A Nonlinear Equalization Technique for Frequency Selective Fading Channels

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***Abstract*—We propose a new equalization technique based on nonlinear Hammerstein type filters to mitigate the intersymbol interference (ISI) effect. This technique is a nonlinear generalization of the linear equalizer. In the present work, linear frequency selective fading channels in presence of additive white gaussian noise are considered and BPSK modulation is employed. Comparison of the simulation results based on our proposed technique with the results obtained when linear equalizer is employed, shows that our technique leads to a considerably better BER performance at moderate and higher SNRs. We also show that our method presents a better MSE estimator than the linear structure.**

***Keywords*— Channel Equalization, Frequency Selective Fading Channels, Hammerstein Filter, Nonlinear Signal Processing**

I. Introduction

In frequency selective channels, the transmitted signal is corrupted by intersymbol interference (ISI) as well as noise. Hence, in these channels, the optimum receiver is based on the maximum likelihood sequence estimation (MLSE) [1]. However, MLSE is a nonlinear method with a high computational complexity that increases exponentially with the channel memory length. As an alternative to MLSE, suboptimum receivers for frequency selective channels have been proposed. Linear and decision feedback equalizers (DFE) are the most common techniques [1]. Linear equalizer (LE) is simply a linear transversal filter with a limited number of taps. Linear transversal filters are also employed in DFE as feedforward and feedback blocks. Many other equalization techniques have also been proposed. Examples of theses methods can be found in the recent published works [1-3].

In this work we offer a generalized nonlinear structure for channel equalization, which is based on Hammerstein type filters. We employ this technique for frequency selective fading channels. Hammerstein filter is a nonlinear polynomial filter used in many applications such as system identification [4], [5], modelling [6], [7], echo cancellation [8], [9], and noise cancellation [10]. Hammerstein decision feedback equalization (HDFE) has been employed in fiber-wireless channel for compensation of nonlinear distortion in the electrical-to-optical converter [11], [12]. Also, in our previous work, we have proposed a new diversity combining technique based on Hammerstein filters to mitigate the fading effects in frequency selective fading channels [13].

This paper is organized as follows. In the next section we present the system model. Section III introduces our nonlinear Hammerstein equalization technique. In section IV, we explain our motivation for using these filters in the proposed system. Simulation results and discussions are presented in section V, before concluding the paper in section VI.

II. System model

In this section the equivalent low-pass discrete time model of the system is presented. In this work we employ BPSK modulation. The transmitted sequence $x(n)\in\{+1,-1\}$ is drawn from an i.i.d. source with equi-probable symbols. We consider a frequency selective fading channel, modeled by a tapped delay line with L taps, as:

$$\mathbf{H}=\left[h_1 \quad h_2 \quad \ldots \quad h_L\right], \tag{1}$$

where h_i is the random gain of the ith tap. These components are assumed to be real-valued zero-mean gaussian random variables with variance σ_{hi}^2. Furthermore, they are assumed uncorrelated and normalized to unity, i.e.:

$$\sum_{i=1}^{L} E\{\left|h_i\right|^2\}=1. \tag{2}$$

The channel fading is assumed sufficiently slow, such that the tap gains do not vary during one data frame. We also assume that the frequency selective fading channel has a specific power delay profile (PDP), which is the profile of the mean square values of the tap gains. Example of these profiles used in our simulations is presented in section V.

The received signal which is corrupted by ISI and noise is expressed as:

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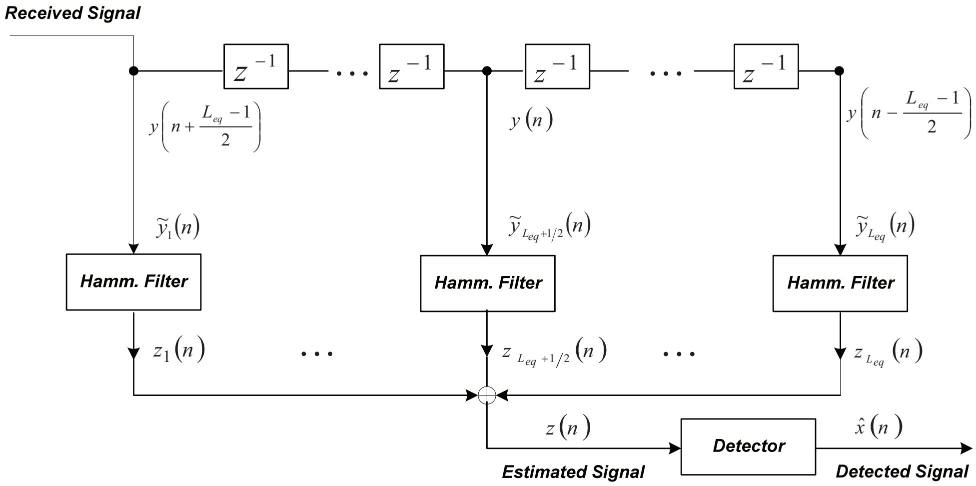

Fig. 1. Generalized Hammerstein Equalization Technique (GHE)

$$y(n)=\sum_{i=1}^{L} h_i\, x(n-i+1)+w(n), \tag{3}$$

where $w(n)$ is a real-valued zero-mean white gaussian noise with variance σ_w^2. Eq. (3) can be expressed in matrix form:

$$y(n)=\mathbf{H}\,\mathbf{X}(n)+w(n), \tag{4}$$

where $\mathbf{H}$ is the channel vector and $\mathbf{X}(n)$ is the received data vector, defined as:

$$\mathbf{X}(n)=[x(n)\quad x(n-1)\ldots x(n-L+1)]^{\mathrm{T}}. \tag{5}$$

In suboptimum receivers, the detected signal is obtained by passing $y(n)$ through an equalizer and a hard detector, respectively.

III. GENERALIZED HAMMERSTEIN EQUALIZATION TECHNIQUE

A. Equalizer Model

The structure of Generalized Hammerstein Equalization technique (GHE) is illustrated in Fig. 1. As shown in this figure, the received signal is passed through a delay line with L_{eq} taps. Then, the signal at each tap is applied to a Hammerstein filter of order D. The output polynomial of the ith filter is then:

$$z_i(n)=\sum_{k=1(k\ odd)}^{D} g_{ik}\,\tilde{y}_i^{\,k}(n) \qquad for \quad i=1,2,\ldots,L_{eq}, \tag{6}$$

where g_{ik} is the kth coefficient of the output polynomial of the ith filter, and $\tilde{y}_i(n)$ is defined as the signal at ith tap, i.e.:

$$\tilde{y}_i(n)=y\left(n-i+\frac{L_{eq}+1}{2}\right) \qquad for \quad i=1,2,\ldots,L_{eq}. \tag{7}$$

Note that only the odd powers appear in the summation of Eq. (6). Similar to our previous work [13], it can be proved that the terms corresponding to the even powers are equal to zero.

The filters outputs are summed to produce the equalizer output $z(n)$:

$$z(n)=\sum_{i=1}^{L_{eq}}\sum_{k=1(k\ odd)}^{D} g_{ik}\,\tilde{y}_i^{\,k}(n). \tag{8}$$

Eq. (8) can be expressed in matrix form:

$$z(n)=\mathbf{G}_{\mathbf{H}}^{\mathrm{T}}\,\mathbf{Y}_{\mathbf{H}}(n), \tag{9}$$

where $\mathbf{G}_{\mathbf{H}}$ is a $L_{eq}(D+1)/2\times 1$ vector that consists of coefficients g_{ik}, and $\mathbf{Y}_{\mathbf{H}}(n)$ is a $L_{eq}(D+1)/2\times 1$ vector defined as:

$$\mathbf{Y}_{\mathbf{H}}(n)=\left[\tilde{\mathbf{Y}}_{\mathbf{1}}^{\mathrm{T}}(n)\quad \tilde{\mathbf{Y}}_{\mathbf{3}}^{\mathrm{T}}(n)\quad \tilde{\mathbf{Y}}_{\mathbf{5}}^{\mathrm{T}}(n)\quad \ldots\quad \tilde{\mathbf{Y}}_{\mathbf{D}}^{\mathrm{T}}(n)\right]^{\mathrm{T}},\ D\quad odd, \tag{10}$$

where $\tilde{\mathbf{Y}}_{\mathbf{p}}(n)$ is a $L_{eq}\times 1$ vector defined by using Eq. (7):

$$\tilde{\mathbf{Y}}_{\mathbf{p}}(n)=\left[\tilde{y}_1^{\,p}(n)\quad \tilde{y}_2^{\,p}(n)\quad \ldots\quad \tilde{y}_{Leq}^{\,p}(n)\right]^{\mathrm{T}}. \tag{11}$$

$z(n)$ is an estimate of the transmitted symbol $x(n)$. Our goal is to find the coefficients g_{ik} such that the mean square error is minimized. $z(n)$ is then passed through a hard detector for making the output decision $\hat{x}(n)$.

B. Calculation of the Coefficients

The coefficients of the Hammerstein filters are found from the training mode by using the MSE criterion. In this mode, the transmitter sends a training sequence that is assumed to be known to the receiver as the desired signal $d(n)$. The error signal is defined as difference between the desired and estimated values:

$$e(n)=d(n)-z(n)=x(n)-z(n). \tag{12}$$

The cost function is defined as below:

$$\zeta=E\left\{e^2(n)\right\}. \tag{13}$$

The coefficients are computed such that to minimize ζ. Using Eqs. (9) and (12) in (13), we get:

$$\begin{aligned}\zeta&=E\left\{\left[\,x(n)-\mathbf{G_H}^{\mathbf{T}}\mathbf{Y_H}(n)\right]\left[\,x(n)-\mathbf{Y_H}^{\mathbf{T}}(n)\mathbf{G_H}\,\right]\right\}\\&=E\left\{x^2(n)\right\}-\mathbf{G_H}^{\mathbf{T}}E\left\{\mathbf{Y_H}(n)x(n)\right\}\\&\quad-E\left\{x(n)\mathbf{Y_H}^{\mathbf{T}}(n)\right\}\mathbf{G_H}+\mathbf{G_H}^{\mathbf{T}}E\left\{\mathbf{Y_H}(n)\mathbf{Y_H}^{\mathbf{T}}(n)\right\}\mathbf{G_H}\end{aligned} \tag{14}$$

If we define the $L_{eq}(D+1)/2\times1$ crosscorrelation vector:

$$\mathbf{P_H}=E\left\{\mathbf{Y_H}(n)x(n)\right\}, \tag{15}$$

and the $L_{eq}(D+1)/2\times L_{eq}(D+1)/2$ autocorrelation matrix:

$$\mathbf{R_H}=E\left\{\mathbf{Y_H}(n)\mathbf{Y_H}^{\mathbf{T}}(n)\right\}, \tag{16}$$

and note that $E\left\{x(n)\mathbf{Y_H}^{\mathbf{T}}(n)\right\}=\mathbf{P_H}^{\mathbf{T}}$, $\mathbf{G_H}^{\mathbf{T}}\mathbf{P_H}=\mathbf{P_H}^{\mathbf{T}}\mathbf{G_H}$, and $E\left\{x^2(n)\right\}=1$, we obtain:

$$\zeta=1-2\,\mathbf{G_H}^{\mathbf{T}}\,\mathbf{P_H}+\mathbf{G_H}^{\mathbf{T}}\,\mathbf{R_H}\,\mathbf{G_H}. \tag{17}$$

This is a quadratic function of vector $\mathbf{G_H}$ with a single global minimum. To minimize ζ, we need to have:

$$\nabla\zeta=\mathbf{0}, \tag{18}$$

where ∇ is the gradient operator. From Eqs. (17) and (18) and using the gradient properties we can write:

$$\nabla\zeta=2\,\mathbf{R_H}\,\mathbf{G_H}-2\,\mathbf{P_H}=\mathbf{0}. \tag{19}$$

Finally, the coefficients of Hammerstein filters are obtained by solving Eq. (19):

$$\mathbf{G_H}_{opt}=\mathbf{R_H}^{-1}\,\mathbf{P_H}, \tag{20}$$

assuming that $\mathbf{R_H}$ is invertible.

IV. Theoretical Basis

In this section we explain our motivation for using Hammerstein filters in the proposed system. For simplicity, we first consider a system with $L=2$ and $L_{eq}=3$. In this case at time *n*, the observed signals at the receiver are:

$$\begin{aligned}\theta_1(n)&=y(n+1)=h_1\,x(n+1)+h_2\,x(n)+w(n+1)\\\theta_2(n)&=y(n)=h_1\,x(n)+h_2\,x(n-1)+w(n)\\\theta_3(n)&=y(n-1)=h_1\,x(n-1)+h_2\,x(n-2)+w(n-1)\end{aligned} \tag{21}$$

Based on these observed data, we would like to estimate the transmitted symbol $x(n)$. Using the MSE criterion, the optimum bayesian estimator $z(n)$, is defined as [14]:

$$z=E\left\{x\mid\theta_1,\theta_2,\theta_3\right\}=\int_{-\infty}^{\infty}x\,p\left(x\mid\theta_1,\theta_2,\theta_3\right)dx, \tag{22}$$

where the notation $p(.)$ represents the probability density function (PDF), and the time index is omitted for notation simplicity. The conditional PDF can be written as:

$$p\left(x\mid\theta_1,\theta_2,\theta_3\right)=\frac{p\left(\theta_1,\theta_2,\theta_3\mid x\right)p(x)}{\int_{-\infty}^{\infty}p\left(\theta_1,\theta_2,\theta_3\mid x\right)p(x)\,dx}. \tag{23}$$

The noise components in Eq. (21) are uncorrelated zero mean gaussian random variables. Hence, for a particular channel occurrence, the joint conditional PDF of the observed data $\{\theta_i\}$, conditioned on the transmitted sequence becomes:

$$\begin{aligned}p\left(\theta_1,\theta_2,\theta_3\mid x\right)&=\prod_{i=1}^{3}p\left(\theta_i\mid x\right)\\&=\frac{1}{(\sqrt{2\pi\sigma_w^2})^3}\exp\left[-\sum_{i=1}^{3}\frac{\left(\theta_i-\alpha_i-\beta_i\,x\right)^2}{2\sigma_w^2}\right]\end{aligned}, \tag{24}$$

where:

$$\begin{cases}\alpha_1=h_1\,x(n+1)\\\alpha_2=h_2\,x(n-1)\\\alpha_3=h_1\,x(n-1)+h_2\,x(n-2)\end{cases}\quad\text{and}\quad\begin{cases}\beta_1=h_2\\\beta_2=h_1\\\beta_3=0\end{cases}, \tag{25}$$

are known values. Also, based on our assumptions, we have:

$$p(x)=\frac{1}{2}\left[\delta(x+1)+\delta(x-1)\right]. \tag{26}$$

Finally, by substituting Eqs. (23), (24), and (26) in Eq. (22), the optimum MSE estimator can be obtained as :

$$z=tgh\left[\,2\sum_{i=1}^{3}\beta_i\left(\theta_i-\alpha_i\right)\right]. \tag{27}$$

Now, we generalize this result for arbitrary values of L and L_{eq}. In this case using Eq. (7), the observed signals at time *n*, are:

$$\theta_i(n) = \widetilde{y}_i(n) \qquad i = 1,2,...,L_{eq}\,. \tag{28}$$

Also, the optimum estimator takes the following form:

$$z(n) = tgh\left[A_0 + \sum_{i=1}^{L_{eq}} A_i\,\theta_i(n)\right], \tag{29}$$

where A_0 and A_i are defined as:

$$\begin{cases} A_0 = -2\sum_{i=1}^{L_{eq}} \alpha_i\,\beta_i \\ A_i = 2\beta_i \qquad i = 1,2,...,L_{eq} \end{cases}, \tag{30}$$

and the values of $\{\alpha_i\}$ and $\{\beta_i\}$ can be found similar to Eq. (25). The Maclaurin expansion of Eq. (29) yields:

$$z(n) = \left[A_0 + \sum_{i=1}^{L_{eq}} A_i\,\theta_i(n)\right] - \frac{1}{3}\left[A_0 + \sum_{i=1}^{L_{eq}} A_i\,\theta_i(n)\right]^3 + \frac{2}{15}\left[A_0 + \sum_{i=1}^{L_{eq}} A_i\,\theta_i(n)\right]^5 - \cdots. \tag{31}$$

We can write this equation as $z(n) = S_1 + S_2$, where using Eqs. (28) and (31) we have:

$$\begin{cases} S_1 = \sum_{i=1}^{L_{eq}} \sum_{k=1}^{\infty} C_{ik}\,\widetilde{y}_i^{\,k}(n) \\ S_2 = Other\ Terms \end{cases}, \tag{32}$$

where the coefficients C_{ik} are known parameters. As can be seen, S_1 is similar to Eq. (8), which is the output of a GHE system. Furthermore, S_1 can in turn be written as $S_1 = S_{11} + S_{12}$, where:

$$\begin{cases} S_{11} = \sum_{i=1}^{L_{eq}} C_{ik}\,\widetilde{y}_i(n) \\ S_{12} = \sum_{i=1}^{L_{eq}} \sum_{k=2}^{\infty} C_{ik}\,\widetilde{y}_i^{\,k}(n) \end{cases}. \tag{33}$$

We can also see that S_{11} is similar to the output of a linear equalizer (LE) system.

From the above discussion we conclude that S_{11} is included in S_1 and also S_1 is a subset of the optimum estimator $z(n)$. This means that GHE system can be considered as a nonlinear generalization of LE system and therefore it is closer to the optimum estimator than LE. This conclusion will be verified by our simulation results in the next section.

V. SIMULATION RESULTS

In this section the average bit error rate (BER) and the average mean square error (MSE) are evaluated numerically for both GHE and LE techniques and the results are compared. The simulations are performed for a frequency selective fading channel with an exponentially decaying power delay profile shown in Fig. 2. This is an example of common profiles used in wireless communication channels [1]. We generate 10,000 random realizations of the channel and obtain the average BER and MSE results by Monte Carlo simulations. We also use a 100-bit sequence for training mode.

In Figs. 3-a and 3-b, the average BER versus SNR are shown for GHE and LE techniques. In these simulations, which are performed for two different number of taps $L_{eq} \in \{3,5\}$, we choose the order of Hammerstein filter $D = 5$. From these figures it is observed that at moderate and higher SNRs, GHE has a considerable better performance than LE.

For example, for $L_{eq} = 3$, when the $SNR = 40dB$, the average BER of GHE is 30 times lower than LE, which is a valuable advantage of our proposed technique. However, at low SNRs the performances of GHE and LE systems are almost the same. In other word, the nonlinear terms in Eq. (33) are not effective at low SNRs. This is due to the inherent property of all nonlinear systems at low signal to noise ratios. Examples of these behaviors are observed in decision feedback equalizers and FM modulators, in which their superiority over linear techniques appears when the SNR is above a threshold level.

Figs. 4-a and 4-b show the average MSE versus SNR for GHE and LE techniques when $L_{eq} \in \{3,5\}$ and $D = 5$. From these figures it is concluded that at moderate and higher SNRs, GHE is a better MSE estimator than LE. This confirms our theoretical discussion in section 4.

To see the effect of polynomial order D on the performance of GHE, we performed the simulations for different values of this parameter and observed that when $D > 5$, the system performance did not change notably. Hence, in this work we choose $D = 5$.

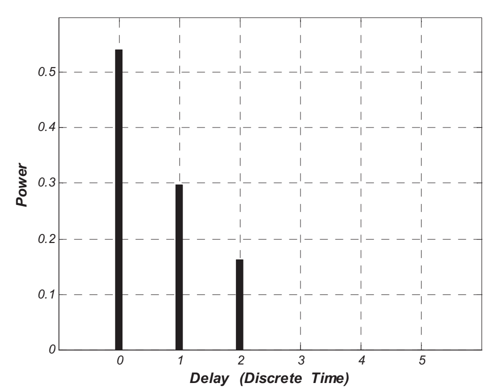

Fig. 2. Example of an exponentially decaying power delay profile

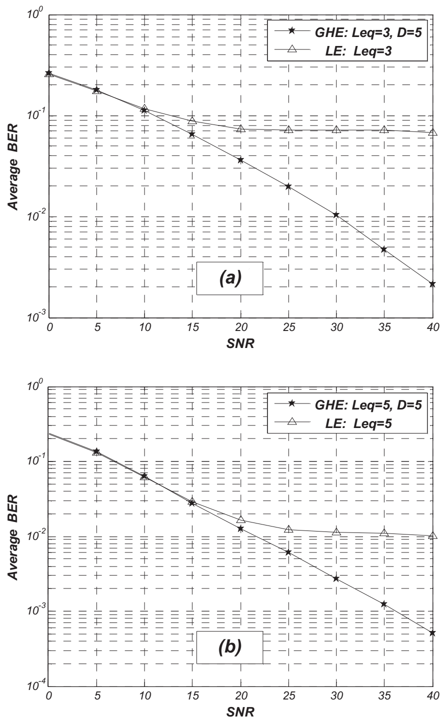

Fig. 3. Average BER for GHE and LE systems with $L_{eq} \in \{3,5\}$ and D=5

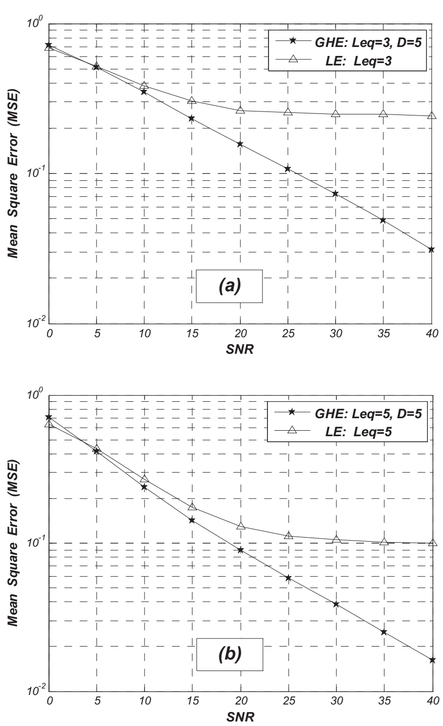

Fig. 4. Average MSE for GHE and LE systems with $L_{eq} \in \{3,5\}$ and D=5

VI. CONCLUSION

In this paper we introduced a nonlinear structure for channel equalization based on Hammerstein type filters. This technique is a generalization of linear technique in which the higher orders of the signal at each tap is used for equalization. We employed BPSK modulation and assumed frequency selective fading channels with a specific power delay profile. Comparison of our simulation results with the results obtained from linear equalizer, shows that:

- At moderate and higher SNRs, GHE is a better MSE estimator than LE.
- At moderate and higher SNRs, the average BER performance of GHE is superior to LE.
- Only the odd powers ($D \leq 5$) of the received signal are required for channel equalization.

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