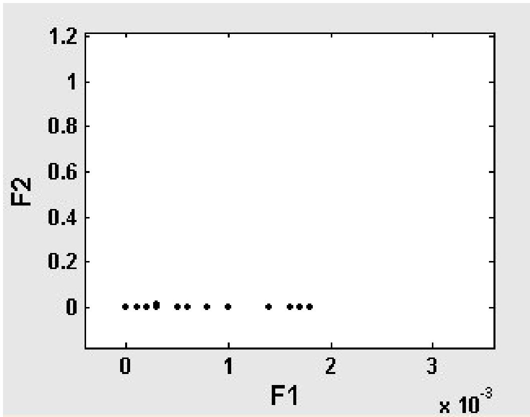

Fig. 4 – The set of Pareto optimal solutions for F1 and F2 (Link 2)

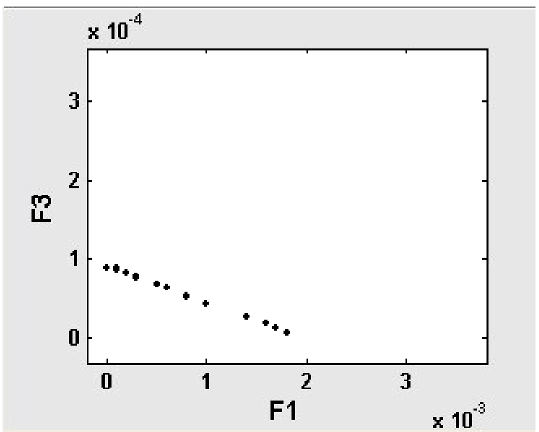

Fig. 5 – The set of Pareto optimal solutions for F1 and F3 (Link 2)

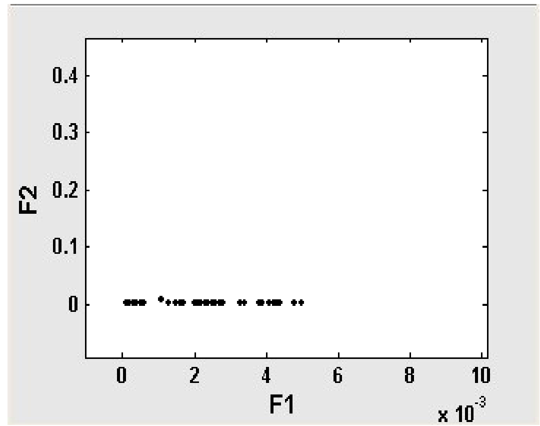

Fig. 6 – The set of Pareto optimal solutions for F1 and F2 (Link 4)

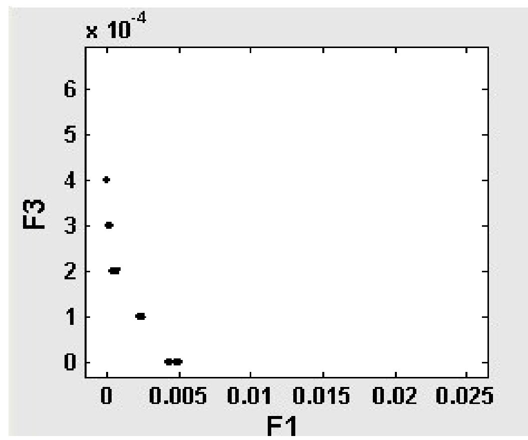

Fig. 7 – The set of Pareto optimal solutions for F1 and F3 (Link 4)