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Vibration and instability of a viscous-fluid-conveying single-walled carbon nanotube embedded in a visco-elastic medium

P Soltani[1,2], M M Taherian[2] and A Farshidianfar[1]

[1] Department of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran
[2] Department of Mechanical Engineering, Islamic Azad University-Semnan Branch, Semnan, Iran

E-mail: payam.soltani@gmail.com

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Abstract
In this study, for the first time, the transverse vibrational model of a viscous-fluid-conveying single-walled carbon nanotube (SWCNT) embedded in biological soft tissue is developed. Nonlocal Euler–Bernoulli beam theory has been used to investigate fluid-induced vibration of the SWCNT while visco-elastic behaviour of the surrounding tissue is simulated by the Kelvin–Voigt model. The results indicate that the resonant frequencies and the critical flow velocity at which structural instability of nanotubes emerges are significantly dependent on the properties of the medium around the nanotube, the boundary conditions, the viscosity of the fluid and the nonlocal parameter. Detailed results are demonstrated for the dependence of damping and elastic properties of the medium on the resonant frequencies and the critical flow velocity. Three standard boundary conditions, namely clamped–clamped, clamped–pinned and pinned–pinned, are applied to study the effect of the supported end conditions. Furthermore, it is found that the visco-elastic foundation causes an obvious reduction in the critical velocity in comparison with the elastic foundation, in particular for a compliant medium, pinned–pinned boundary condition, high viscosity of the fluid and small values of the nonlocal coefficient.

Nomenclature

c	dimensionless damping coefficient of visco-elastic medium
d_0	outer diameter of SWCNT
e_0a	nonlocal parameter
e_n	dimensionless nonlocal parameter
k	dimensionless elastic stiffness of the visco-elastic medium
m	mass ratio
m_c	mass per unit length of SWCNT
m_f	mass per unit length of flow fluid in the SWCNT
t	time
v	uniform mean velocity of flow fluid in the SWCNT
v_n	dimensionless uniform mean velocity of flow fluid in the SWCNT
v_{crt}	dimensionless critical flow velocity for the first mode
$\%\Delta v_{crt}$	variation ratio of the critical flow velocity for the first mode
x	dimensionless axial coordinate
$y(x)$	dimensionless mode shape
$\bar{y}(x)$	mode shape
A_f	cross-sectional area of flow fluid in the SWCNT
C	damping coefficient of the visco-elastic medium
E	Young's modulus of SWCNT
EI	bending rigidity of SWCNT
I	Moment of inertia of the cross-sectional area of the SWCNT
K	elastic stiffness of the visco-elastic medium
L	length of the SWCNT
X	axial coordinate
$Y(X,t)$	transverse deflection of the SWCNT

β dimensionless complex circular frequency
η dimensionless viscosity of flow fluid in the SWCNT
μ viscosity of flow fluid in the SWCNT
ρ_c mass density of SWCNT
ρ_f mass density of flow fluid in the SWCNT
ω complex circular frequency

1. Introduction

In recent years, extensive research has shown that carbon nanotubes (CNTs) with hollow cylindrical geometry exhibit exceptional mechanical properties and chemical and thermal stability [1–4]. These properties cause CNTs to be of great scientific interest for many applications in nanomechanical systems and nanobiological devices; such as fluid storage, fluid transport and drug delivery [5–8]. In nanomedicine, nanotubes can be used as biocompatible and supportive substrates, and as pharmaceutical excipients for creating versatile drug delivery systems (DDS) [9]. In this way, single-walled carbon nanotubes (SWCNTs) act as tiny straws or nanochannels to deliver drugs directly into the target cells and consequently, the therapeutic properties of the drug will be improved with a reduction of undesirable side effects [9, 10]. For instance, these kinds of SWCNT-based DDS have been successfully trialled for use in treating Alzheimer disease [11] and cancer [12]. Since CNT is intended to be used as a drug carrier in DDS, nanotubes should be embedded in soft biological tissues of the body. Therefore, the mechanical behaviour of the medium and their consequent interactions with the fluid-induced vibration of the nanotube become very important. Soft organic tissues are in general characterized by visco-elastic models, and usually a Kelvin–Voigt model (a simple and efficient mechanical model consisting of springs and dashpots in parallel) can be employed to simulate the interactions [13–16].

Recently, many studies have been devoted to the modelling of CNTs using some types of continuum-based elasticity theories. The Euler–Bernoulli beam theory has been used to predict the mechanical and vibrational characteristics of CNTs [17–20]. Moreover, in several investigations, instability and flow-induced vibration of CNTs have been discussed using continuum approaches. For instance, Yoon *et al* [21] studied the influence of an internal moving fluid on the free vibration and flow-induced structural instability of CNTs. The paper indicates that the internal moving fluid could substantially affect resonant frequencies, in particular for longer CNTs of larger innermost radius at higher flow velocities. They also considered the influence of fluid velocity on the free vibration and flutter instability of cantilever CNTs based on a continuum elastic model [22]. The results have shown that a moderately stiff surrounding elastic medium (such as polymers) could significantly mitigate the effect of an internal moving fluid and eliminate flutter instability within the practical range of flow velocities. The effects of a nonviscous fluid flow on the vibration and instability of a multi-walled CNT have been studied using both Timoshenko and Euler–Bernoulli classical beam theories [10]. The main finding of this work has been that the Timoshenko theory predicts the loss of stability at lower fluid flow velocities compared with the Euler–Bernoulli beam model, in particular for stubby nanotubes. Wang *et al* [23] developed a model for investigating the coupling vibrational behaviour of a fluid-filled CNT in a Winkler-like foundation. The effects of the different boundary conditions, aspect ratio, surrounding elastic medium, mass density of the fluid and layer number on the fluid-induced vibrations of CNTs have been discussed widely. Nonlocal elastic beam theory has been applied to the analysis of free transverse vibration of the fluid-conveying SWCNTs and the significant influences of the nonlocal parameter on the natural frequencies and mode shapes have been evaluated [24]. Divergence, flutter and coupled-mode flutter instability have been considered for a SWCNT conveying fluid with simply supported and clamped ends [25]. Furthermore, the nonlocal Euler–Bernoulli beam theory has been employed to study the fundamental frequency of a viscous-fluid-conveying SWCNT embedded in an elastic medium and the effects of the nonlocal parameter, viscosity, aspect ratio and elastic medium constant have been shown [26].

However, to the best of our knowledge, no investigation has been performed on fluid-induced vibration and instability of CNTs on visco-elastic foundations. In this study, for the first time, a nonlocal Euler–Bernoulli model has been applied to analyse the free transverse vibration of a viscous-fluid-conveying SWCNT embedded in a Kelvin–Voigt medium. The effects of damping and stiffness of the foundation, viscosity of the moving fluid, nonlocal parameter and boundary conditions on the resonant frequencies and the structural instability of the SWCNT are considered in detail. Moreover, the variation of the critical velocity, as a divergence instability threshold, has been discussed based on the various parameters.

2. The model for CNTs conveying fluid

Figure 1 shows a SWCNT, as a hollow cylindrical tube in a Kelvin–Voigt visco-elastic medium. By neglecting gravity effect and axial loads, the governing equation of motion of a SWCNT conveying fluid using nonlocal Euler–Bernoulli theory can be expressed as

$$EI\frac{\partial^4 Y}{\partial X^4} + 2m_f v\frac{\partial^2 Y}{\partial X\partial t} + m_f v^2\frac{\partial^2 Y}{\partial X^2} - \mu A_f\left(v\frac{\partial^3 Y}{\partial X^3} + \frac{\partial^3 Y}{\partial t\partial X^2}\right) + \frac{\partial^2}{\partial t^2}\left[(m_c + m_f)Y - m_c(e_0a)^2\frac{\partial^2 Y}{\partial X^2}\right] + KY + C\frac{\partial Y}{\partial t} = 0, \quad (1)$$

where X is the axial coordinate, t is the time and $Y(X, t)$ is the transverse deflection of the CNT. EI, L and m_c are the bending rigidity, length and the mass per unit length of the SWCNT, respectively. In addition, v, μ, A_f and m_f are the uniform mean velocity, viscosity, the cross-sectional area and the mass per unit length of fluid flow in the SWCNT, in that order. e_0a represents a nonlocal parameter revealing the nanoscale effect on the response of the structures [24]. K and C represent the elastic stiffness and damping coefficient of the visco-elastic medium around the SWCNT based on the Kelvin–Voigt model. The two last terms in equation (1) are the forces exerted by the surrounding medium.

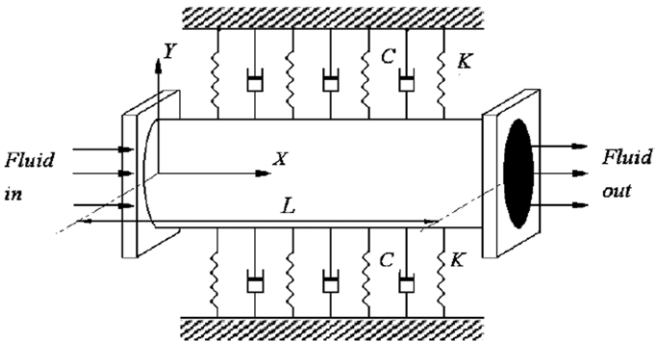

Figure 1. A fluid-conveying SWCNT embedded in a Kelvin–Voigt visco-elastic foundation.

For a self-excited vibration, the solution of equation (1) can be written as

$$Y(X,t) = \bar{y}(X)\mathrm{e}^{\mathrm{i}\omega t} \tag{2}$$

and ω is the complex circular frequency.

Defining the following dimensionless parameters:

$$x = \frac{X}{L}, \quad y = \frac{\bar{y}}{L}, \quad m = \frac{m_\mathrm{f}}{m_\mathrm{c}+m_\mathrm{f}}, \quad v_n = \sqrt{\frac{m_\mathrm{f}}{EI}}vL,$$
$$e_n = \frac{e_0 a}{L}, \quad \eta = \frac{\mu A_\mathrm{f}}{\sqrt{EIm_\mathrm{f}}}, \quad \beta = \frac{\omega}{\sqrt{EI/(m_\mathrm{c}+m_\mathrm{f})L^4}}, \tag{3}$$
$$k = \frac{KL^4}{EI}, \quad c = \frac{CL^2}{\sqrt{EI(m_\mathrm{c}+m_\mathrm{f})}}.$$

Equation (1) may be written in the dimensionless form

$$\frac{\mathrm{d}^4 y}{\mathrm{d}x^4} + 2\mathrm{i}\beta\sqrt{m}v_n\frac{\mathrm{d}y}{\mathrm{d}x} + [v_n^2 + (1-m)\beta^2 e_n^2]\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$$
$$-\eta\left(v_n\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} + \mathrm{i}\beta\sqrt{m}\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right) - \beta^2 y + ky + \mathrm{i}\beta c y = 0. \tag{4}$$

The solution of the complex ordinary differential equation given in equation (4) is

$$y(x) = A\mathrm{e}^{\mathrm{i}\alpha x}. \tag{5}$$

where A is a constant.

By substituting equation (5) into equation (4), the following equation is obtained:

$$\alpha^4 - 2\alpha\beta\sqrt{m}v_n - [v_n^2 + (1-m)\beta^2 e_n^2]\alpha^2$$
$$-\eta(-\mathrm{i}v_n\alpha^3 - \mathrm{i}\beta\sqrt{m}\alpha^2) - \beta^2 + k + \mathrm{i}\beta c = 0. \tag{6}$$

Boundary conditions. Three typical boundary conditions have been considered, a simply supported beam at both ends or pinned–pinned condition (P–P):

$$\frac{\partial^2 Y(0,t)}{\partial X^2} = Y(0,t) = \frac{\partial^2 Y(L,t)}{\partial X^2} = Y(L,t) = 0; \tag{7}$$

a beam clamped at one end and simply supported at the other end, i.e. clamped–pinned condition (C–P):

$$\frac{\partial Y(0,t)}{\partial X} = Y(0,t) = \frac{\partial^2 Y(L,t)}{\partial X^2} = Y(L,t) = 0 \tag{8}$$

and a beam clamped at both ends or the clamped–clamped boundary condition (C–C):

$$\frac{\partial Y(0,t)}{\partial X} = Y(0,t) = \frac{\partial Y(L,t)}{\partial X} = Y(L,t) = 0. \tag{9}$$

In addition, the dimensionless form of the boundary conditions will be as follows:

For P–P condition

$$\frac{\mathrm{d}^2 y(0)}{\mathrm{d}x^2} = y(0) = \frac{\mathrm{d}^2 y(1)}{\mathrm{d}x^2} = y(1) = 0. \tag{10}$$

For C–P condition

$$\frac{\mathrm{d}y(0)}{\mathrm{d}x} = y(0) = \frac{\mathrm{d}^2 y(1)}{\mathrm{d}x^2} = y(1) = 0. \tag{11}$$

For C–C condition

$$\frac{\mathrm{d}y(0)}{\mathrm{d}x} = y(0) = \frac{\mathrm{d}y(1)}{\mathrm{d}x} = y(1) = 0. \tag{12}$$

Using a similar method to the one described in [24], the characteristic equation in dimensionless frequencies β can be obtained for each type of the boundary conditions.

3. Results and discussion

In most of the previous papers related to the self-excited vibration of fluid-conveying SWCNTs, it was assumed that Young's modulus and the thickness of the SWCNT were around 1 TPa and 0.3 nm, respectively [21, 22, 24–26]. However, in this paper, we use 3.4 TPa for Young's modulus and 0.1 nm for the effective thickness of SWCNT as the most recent investigations indicate [27, 28]. As a case study, mechanical and dimensional properties of the SWCNT and the fluid are shown in table 1. The elastic and damping coefficients of the Kelvin–Voigt foundation are defined in table 2.

β is the dimensionless complex frequency of the SWCNT conveying fluid. The real component of frequency Re[β] represents the dimensionless resonant frequency while the imaginary component Im[β] is related to the damping of the model and called the decaying rate of amplitude [22]. In addition, when $v_n = k = c = \mu = 0$, in our study, β reaches a real number, equal to the natural frequency of a SWCNT and exactly the same as in previous studies [24–26].

3.1. Resonant frequencies

Various parameters influence the flow-induced resonant frequencies of a fluid-conveying SWCNT. In this section, we have focused on the mean velocity of the internal flow, the mechanical behaviour of the surrounding medium, the boundary conditions and the nonlocal parameter.

3.1.1. The effects of the visco-elastic foundation. Figure 2 shows the real part of the dimensionless frequency β against the dimensionless flow velocity v_n for the first three vibration modes. The SWCNT is assumed to have a C–C boundary condition, and different dimensionless damping and stiffness

Table 1. The parameters used for the model (SWCNT and fluid) used as a case study.

	Fluid			SWCNT					
Parameter	m_f	ρ_f	μ	m_c	ρ_c	E	EI	e_0a/L	L/d_0
Dimension	kg m^{-1}	kg m^{-3}	Pa s	kg m^{-1}	kg m^{-3}	TPa	N m	—	—
Value	2.64×10^{-14}	1×10^3	1.12×10^{-3}	4.26×10^{-15}	2.3×10^3	3.5	2.74×10^{-23}	0.05	50

Table 2. The elastic and damping constants used for the foundation of the SWCNT as a Kelvin–Voigt model.

	Medium			
Parameter	K	k	C	c
Dimension	MPa	—	Pa s	—
Value	0.1	29.5	1.02×10^{-4}	10

parameters (c and k) are used to represent the effects of the visco-elastic behaviour of the foundation. The figures indicate that the resonant frequencies decrease parabolically with the dimensionless flow velocity of the internal moving fluid so that, as the flow velocity increases, the resonance frequency reduces to zero. The vanishing frequency and the existence of adjacent neutral equilibrium states cause structural or divergence instability [21]. In this study, this dimensionless flow velocity for the first mode is called the dimensionless critical flow velocity v_{crt}. As seen in figure 2, divergence instability for the first mode occurs when $v_{crt} \cong 6$ but by increasing the dimensionless damping parameter c of the visco-elastic foundation and/or decreasing the dimensionless stiffness parameter k, the divergence instability takes place at lower flow velocities. A previous study [24] predicts divergence instability for the second and third modes at flow velocities of about 9 and 12, respectively. However, in this study, with the new bending stiffness EI of SWCNT, the divergence instability in the second mode takes place when the flow velocity reaches about 12 and in the third mode, the resonant frequency does not vanish within the range of flow velocities. Furthermore, it is apparent from the figures that the resonant frequencies are significantly influenced by the parameters k and c. In fact, the compliant visco-elastic media with high damping properties cause a decrease in the induced frequencies, in particular for the first and second modes.

Moreover, damping of the visco-elastic foundation and fluid viscosity make the fluid-induced vibration of the SWCNT nonconservative in nature. This means that, unlike previous studies [10, 24, 25], when resonant frequencies are not zero ($\mathrm{Re}(\beta) \neq 0$), the decaying rate of amplitude $\mathrm{Im}[\beta]$ does not remain zero anymore and consequently, the fluid flow in the nanotube contributes to the vibration of the SWCNT. On the other hand, when the decaying rate of amplitude is positive ($\mathrm{Im}[\beta]\rangle 0$), vibration of the SWCNT is damped with decaying amplitude for a low flow velocity while for a sufficiently high flow velocity, the imaginary part of frequency changes sign to negative ($\mathrm{Im}[\beta]\langle 0$) and the amplitude starts to grow with time [22]. To analyse the effects of damping, figure 3 shows the decaying rate of amplitude $\mathrm{Im}[\beta]$ as a function of dimensionless flow velocity v_n for the first three modes of the case study. From this figure, it is clear that, in the first mode, when the dimensionless flow velocity v_n is lower than the dimensionless critical flow velocity ($v_n < v_A = v_{crt}$), the decaying rate is positive, the amplitude decreases with time and the CNT is damped. However, as the dimensionless flow velocity increases beyond v_B, the decaying rate of amplitude will be negative. Hence, the SWCNT can gain energy from the flow and the induced vibration would be amplified for a sufficiently high flow velocity (i.e. $v_n > v_B$). Furthermore, when the dimensionless flow velocity is within the range $v_A < v_n < v_B$, the imaginary part of frequency is nonzero (both negative and positive) while the real part is exactly zero (see figure 2(*a*)). Thus, the nanotube does not vibrate in this range of flow velocities in the first mode. Similarly, for the second and third mode, the decaying rate of amplitude is positive when dimensionless flow velocity is lower than v_C and v_D, respectively and the vibrations are damped in these ranges. Furthermore, the second mode of vibration becomes unstable for the flow velocities higher than v_C.

3.1.2. The effects of the boundary conditions. Vibration behaviour of a fluid-conveying nanotube is completely related to the boundary conditions. However, getting the theoretical boundary conditions to simulate the actual stiffness of the supports is still a challenge, in particular for nanostructures [30]. Consequently, to show the effects of the boundary stiffness on the vibration characteristics of CNTs, typical and standard boundary conditions are usually applied [21–26]. As mentioned above, this model has been solved for P–P, C–P and C–C conditions. The natural frequencies have been shown as a function of flow velocity v_n and boundary conditions in figure 4. The results indicate that while the bending stiffness of the SWCNT rises from P–P to C–C, the associated resonant frequencies and the dimensionless critical flow velocity v_{crt} increase.

3.1.3. The effects of the nonlocal parameter. As the small-sized scale becomes significant for nanostructures like CNTs, the local or classical continuum theories may be no longer accurate enough and cannot predict the behaviour of nanoscale materials anymore. Unlike the local continuum elasticity, the nonlocal elasticity theory regards that the stress at a given point in a body depends not only on the strain at that point but also on those at all points of the body [31]. This definition of nonlocal elasticity is based on the lattice dynamics and observations on photon dispersion. Nonlocal continuum models admit size dependence and small-scale effects in the elastic solutions of nanostructures, and consider forces between atoms and internal length scale in the construction of constitutive equations. Therefore, the nonlocal elasticity theories represent a more accurate mechanical model on the nanoscale [29]. In order to

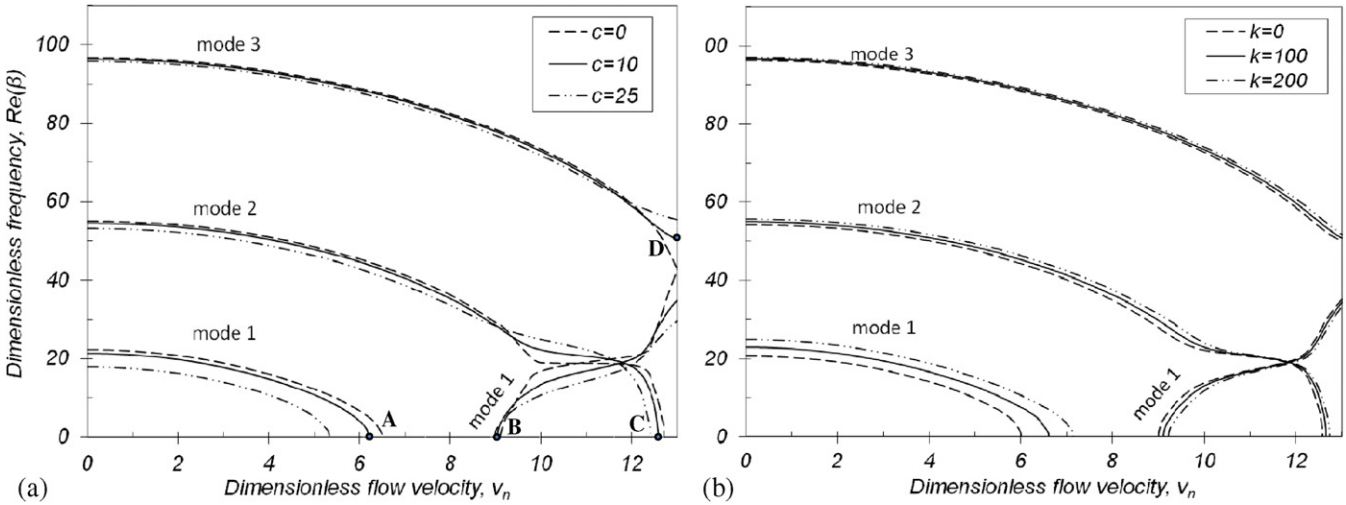

Figure 2. The dimensionless resonant frequency against the dimensionless flow velocity v_n for a clamped–clamped SWCNT (*a*) with different values of the dimensionless damping parameter c, (*b*) with different values of the dimensionless elastic parameter k.

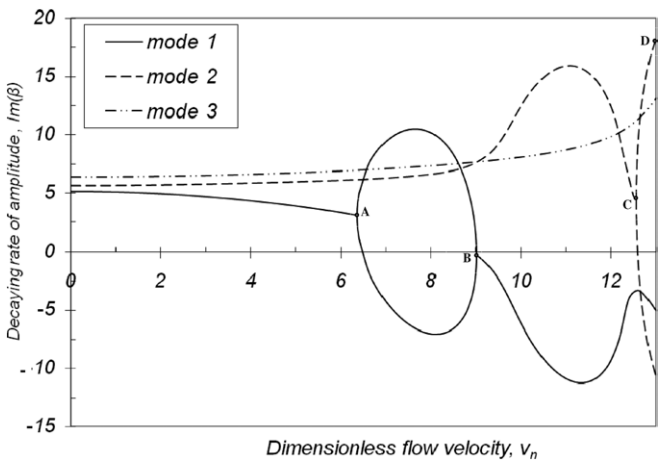

Figure 3. The decaying rate of amplitude against the dimensionless flow velocity v_n for the case study.

study the small-scale effect, the induced resonant frequencies are plotted against the dimensionless flow velocity v_n with certain nonlocal parameters e_n (figure 5). According to this figure, the nonlocal Euler–Bernoulli theory predicts smaller resonant frequencies compared with the local continuum results ($e_n = 0$). Actually, the nonlocal theory introduces a more flexible model as the CNT can be viewed as atoms linked by elastic springs while the local model assumes spring constants to take on an infinite value [32]. Consequently, the frequency reduction in the nonlocal model is physically justifiable. Furthermore, the difference between the nonlocal and local results will be significant for higher vibration modes and thus, the small-scale effects cannot be ignored.

4. Structural instability and critical velocity

As stated above, for the first mode, the critical velocity v_{crt} is the flow velocity at which the resonant frequency reduces to zero (i.e. the vibrations cease). At this flow velocity, the nanotube undergoes structural or divergence instability. As with the resonant frequency, the critical flow velocity is a function of the visco-elastic behaviour of the foundation, the boundary conditions, the viscosity of the fluid and the nonlocal parameter. To see the effects of damping due to the visco-elastic foundation clearly, 'the variation ratio' of the critical flow velocity $\%\Delta v_{\mathrm{crt}}$ has been defined to be the percentage of critical flow velocity increment for the nanotube embedded in the elastic foundation (with $C = 0$) compared with the visco-elastic foundation (with $C \neq 0$).

$$\%\Delta v_{\mathrm{crt}} \equiv \frac{100 \times (v_{\mathrm{crt\ at\ C=0}} - v_{\mathrm{crt\ at\ C=C}})}{v_{\mathrm{crt\ at\ C=0}}}. \qquad (13)$$

Certainly, variation ratio $\%\Delta v_{\mathrm{crt}}$ provides a better illustration for the pure effects of the damping nature of the foundation. In addition, increasing this parameter implies that the critical flow velocity shifts to lower values as the nanotube vibrates in a visco-elastic medium.

Figures 6–9 represent the variation ratio $\%\Delta v_{\mathrm{crt}}$ as a function of the dimensionless damping parameter c of the Kelvin–Voigt medium, while the effects of a certain parameter have been evaluated in each figure. All the figures reveal that with increasing damping property of the medium, the variation ratio $\%\Delta v_{\mathrm{crt}}$ increases. In other words, in all the following conditions, divergence instability occurs at lower flow velocities when a SWCNT vibrates in a visco-elastic foundation with higher damping characteristics. This means that the visco-elastic medium with a high damping property (such as a soft biological tissue) dissipates the vibrational energy quickly and the resonant frequency vanishes at a lower flow velocity.

The variation ratio $\%\Delta v_{\mathrm{crt}}$ is very sensitive to the stiffness, due to the boundary conditions and the foundation. As the stiffness of the model decreases from C–C to P–P, the jump of the variation ratio $\%\Delta v_{\mathrm{crt}}$ is considerable (figure 6). This shows the significant influence of the boundary condition on the critical flow velocity of a SWCNT on a visco-elastic foundation. Figure 7 represents the effects of medium stiffness on the divergence instability. The figure indicates that with

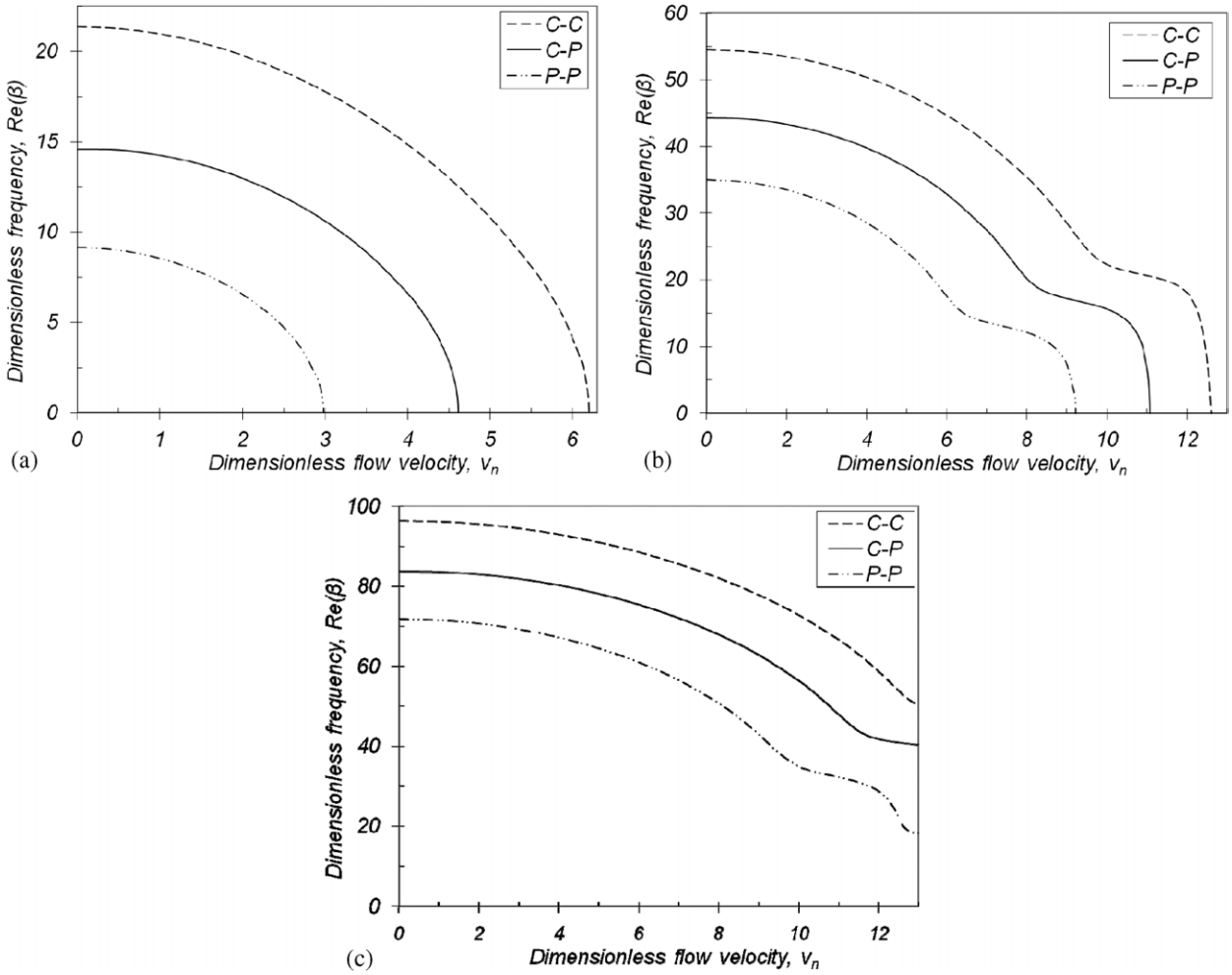

Figure 4. The dimensionless resonant frequency against the dimensionless flow velocity v_n for clamped–clamped, clamped–pinned and pinned–pinned boundary conditions: (*a*) in the first mode, (*b*) in the second mode, (*c*) in the third mode.

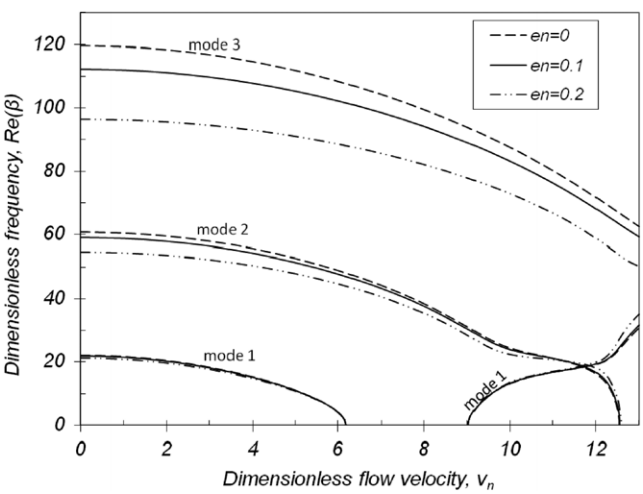

Figure 5. The dimensionless resonant frequency against the dimensionless flow velocity v_n for a clamped–clamped SWCNT with different values of the dimensionless nonlocal parameter e_n.

an increase in the dimensionless stiffness parameter k, the variation ratio $\%\Delta v_{\mathrm{crt}}$ decreases and the effect of the damping properties of the foundation on the critical velocity is reduced. In addition, for the Kelvin–Voigt foundation (for $C > 20$), the present model predicts a reduction in the critical velocity by more than about 10%, compared with previous studies [25, 26].

The importance of nonlocal elasticity theory in the divergence instability and the associated critical flow velocity is demonstrated in figure 8. According to this figure, the nonlocal Euler–Bernoulli theory estimates smaller values of $\%\Delta v_{\mathrm{crt}}$ than the local theory does. Moreover, the effect of the dimensionless damping parameter c on the critical velocity diminishes with the increase in the dimensionless nonlocal parameter.

Finally, to investigate the effect of fluid viscosity, the variation ratio $\%\Delta v_{\mathrm{crt}}$ is plotted as a function of the dimensionless damping parameter c and the dimensionless viscosity parameter η (figure 9). The results reveal that for media with high damping properties, reduction in the critical flow velocity will be significant, in particular for high viscosity fluids.

5. Conclusion

Transverse vibration and divergence instability of a viscous-fluid-conveying SWCNT are represented using the nonlocal Euler–Bernoulli beam theory. Pursuant to the visco-elastic

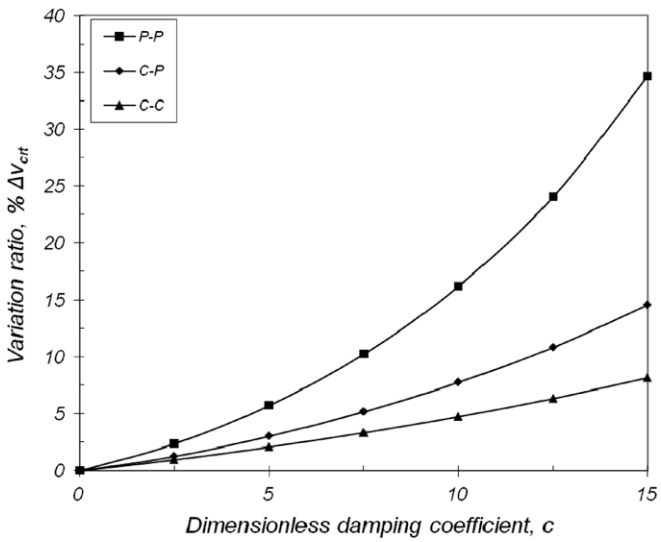

Figure 6. The variation ratio $\%\Delta v_{\mathrm{crt}}$ against the dimensionless damping constant of the Kelvin–Voigt foundation c with different types of boundary conditions.

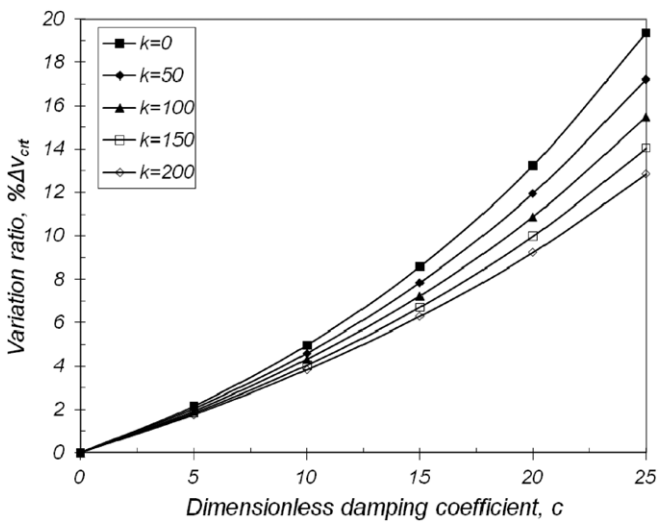

Figure 7. The variation ratio $\%\Delta v_{\mathrm{crt}}$ against the dimensionless damping constant of the Kelvin–Voigt foundation c with different values of the dimensionless elastic parameter k.

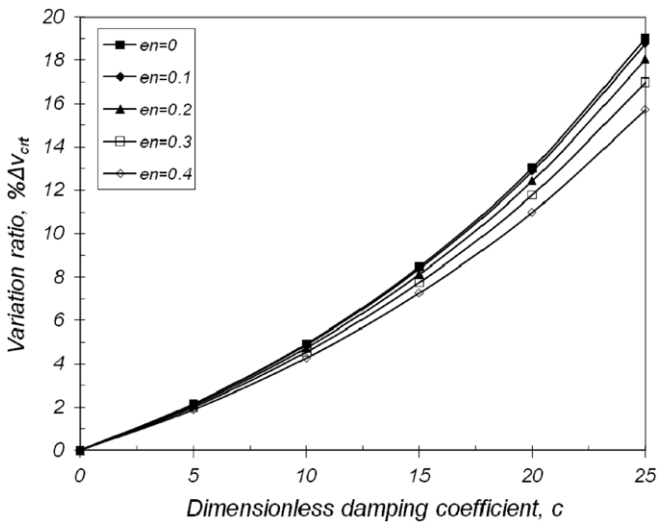

Figure 8. The variation ratio $\%\Delta v_{\mathrm{crt}}$ against the dimensionless damping constant of the Kelvin–Voigt foundation c with different values of the dimensionless nonlocal parameter e_n.

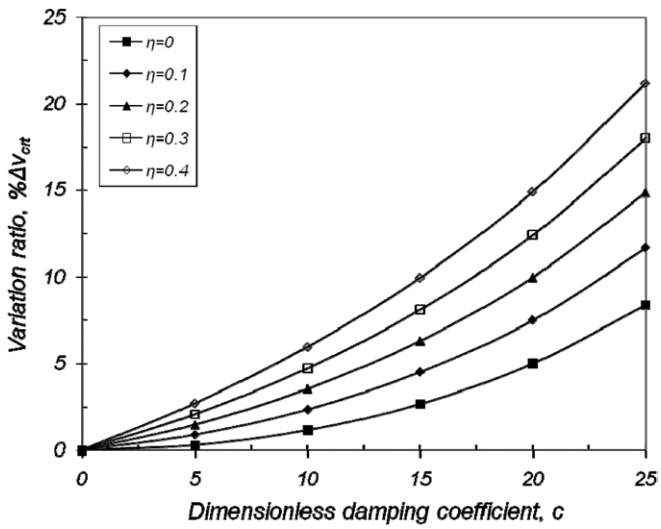

Figure 9. The variation ratio $\%\Delta v_{\mathrm{crt}}$ against the dimensionless damping constant of the Kelvin–Voigt foundation c with different values of the dimensionless viscosity parameter of the fluid η.

behaviour of soft tissues, the nanotube is assumed to be embedded in a Kelvin–Voigt foundation. It is shown that the damping nature of the medium substantially affects the natural frequencies, in particular for the first and second modes. Moreover, our investigation determines that structural instability and the associated critical flow velocity can be affected by the elastic and damping characteristics of the medium, the end conditions, the viscosity of the fluid and the nonlocal parameter. Increasing the stiffness of the model, due to the boundary condition or to the foundation, causes the critical flow velocity to increase, whereas for a medium with a higher damping property, the divergence instability occurs at a lower flow velocity. Furthermore, with a decrease in the nonlocal constant, as the viscosity of the fluid increases, the divergence instability happens at a lower flow velocity for a visco-elastic medium in comparison with an elastic medium.

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