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VIBRATION CONTROL OF SUPER TALL BUILDINGS BY TUNED LIQUID COLUMN DAMPER

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The tuned liquid column damper (TLCD) is a passive control device that consists of tube like containers filled with liquid, preferably water. The damping in a liquid column is amplitude-dependent and consequently nonlinear. Most of the previous studies concentrated on the equivalent linearization technique and replace the nonlinear damping term with the linear one. This may cause that the prediction of the optimal parameters for the TLCD design does not be accurate.

In this study He's homotopy perturbation method (HPM) and variational iteration method (VIM) are applied to find better approximate solution for the structure-TLCD equations. This new method is utilized to solve the equation of motion of a building modeled as a single degree of freedom (SDOF) system coupled with the equation of motion of the liquid in the column of the TLCD and subjected to a harmonic type of wind excitation. This new technique leads to achieve more reliable solution for the TLCD design. Present solution gives an expression which can be used in wide range of time for all domain of response. After that the Citicorp Center which has a very flexible structure is used as an example to illustrate the design procedure of the TLCD and demonstrate the performance of the TLCD in effectively reducing vibrational response under the wind excitation.

1. Introduction

The current trend towards structure with increasing height and advanced construction techniques has led to lighter and highly flexible civil engineering structures in many urban areas, such as PETRONAS Twin Tower in Malaysia, and Shanghai World Financial Center and Jin Mao Building in China [1]. These structures are vulnerable to dynamic loads, such as wind gusts, ocean waves and earthquakes. It is thus necessary to find a cost effective solution for suppressing the vibration of structures. Among many varieties of control devices [2, 3] the tuned liquid column damper (TLCD) as an energy absorbing device which does not require an external power source for operation and utilizes the motion of the structure to develop the control forces is a good candidate. A TLCD consists of tube like containers filled with liquid (commonly water) where energy is dissipated by the movement of the liquid through an orifice. It can provide the same level of vibration suppression as conventional tuned mass damper (TMD) systems [4, 5] but with following advantages:

1) The TLCD parameters (frequency and mass) can be easily tuned by adjusting the height of the liquid in the tube.
2) The required level of damping can be readily achieved and controlled through the orifice.
3) It can utilize as a water storage facility at the top of the buildings for an emergency such as fire.
4) Easy installation and little maintenance needed.

Moreover, TLCD can dissipate energy in vertical direction [6] similar to horizontal motion of structures [7]; also it can reduce rotational and pitching vibration of structures [8, 9].

In the last two decades, there have been several studies under taken on the evaluation of TLCD performance in suppressing the vibration of structures under wind excitations [10-12]. Since, the damping of the liquid motion is nonlinear; most previous research uses linearization techniques and until now, there have been few studies examining the nonlinear vibrations of a structure equipped with the TLCD [13]. As an accurate prediction of the TLCD parameters is crucial in control problems, the nonlinear analysis of the TLCD is clearly needed. Hence, this research utilizes an analytical approach base on the homotopy theory and the perturbation technique known as the homotopy perturbation method (HPM) [14, 15]. Unlike the other analytical methods that are used in nonlinear analysis of engineering problems such as perturbation techniques [16, 17], HPM does not depend on small parameter and few iterations would acquire precise solutions.

The intention of this paper is to find an analytical solution of a TLCD-structure system under a harmonic type of wind excitation. Due to inherent nonlinear damping, iterations is generally required in order to obtain the frequency domain response of a structure equipped with a TLCD. It would be quite a time consuming task to carry out a detailed design of the mass damper. Therefore to facilitate the design of the damper and to improve the approximate solution, the nonlinear equation of motion of the structure equipped with the TLCD is solved by using the homotopy perturbation method and the variational iteration technique [18]. Finally the Citicorp Center is used as an example to illustrate the design procedure for the TLCD under the harmonic type of wind excitation.

2. Equation of motion

Let it be assumed that the motion of a building modeled as a single degree of freedom (SDOF) system, is to be mitigated using tuned liquid column damper (TLCD). A structure-TLCD system under ground motion is shown in Fig. 1. $r = A_v/A_h$ is the cross sectional area ratio of the mass damper. It can be uniform ($r = 1$) which corresponds to the tuned liquid column damper and non-uniform ($r \neq 1$) which corresponds to the liquid column vibration absorber (LCVA) [19], where A_v and A_h are the vertical and horizontal column cross sectional area, respectively. In consideration of dynamic equilibrium condition and the interaction between the structure and the liquid column in TLCD, the equation of motion of a structure equipped with the TLCD for lateral vibration control under wind excitation expressed as

$$M\ddot{X} + \rho A_h L_e \ddot{X} + C\dot{X} + KX + \rho A_h r L_h \ddot{Y} = F(t) \quad (1)$$

$$\rho A_h r L_e \ddot{Y} + \tfrac{1}{2}\rho A_h r^2 \eta \left|\dot{Y}\right| \dot{Y} + 2\rho A_h g r Y + \rho A_h r L_h \ddot{X} = 0 \quad (2)$$

where M,C,K are the structural mass, damping and stiffness constant; X is the lateral displacement of the structure; Y is the motion of the liquid surface inside the TLCD; $F(t)$ is the external excitation force (of wind load type); L_h and L_v are the horizontal and vertical column length, respectively; ρ is the liquid density in the TLCD; g is the acceleration due to gravity; $L_e = 2L_v + rL_h$

is defined as the effective length which means the length of an equivalent uniform liquid column having the same circular frequency, if r = 1 (for TLCD), then $L_e = L = 2L_v + rL_h$ that represents the total length of the TLCD.

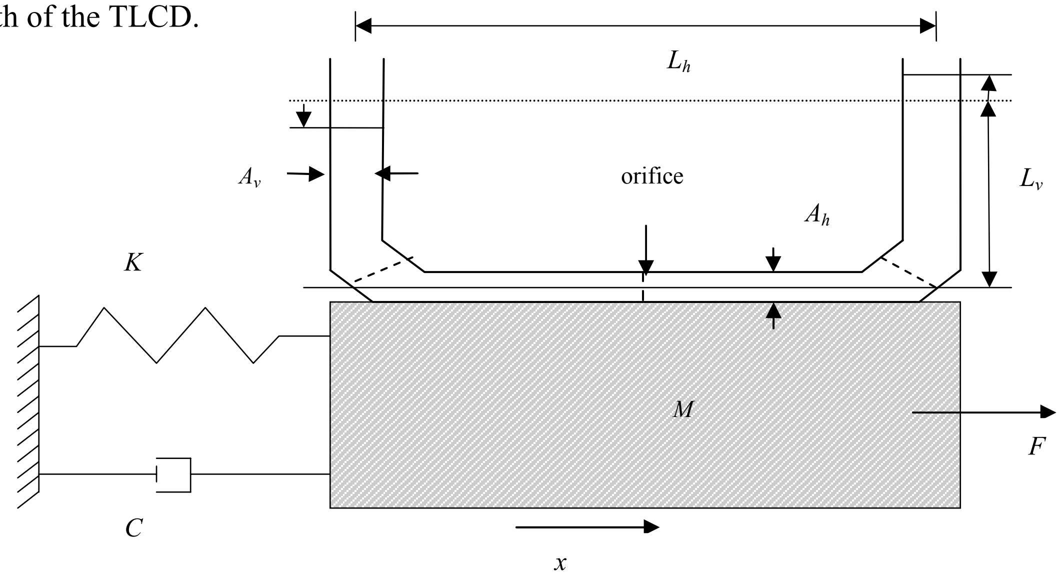

Figure 1: structure equipped with the tuned liquid column damper (TLCD)

It is convenient to work with dimensionless position and time, according to $x = \frac{X}{L_h}$; $y = \frac{Y}{L_h}$; $\tau = \frac{t}{T_d}$. It is easily observed that the natural frequency of a TLCD is $\omega_d = \sqrt{\frac{2g}{L_e}}$, and accordingly the natural period is $T_d = 2\pi\sqrt{\frac{L_e}{2g}}$. In such a way Eq. (1) and Eq. (2) are rewritten in the following form

$$\mu n y'' + (1+\mu)x'' + 4\pi\zeta\gamma x' + 4\pi^2\gamma^2 x = f(\tau) \quad (3)$$

$$y'' + \tfrac{1}{2} rn\eta |y'| y' + 4\pi^2 y + nx'' = 0 \quad (4)$$

where the notation prime (") stands for differentiation with respect to the scaled time τ, and the following abbreviation were introduced

$\mu = \frac{\rho A_h L_e}{M}$ is the mass ratio of the liquid column to the structure. $\zeta = \frac{C}{2M\omega_s}$ is the damping ratio of the structure. $\omega_s = \sqrt{\frac{K}{M}}$ is the natural frequency of the structure. $\gamma = \frac{\omega_s}{\omega_d}$ is the frequency ratio of the structure versus TLCD and for the TLCD (r = 1) $n = \frac{L_h}{L}$.

3. Homotopy perturbation method

To explain the basic ideas of the homotopy perturbation method, we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0, \quad r \in \Omega \quad (5)$$

with boundary conditions:

$$B(u, \partial u/\partial n) = 0, \quad r \in \Gamma \quad (6)$$

where A, B, f(r) and Γ are a general differential operator, a boundary operator, a known analytical function and the boundary of the domain Ω, respectively.

The operator A can be divided into a linear part L and a nonlinear part N. Therefore, Eq. (5) can be written as follows

$$L(u)+N(u)-f(r)=0 \tag{7}$$

By the homotopy technique, we construct a homotopy $v(r,p): \Omega\times[0,1]\rightarrow\Re$ which satisfies

$$H(v,p)=(1-p)[L(v)-L(u_0)]+p[A(v)-f(r)]=0,\ p\in[0,1], r\in\Omega \tag{8}$$

or

$$H(v,p)=L(v)-L(u_0)+pL(u_0)+p[N(v)-f(r)]=0 \tag{9}$$

where $p\in[0,1]$ is an embedding parameter, u_0 is an initial approximation of Eq. (5), which satisfies the boundary conditions. Obviously, Eq. (8) results in:

$$H(v,0)=L(v)-L(u_0)=0 \tag{10}$$

$$H(v,1)=A(v)-f(r)=0 \tag{11}$$

Due to the fact that $0\le p\le 1$, the embedding parameter p can be used as a small parameter. Applying the perturbation technique [14], the approximation of Eq. (8) can be written as a power series in p:

$$v=v_0+pv_1+p^2v_2+\cdots \tag{12}$$

When $p\rightarrow 1$, Eq. (12) leads to the approximate solution of Eq. (7). The combination of the perturbation technique and the homotopy method not only can eliminate the limitations of the traditional perturbation methods, but also take full advantage of the traditional perturbation techniques.

4. Variational iteration method

Another analytical technique called the variational iteration method (VIM) based on the use of restricted variations and correction functions, is described by He in 1999 [19] and is used to give approximate solutions of nonlinear ordinary and partial differential equations. In this method the solution of a linearization assumption is used as an initial estimation, and then a more highly precise approximation is obtained via the variational theory.

To clarify the basic concept of He's variational iteration method, let us consider the following general nonlinear system:

$$L(u)+N(u)=g(t) \tag{13}$$

where L is a linear operator, N is a nonlinear operator and $g(t)$ is a known analytical function.

According to VIM, the general Lagrange multiplier method is modified into an iteration method, in the following way [19]:

$$u_{n+1}(t)=u_n(t)+\int_0^t \lambda\{Lu_n(\tau)+N\tilde{u}_n(\tau)-g(\tau)\}d\tau \tag{14}$$

where λ is a general Lagrange multiplier which can be determined optimally via the variational theory. The subscript n denotes the nth approximation and $\tilde{u}_n$ is considered as a restricted variation, i.e. $\delta\tilde{u}_n=0$.

Eq. (14) is called a correction functional. Since λ can be exactly identified for linear problems, the exact solutions can be obtained by only one iteration step. This method is also very effective and accurate for nonlinear problems [20, 21].

5. Analytical solution to wind loading

According to the homotopy perturbation, after separating the linear and nonlinear parts of Eq. (4), a homotopy can be constructed which satisfies Eq. (9).

in which

$$L(x,y)=y''+nx''+4\pi^2y,\ N(x,y)=\frac{1}{2}rn\eta y'^2,\ f(r)=0 \tag{15}$$

Replacing Eq. (15) into Eq. (9) results in

$$H(u,v,p)=L(u,v)-L(x_0,y_0)+pL(x_0,y_0)+p[N(u,v)-f(r)]=0 \tag{16}$$

$$v''+nu''+4\pi^2 v-y_0''-nx_0''-4\pi^2 y_0+p\left[y_0''+nx_0''+4\pi^2 y_0\right]+p\left[\frac{1}{2}rn\eta v'^2\right]=0 \tag{17}$$

$p\in[0,1]$ is the embedding parameter. As the embedding parameter increases from 0 to 1, x and y vary from the initial guesses $x_0(t)$ and $y_0(t)$ to the exact solutions $x(t)$ and $y(t)$. Assumes that the approximations of Eq. (17) can be expressed as a series of the power of p,

$$\begin{cases} u=u_0+pu_1+p^2u_2+\cdots \\ v=v_0+pv_1+p^2v_2+\cdots \end{cases} \tag{18}$$

Substituting Eq. (18) into Eq. (17) and rearranging the resultant equation based on power of p-terms, one has:

$$p^0: v_0''+nu_0''+4\pi^2 v_0-y_0''-nx_0''-4\pi^2 y_0=0 \tag{19a}$$

or

$$L(u_0,v_0)-L(x_0,y_0)=0 \tag{19b}$$

$$p^1: v_1''+nu_1''+4\pi^2 v_1+y_0''+nx_0''+4\pi^2 y_0+\frac{1}{2}rn\eta v_0'^2=0 \tag{20a}$$

or

$$L(u_1,v_1)+L(x_0,y_0)+\frac{1}{2}rn\eta v_0'^2=0 \tag{20b}$$

Then by seperating the linear and nonlinear parts of Eq. (3) and using the homotopy perturbation technique, Eqs. (22) and (23) would be obtained.

$$L(x,y)=\mu ny''+(1+\mu)x''+4\pi\zeta\gamma x'+4\pi^2\gamma^2 x,\ N(x,y)=0,\ f(r)=f \tag{21}$$

$$p^0: \mu nv_0''+(1+\mu)u_0''+4\pi\zeta\gamma u_0'+4\pi^2\gamma^2 u_0-\mu ny_0''-(1+\mu)x_0''-4\pi\zeta\gamma x_0'-4\pi^2\gamma^2 x_0=0 \tag{22a}$$

or

$$L(u_0,v_0)-L(x_0,y_0)=0 \tag{22b}$$

$$p^1: \mu nv_1''+(1+\mu)u_1''+4\pi\zeta\gamma u_1'+4\pi^2\gamma^2 u_1+\mu ny_0''+(1+\mu)x_0''+4\pi\zeta\gamma x_0'+4\pi^2\gamma^2 x_0=f \tag{23a}$$

or

$$L(u_1,v_1)+L(x_0,y_0)=f \tag{23b}$$

With the effective initial approximation, the solution of Eqs. (19) and (22) may be written as follows:

$$u_0=v_0=x_0=y_0=\cos(\omega t) \tag{24}$$

Using trigonometric relations and substituting Eq. (24) into Eqs. (20a) and (23a) result in:

$$\begin{cases} \mu nv_1''+(1+\mu)u_1''+4\pi\zeta\gamma u_1'+4\pi^2\gamma^2 u_1-4\pi\zeta\gamma\omega\sin(\omega t)+\left[-\omega^2(1+\mu+\mu n)+4\pi^2\gamma^2\right]\cos(\omega t)=f \\ v_1''+nu_1''+4\pi^2 v_1+\left[-\omega^2(n+1)+4\pi^2\right]\cos(\omega t)-\frac{1}{4}rn\eta\omega^2\cos(2\omega t)+\frac{1}{4}rn\eta\omega^2=0 \end{cases} \tag{25}$$

The solution of Eq. (25) can be readily obtained by the so-called variational iteration method which mentioned before.

To solve Eq. (25), following assumption should be considered:

$$w_1=x;\quad w_2=x';\quad w_3=y;\quad w_4=y' \tag{26}$$

$$\{w_1'=w_2,\quad (1+\mu)w_2'=-\mu nw_4'-4\pi\zeta\gamma w_2-4\pi^2\gamma^2 w_1+f(\tau),\quad w_3'=w_4,\quad w_4'=-\tfrac{1}{2}rn\eta n_4{}^2-4\pi^2 w_3-nw_2' \tag{27}$$

Then the correction functionals can be written as follows:

$$\begin{cases} w_1^{(n+1)}=w_1^{(n)}+\int_0^t\lambda_1\left(w_1'^{(n)}-\tilde{w}_2^{(n)}\right)d\tau,\quad w_2^{(n+1)}=w_2^{(n)}+\dfrac{1}{1+\mu}\int_0^t\lambda_2\left(w_2'^{(n)}+\mu n\tilde{w}_4'^{(n)}+4\pi\zeta\gamma\tilde{w}_2^{(n)}+4\pi^2\gamma^2\tilde{w}_1^{(n)}-f(\tau)\right)d\tau \\ w_3^{(n+1)}=w_3^{(n)}+\int_0^t\lambda_3\left(w_3'^{(n)}-\tilde{w}_2^{(n)}\right)d\tau,\quad w_4^{(n+1)}=w_4^{(n)}+\int_0^t\lambda_4\left(w_4'^{(n)}+\tfrac{1}{2}rn\eta\tilde{w}_4^{(n)}+4\pi^2\tilde{w}_3^{(n)}+n\tilde{w}_2'^{(n)}\right)d\tau \end{cases} \tag{28}$$

Making the above correction functionals stationary and knowing that δ v (0) = 0:

$$\begin{cases} \delta w_1^{(n+1)}(t) = \delta w_1^{(n)}(t) + \lambda_1(\tau)\delta w_1^{(n)}(\tau)\Big|_{\tau=t} + \int_0^t \lambda_1' \delta w_1^{(n)}(\tau) d\tau \\ \delta w_2^{(n+1)}(t) = \delta w_2^{(n)}(t) + \frac{1}{1+\mu}\lambda_2(\tau)\delta w_2^{(n)}(\tau)\Big|_{\tau=t} + \frac{1}{1+\mu}\int_0^t \lambda_2' \delta w_2^{(n)}(\tau) d\tau \\ \delta w_3^{(n+1)}(t) = \delta w_3^{(n)}(t) + \lambda_3(\tau)\delta w_3^{(n)}(\tau)\Big|_{\tau=t} + \int_0^t \lambda_3' \delta w_3^{(n)}(\tau) d\tau \\ \delta w_4^{(n+1)}(t) = \delta w_4^{(n)}(t) + \lambda_4(\tau)\delta w_4^{(n)}(\tau)\Big|_{\tau=t} + \int_0^t \lambda_4' \delta w_4^{(n)}(\tau) d\tau \end{cases} \quad (29)$$

Thus the stationary condition can be obtained as follows:

$$\lambda_1' = 0, \quad 1 + \lambda_1(\tau)\big|_{\tau=t} = 0 \quad (30)$$

$$\lambda_2' = 0, \quad 1 + \lambda_2(\tau)\big|_{\tau=t} = 0 \quad (31)$$

$$\lambda_3' = 0, \quad 1 + \lambda_3(\tau)\big|_{\tau=t} = 0 \quad (32)$$

$$\lambda_4' = 0, \quad 1 + \frac{1}{1+\mu}\lambda_4(\tau)\big|_{\tau=t} = 0 \quad (33)$$

Solving the systems of Eqs. (30)-(33) yields:

$$\lambda_1 = -1, \lambda_2 = -1, \lambda_3 = -1, \lambda_4 = -\mu - 1 \quad (34)$$

As a result the following variational iteration formula can be obtained:

$$u_1 = \int_0^t (-1)\left\{\left[-\omega^2(1+\mu+\mu n) + 4\pi^2\gamma^2\right]\cos(\omega t) - 4\pi\zeta\gamma\omega\sin(\omega t) - f\right\} d\tau \quad (35)$$

$$v_1 = \int_0^t (-1)\left\{\left[-\omega^2(n+1) + 4\pi^2\right]\cos(\omega t) - \frac{1}{4}rn\eta\omega^2\cos(2\omega t) + \frac{1}{4}rn\eta\omega^2\right\} d\tau \quad (36)$$

Therefore the first order approximate solution can be obtained:

$$x = u_0 + u_1, \; y = v_0 + v_1 \quad (37)$$

6. Design procedure

In this section the process of designing the TLCD is investigated . To illustrate the design procedure of this mass damper, the Citicorp Center [22] which has a very flexible structure is used as an example in this study. A TLCD is to be installed at the top of the building to abating the vibration induced by the harmonic type of wind excitation. The mass, stiffness and damping constants of this building are, $1.8\times10^7\ N.s^2/m$, $1.82\times10^7\ N/m$ and $0.36\times10^6\ N.s/m$ $(\zeta = 0.01)$ respectively.

These properties represented the first mode of natural frequency. The step by step procedure for the TLCD design is stated as follows:

1) The first step of designing a TLCD is to select a proper mass ratio. The larger mass ratio, results on better control performance but by increasing the mass ratio, the geometry of the TLCD becomes larger and may require a stronger supporting system at the top of the structure, which would increase the installation cost. In this example, a mass ratio of 0.025 is selected and with a horizontal length ratio, n, chosen as 0.6, the tuning frequency ratio $\gamma = \frac{\omega_s}{\omega_d}$ of 1 is selected. Therefore, the total length, L, the horizontal length, L_h and the vertical length, L_v, of the TLCD, can be obtained as follows:

$$\omega_s = \sqrt{\frac{K}{M}} = 1.006\ rad/s, \omega_d = \omega_s = 1.006\ rad/s, \sqrt{\frac{2g}{L}} = 1.006, L = 19.4\ m$$

$$n = \frac{L_h}{L}, 0.6 = \frac{L_h}{L}, L_h = 11.6\ m$$

$$L = L_h + 2rL_v \;; r = 1\,(for\;the\;TLCD),\; L_v = 3.88\; m$$

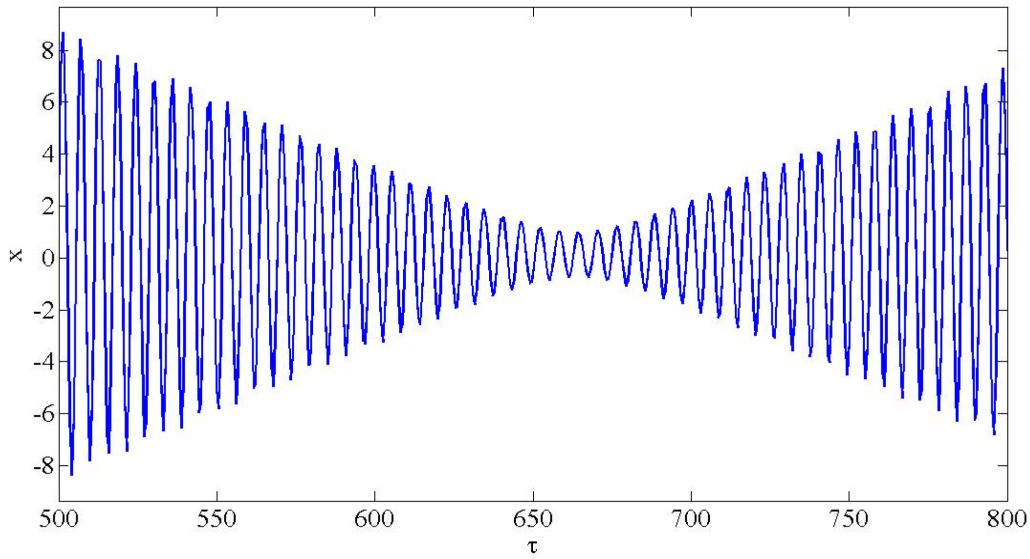

Figure 2: Effect of the TLCD on reducing the vibration of buildings

2) By $\mu = \frac{\rho A_h (L_h + 2L_v)}{M}$ and knowing that the density of the water is about 997 kg/m^3, a cross section A=23.26 m^2 is thus determined. This huge cross section is due to the large first modal mass and the mass ratio that is chosen. One possible solution to create a space for such a huge TLCD is to divide it into few smaller TLCDs with identical configurations.

Fig. 2 shows the effects of the TLCD on reducing the vibration of the Citicorp Center building when subjected to the external loading (wind excitation) with the amplitude f = 0.053.

7. Conclusion

The main objective of this paper is to develop some analytical formulas and closed form solution to obtain more accurate response of a building equipped with the tuned liquid column damper (TLCD) for suppressing horizontal vibration of the structure under a harmonic type of wind excitation. The TLCD is a passive control device that dissipates energy by the movement of the liquid through an orifice and improves dynamics of structures. The damping in a liquid column is regulated by the orifice and consequently nonlinear. Most of the researchers have been focused on using the linearization or numerical techniques and try to find an equivalent term for the inherent nonlinear damping. This may cause that the obtained response of the structure do not be accurate enough to decide whether the TLCD is effective or not and since the prediction of optimal parameters such as mass ratio, frequency tuning ratio, head loss coefficient and damping ratio that minimize the structural response is dependent to the solution of the equation of motion, determining the precise answers is essential.

Hence, the homotopy perturbation method (HPM) and variational iteration method (VIM) are applied to obtain an analytical approximate solution for the coupled equation of motion of the building that is modeled as a single degree of freedom (SDOF) system and the TLCD that is installed on top of the building.

Finally, the Citicorp Center, a flexible skyscraper is used as an example to illustrate the design procedure for the TLCD under the harmonic type of wind excitation and to demonstrate the influence of the TLCD on mitigating the oscillation of super tall buildings.

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