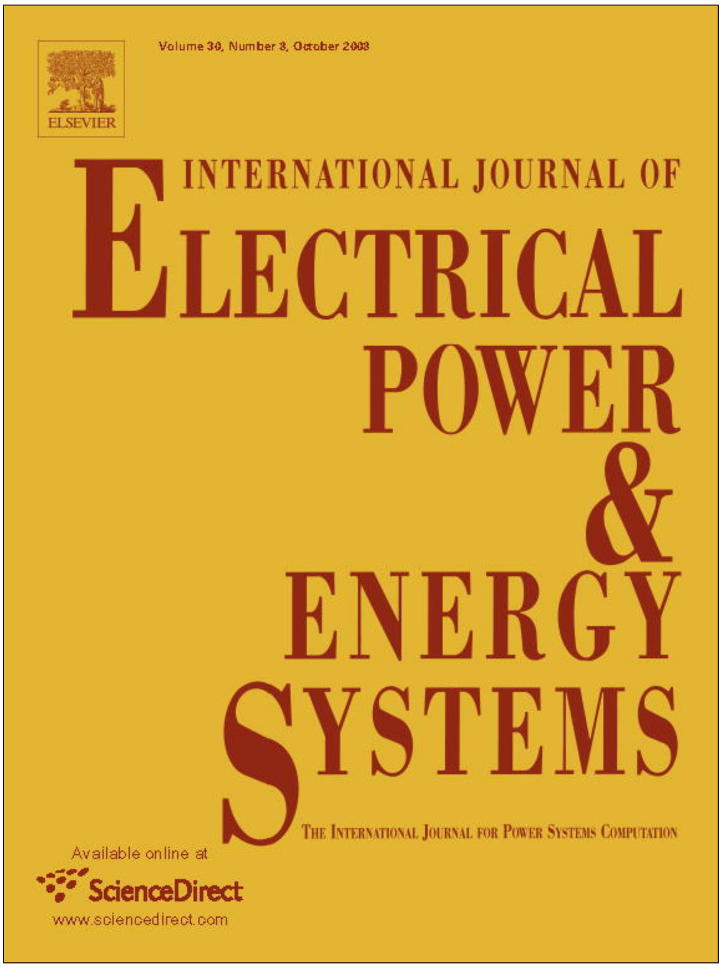
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Considering system non-linearity in transmission pricing

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ABSTRACT

In this paper a new approach for transmission pricing is presented. The contribution of a contract on power flow of a transmission line is used as extent-of-use criterion for transmission pricing. In order to determine the contribution of each contract on power flow of each transmission line, first the contribution of each contract on each voltage angle is determined, which is called voltage angle decomposition. To this end, DC power flow is used to compute a primary solution for voltage angle decomposition. To consider the impacts of system non-linearity on voltage angle decomposition, a method is presented to determine the share of different terms of sine argument in sine value. Then the primary solution is corrected in different iterations of decoupled Newton–Raphson power flow using the presented sharing method. The presented approach is applied to a 4-bus test system and IEEE 30-bus test system and the results are analyzed.

1. Introduction

Transmission pricing is an important issue in restructured power systems. Different usage-based methods have been presented for transmission pricing [1–5]. Many networks use *postage stamp rate* method for transmission pricing [1,4]. This method is like mailing a letter within a country. This method assumes that the entire transmission system is used, regardless of the actual facilities that carry the transmission services. In *postage stamp* method network users are charged based on the magnitude of their transacted power and average embedded cost of the network. In this method users are not differentiated by the "extent-of-use" of network facilities. *Contract path* method is based on the assumption that power flows through a certain, prespecified path [1–4]. In this method first the least cost electrical path between generation and load points is determined for a given transaction. The transaction is charged a *postage stamp rate* that is computed either separately for each transmission system or as a grid average. In reality the actual path taken by a transaction may be quit different from the specified contract path. In *MW mile* method MW flows related to each transaction are computed in all transmission lines using DC power flow. To compute the transmission charge of a given transaction, the magnitude of its MW flow in every line is multiplied by its length and a weighting factor reflecting the cost per unit capacity of the line and summed over all transmission lines. This method ensures the full recovery of fixed transmission cost and approximately reflects the actual usage of transmission network. *MVA mile* method is an extended version of the *MW mile* method [1,8]. This method includes charging for reactive power in addition to the charging for active power. In this method MVA flows related to each transaction are computed in all transmission lines using tracing methods or sensitivity factors. In *Distribution factors* method distribution factors are computed using linear power flow [1,6,9–11]. In general, generation distribution factors are used to analyze system security and contingency. They are used to approximately determine the contribution of generations and loads on transmission line flows. Distribution factors can be used to allocate transmission cost to transactions, generators, or loads. In *tracing algorithms* first contribution of transmission users in network usage is determined based on proportional sharing principle [1,8,12–16]. There are two tracing algorithms, which are recognized as Bialek's and Kirchen's tracing algorithms. Tracing algorithms are extended to allocate fixed transmission costs based on contribution of transmission users in network usage. Some *AC Power Flow* methods including *AC Flow Sensitivity*, *Full AC Power Flow Solution*, and *Power Flow Decomposition* have been proposed to allocate transmission cost [1,3,7,17–20]. In *AC Flow Sensitivity* method the sensitivity of transmission line flows to the bus power injections are derived from AC power flow models. This method uses the same logic of the DC flow distribution factors. In *Full AC Power Flow Solution* two power flow simulations is performed to determine the combined impacts caused by the transactions on the system: one for base case, no transactions, and one for the operating case including all the transactions. For each transaction t two power flow simulations are also performed. In one case only transaction t is included and in the other all transactions except for

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t are included. Marginal and incremental impact of each individual transaction on the system is obtained by comparing the results of these two simulations with the base case. Then the "fair resource allocation" problems are solved to distribute the MW/MVAR line flows to each transaction. The *Power Flow Decomposition Method* is based on superposition of all transactions on the system. In this method the network flows are decomposed into components associated with individual transactions plus one interaction component to account for the nonlinear nature of power flow models. In [21,22] a method is presented for voltage angle decomposition. The presented method was used for loss allocation and transmission pricing. The method does not consider system non-linearity in voltage angle decomposition. In this paper in order to take into account system non-linearity in voltage angle decomposition, first a method is presented for determining the share of x_k in $\sin(x_1 + \ldots + x_m)$. Based on the proposed sharing method, a decomposed decoupled power flow approach is presented. Then the contribution of each transaction in power flow of each transmission line is computed based on voltage angles decomposition. The contribution of a transaction in power flow of a transmission line is used as extent-of-use criterion for transmission pricing.

The paper is organized as follows: In Section 2, a method is presented for determining the share of x_k in $\sin(x_1 + \ldots + x_m)$. Based on the presented sharing method, a decomposed decoupled power flow approach is presented in Section 3 to compute the contributions of contracts on voltage angles. In Section 4 contribution of contracts in line power flows is computed. Transmission pricing based on decomposed line power flows is described in Section 5. The proposed method is applied to a 4-bus test system and IEEE 30-bus test system in Section 6. Conclusion in Section 7 closes the paper.

2. How much the share of x_k in $A = \sin\left(\sum_{k=1}^{m} x_k\right)$ is?

Suppose x_k and $(x_1 + x_2 + \cdots + x_m)$change in the range of [-π/2 π/2]. In order to determine the share of x_k in A, the sine function is approximated with finite terms of its Maclaurin series:

$$A = \sin\left(\sum_{k=1}^{m} x_k\right) \cong \sum_{i=1}^{M} (-1)^{i+1} \frac{1}{(2i-1)!} \left(\sum_{k=1}^{m} x_k\right)^{2i-1} \tag{1}$$

As M tends to infinite, the right hand side of (1) tends to A. If $M = 5$, the maximum error between A and the right hand side of (1) is 3.5e − 6 which is a proper approximation. In order to determine the share of x_k in A, the share of x_k in $B = (x_1 + x_2 + \ldots + x_m)^n$ should be determined. B can be expanded as follows:

$$B = \left(\sum_{k=1}^{m} x_k\right)^n = \sum_{r_m=0}^{n} \cdots \sum_{r_1=0}^{n} \binom{n}{r_1} \binom{n-r_1}{r_2} \cdots \binom{n - \sum_{j=1}^{m-1} r_j}{r_m} x_1^{r_1} x_2^{r_2}, \ldots, x_m^{r_m} \delta\left[n - \sum_{j=1}^{m} r_j\right] \tag{2}$$

where $\delta[t] = 1$ if $t = 0$ and $\delta[t] = 0$ otherwise. There is symmetry in terms of (2). Hence, the share of x_k in B is equal to:

$$S(x_k, B) = \sum_{r_m=0}^{n} \cdots \sum_{r_1=0}^{n} \binom{n}{r_1} \binom{n-r_1}{r_2} \cdots \binom{n - \sum_{j=1}^{m-1} r_j}{r_m} x_1^{r_1} x_2^{r_2} \ldots x_m^{r_m} \delta\left[n - \sum_{j=1}^{m} r_j\right] \frac{u(r_k)}{\sum_{i=1}^{m} u(r_i)} \tag{3}$$

where $S(x_k,B)$ is the share of x_k in B, $u(t) = 1$ if $t > 0$ and $u(t) = 0$ otherwise. Combining (1) in (3) yields:

$$S(x_k, A) \cong \sum_{i=1}^{M} (-1)^{i+1} \frac{1}{(2i-1)!} \sum_{r_m=0}^{2i-1} \cdots \sum_{r_1=0}^{2i-1} \binom{2i-1}{r_1} \binom{2i-1-r_1}{r_2} \cdots \binom{2i-1 - \sum_{j=1}^{m-1} r_j}{r_m} x_1^{r_1} x_2^{r_2} \ldots x_m^{r_m} \delta\left[n - \sum_{j=1}^{m} r_j\right] \frac{u(r_k)}{\sum_{i=1}^{m} u(r_i)} \tag{4}$$

where $S(x_k,A)$ is the share of x_k in A. Therefore, A can be expanded as follows:

$$A = \sin\left(\sum_{k=1}^{m} x_k\right) = \sum_{k=1}^{m} S(x_k, A) \tag{5}$$

Example. How much is the contribution of x_1 in $A = \sin(x_1 + x_2 + x_3)$?

For simplicity suppose M=2 in Maclaurin series, then:

$$A = \sin(x_1 + x_2 + x_3) \cong (x_1 + x_2 + x_3) - 0.1667(x_1 + x_2 + x_3)^3$$

According to (4) the share of x_1 in $(x_1 + x_2 + x_3)$ is equal to x_1 and the share of x_1 in $(x_1 + x_2 + x_3)^3$ is equal to $x_1^3 + 1.5x_1^2x_2 + 1.5x_1^2x_3 + 2x_1x_2x_3 + 1.5x_1x_2^2 + 1.5x_1x_3^2$. Hence, the share of x_1 in $A = \sin(x_1 + x_2 + x_3)$ is equal to:

$$S(x_1, A) = x_1 - 0.1667(x_1^3 + 1.5x_1^2x_2^+ 1.5x_1^2x_3 + 2x_1x_2x_3 + 1.5x_1x_2^2 + 1.5x_1x_3^2).$$

3. Voltage angle decomposition

In order to compute the contribution of each contract on the power flow of each line, first the contribution of each contract on the voltage angle of each bus is determined. A primary solution for angle allocation is computed by DC power flow:

$$\delta^0 = \delta^{dc} = B^{-1}\mathbf{P}_{sch} \tag{6}$$

where δ^0 is voltage angles vector and is equal to $\delta^0 = [\delta_2^0 \delta_3^0 \ldots \delta_{nb}^0]^T$, $\mathbf{P}_{sch}$ is vector of scheduled power and is equal to $\mathbf{P}_{sch} = [P_{sch2}\ P_{sch3} \ldots P_{schnb}]^T$, nb is number of buses, and B is the imaginary part of admittance matrix if all lines are assumed lossless. It is assumed that bus 1 is slack bus. Total scheduled power in each bus is equal to the sum of scheduled power of all contracts at that bus:

$$\mathbf{P}_{sch} = \sum_{k=1}^{nc} \mathbf{P}_{sch}^{(k)} \tag{7}$$

where nc is number of contracts and $\mathbf{P}_{sch}^{(k)}$ is the vector of scheduled power of contract k. Here a contract means a bilateral contract, set of transactions of a scheduling coordinator, or set of transactions of a power pool. Substituting (7) in (6) yields:

$$\delta^0 = \sum_{k=1}^{nc} \delta^{0,(k)} \tag{8}$$

where $\delta^{\nu,(k)}$ is the contribution of contract k in voltage angles in iteration ν and $\delta^{0,(k)}$ is defined as follows:

$$\delta^{0,(k)} = B^{-1}\mathbf{P}_{sch}^{(k)} \tag{9}$$

using (9), the contribution of each contract in voltage angle of each bus is computed. In computing $\delta^{0,(k)}$, it is assumed that the system is linear. To take into account system non-linearity, $\delta^{0,(k)}$ is corrected using decoupled Newton–Raphson power flow. If resistance of transmission lines is neglected and voltage is assumed to be 1 pu at each bus, injection power of bus i at iteration ν can be computed as follows:

$$P^{v}_{\text{cal } i} = \sum_{j=1}^{\text{nb}} \frac{1}{x_{ij}} \sin(\delta^{v}_{ij}) \quad (10)$$

where $\delta^{v}_{ij} = \delta^{v}_{i} - \delta^{v}_{j}$, and x_{ij} is the series reactance of line*ij*.
Substituting $\delta^{v} = \sum_{k=1}^{\text{nc}} \delta^{v,(k)}$ in (10), yields:

$$P^{v}_{\text{cal } i} = \sum_{j=1}^{\text{nb}} \frac{1}{x_{ij}} \sin\left(\sum_{k=1}^{\text{nc}} \delta^{v,(k)}_{ij}\right) \quad (11)$$

where $\delta^{v,(k)}_{ij} = \delta^{v,(k)}_{i} - \delta^{v,(k)}_{j}$. According to (5), (11) can be written as follows:

$$P^{v}_{\text{cal } i} = \sum_{j=1}^{\text{nb}} \frac{1}{x_{ij}} \sum_{k=1}^{\text{nc}} S(\delta^{v,(k)}_{ij}, A_{ij}) \quad (12)$$

where $S(\delta^{v,(k)}_{ij}, A_{ij})$ is the share of $\delta^{v,(k)}_{ij}$ in $A_{ij} = \sin\left(\sum_{k=1}^{\text{nc}} \delta^{v,(k)}_{ij}\right)$ and is computed using the approach presented in Section 2. Since in power system all quantities are expressed in perunit, a high accurate approximation is needed for sine function in (1) and (4). Suppose base of power is 1000 MVA and x_{ij} is 10^{-3} pu. According to (10) in order to power error is less than 10^{-5} pu (10^{-2} MW), sine approximation error must be less than 10^{-8}. Hence, M in (1) must be greater than 5 for $\sum_{k=1}^{m} x_k \leqslant \pi/3$. Eq. (12) can be rewritten as follows:

$$P^{v}_{\text{cal } i} = \sum_{k=1}^{\text{nc}} \sum_{j=1}^{\text{nb}} S\left(\delta^{v,(k)}_{ij}, \frac{1}{x_{ij}} A_{ij}\right) = \sum_{k=1}^{\text{nc}} S(\delta^{v,(k)}_{ij}, P^{v}_{\text{cal } i}) = \sum_{k=1}^{\text{nc}} P^{v,(k)}_{\text{cal } i} \quad (13)$$

where $P^{v,(k)}_{\text{cal } i}$ is the contribution of contract k in injection power of bus i at iteration v. In each iteration of decoupled Newton–Raphson $\Delta\delta^{v}$ can be calculated as follows:

$$\Delta\delta^{v} = J_{11}^{-1}(\mathbf{P}_{\text{sch}} - \mathbf{P}^{v}_{\text{cal}}) \quad (14)$$

substituting (7), (13), and $\delta^{v} = \sum_{k=1}^{\text{nc}} \delta^{v,(k)}$ in (14) yields:

$$\Delta\delta^{v,(k)} = J_{11}^{-1}(\mathbf{P}^{(k)}_{\text{sch}} - \mathbf{P}^{v,(k)}_{\text{cal}}) \quad (15)$$

The primary solution of (9) is corrected at each iteration of decoupled Newton–Raphson using (15) and (16):

$$\delta^{v,(k)} = \delta^{v-1,(k)} + \Delta\delta^{v,(k)} \quad (16)$$

in this way the linear DC power flow solution is forced to go toward non-linear AC solution through piecewise lines. System non-linearity is taken into account by correcting the contribution of contracts in voltage angles in each iteration of decoupled Newton–Raphson. As decoupled Newton–Raphson converges, the contribution of each contract in each voltage angle is computed considering system non-linearity.

Note that in (9) and (15) it is assumed that $\delta^{(k)}_{1} = 0$ i.e. the share of each contract in voltage angle of slack bus is zero. Note that no assumption was made for selecting slack bus. Hence any bus can be selected as slack bus even if some contracts have load at it.

4. Contribution of contracts in line power flows

After computing the contribution of each contract in each voltage angle, the contribution of each contract in power flow of each transmission line can be computed as follows:

$$P_{\text{line}ij} = \frac{1}{x_{ij}} \sin(\delta_{ij}) = \frac{1}{x_{ij}} \sin\left(\sum_{k=1}^{\text{nc}} \delta^{(k)}_{ij}\right) = \frac{1}{x_{ij}} \sum_{k=1}^{\text{nc}} S(\delta^{(k)}_{ij}, A_{ij}) \quad (17)$$

assuming $P_{\text{line}ij} = \sum_{k=1}^{\text{nc}} P^{(k)}_{\text{line}ij}$, (17) yields:

$$P^{(k)}_{\text{line}ij} = \frac{1}{x_{ij}} S(\delta^{(k)}_{ij}, A_{ij}) \quad (18)$$

where $P_{\text{line}ij}$ is total power of line*ij*, $P^{(k)}_{\text{line}ij}$ is the contribution of contract k in power flow of line*ij*, $\delta^{(k)}_{ij} = \delta^{(k)}_{i} - \delta^{(k)}_{j}$, and $\delta^{(k)}_{i}$ is the contribution of contract k in voltage angle of bus i. In order to measure how much the presented approach takes into account the effects of non-linearity, Voltage Angle Error (VAE) at iteration v at bus i and Line Power Error (LPE) at iteration v at line*ij* is defined as follows:

$$\text{VAE}^{v}_{i} = \left|\delta_i - \sum_{k=1}^{\text{nc}} \delta^{v,(k)}_{i}\right| \quad (19)$$

$$\text{LPE}^{v}_{ij} = \left|P_{\text{line}ij} - \sum_{k=1}^{\text{nc}} P^{v,(k)}_{\text{line}ij}\right| \quad (20)$$

Improvement of these criteria at an iteration shows how much non-linearity is considered at this iteration.

5. Transmission pricing

The first step for transmission pricing is to determine the extent-of-use criterion. In the proposed method, the extent-of-use of contract k in transmission line*ij* is equal to the contribution of contract k in power of line*ij*. Some contracts produce counter flow in some lines. Counter flows not only does not occupy transmission capacity but also release transmission capacity. Therefore, it is assumed that the extent-of-use of contracts in lines in which create counter flow is zero. Hence the extent-of-use of contract k in line*ij* at hour h, $U^{(k)}_{ij}(h)$, is defined as follows:

$$U^{(k)}_{ij}(h) = \begin{cases} P^{(k)}_{\text{line}ij}(h) & \text{if } P^{(k)}_{\text{line}ij}(h) \cdot P_{\text{line}ij}(h) > 0 \\ 0 & \text{if } P^{(k)}_{\text{line}ij}(h) \cdot P_{\text{line}ij}(h) \leqslant 0 \end{cases} \quad (21)$$

Suppose R_{ij} is the value that must be returned in 1 h due to investment and operation cost of line*ij*. Assume $P_{\text{line}ij}(h)$, $P^{(k)}_{\text{line}ij}(h)$, and $U^{(k)}_{ij}(h)$ are constant during hour h. The share of contract k in return value of line*ij* at hour h, $R^{(k)}_{ij}(h)$, is equal to:

$$R^{(k)}_{ij}(h) = \frac{U^{(k)}_{ij}(h)}{\sum_{j=1}^{\text{nc}} U^{(j)}_{ij}(h)} R_{ij} \quad (22)$$

The share of contract k in return value of line*ij* from hour h_1 to hour h_2, $R^{(k)}_{ij}(h_1, h_2)$, is equal to:

$$R^{(k)}_{ij}(h_1, h_2) = \sum_{h=h_1}^{h_2} R^{(k)}_{ij}(h) \quad (23)$$

In (23) investment and operation cost of line*ij* is allocated to different contracts base on hourly network usage. Costs can be allocated based on the use of contracts from the network capacity at daily, weakly, monthly or yearly peak load.

6. Numerical results

6.1. 4-Bus test system

Consider the 4-bus test system that shown in Fig. 1. Parameters of transmission lines, generation data, and load data are given in Tables 1 and 2. Table 3 shows the active power contracts. Tables 4 and 5 show voltage angles, line power flows, and the share of

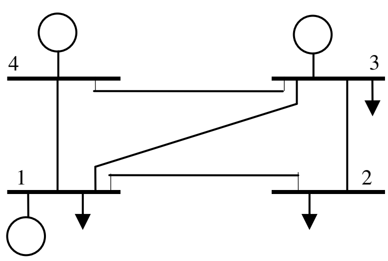

Fig. 1. 4-bus test system.

Table 1
Parameters of transmission lines of 4-bus test system

From bus	To bus	Resistance (pu)	Inductance (pu)	Limit (MW)
1	2	0.02	0.08	250
1	3	0.03	0.12	250
1	4	0.01	0.05	150
2	3	0.02	0.06	150
3	4	0.01	0.03	150

Table 2
Generation and load data of 4-bus test system

Bus no.	Bus type	Generation (MW)	Load (MW)
1	PV	1000	400+j100
2	PV	0	700+j50
3	PV	400	300+j30
4	Slack	–	0+j0

Table 3
Active power contracts of 4-bus test system in MW

	Bus 1	Bus 2	Bus 3	Bus4
1-Power pool contracts	400	−300	−100	0
2-Bilateral contract 1	−400	0	400	0
3-Bilateral contract 2	200	−200	0	0
4-Bilateral contract 3	400	−200	−200	0

Table 4
The share different contracts in voltage angles of 4-bus test system computed using decomposed DC power flow

Bus no.	$\delta_i^{(0)}$ (°)	$\delta_i^{(0,1)}$ (°)	$\delta_i^{(0,2)}$ (°)	$\delta_i^{(0,3)}$ (°)	$\delta_i^{(0,4)}$ (°)
1	3.902	3.5793	−5.3396	1.4963	4.1661
2	−14.6717	−6.1347	−0.4288	−4.1613	−3.9469
3	−2.5013	−2.2944	3.4228	−0.9591	−2.6705
4	0	0	0	0	0

Table 5
The share different contracts in line power flows of 4-bus test system computed using decomposed DC power flow

Line no.	P_{lineij} (pu)	$P^{(1)}_{lineij}$ (pu)	$P^{(2)}_{lineij}$ (pu)	$P^{(3)}_{lineij}$ (pu)	$P^{(4)}_{lineij}$ (pu)
1–2	3.7473	1.9581	−0.9830	1.1382	1.6341
1–3	0.8747	0.7998	−1.1903	0.3342	0.9311
1–4	1.3087	1.1990	−1.7871	0.5011	1.3956
2–3	−3.1623	−0.9980	−1.0009	−0.8320	−0.3314
3–4	−1.3093	−1.2004	1.7901	−0.5018	−1.3972

contracts on voltage angles and line power flows which are computed using decomposed DC power flow. The computed voltage angles and the share of contracts in voltage angles are used as initial solution for decomposed Newton–Raphson power flow. Decomposed decoupled Newton–Raphson power flow converges in five iterations. The process stops when the maximum of absolute of voltage angle deviations is less that 1e−15 radian. Tables 6 and 7 show the voltage angles, line power flows, and the share of contracts on voltage angles and line power flows. Figs. 2 and 3 show voltage angle error and line power flow error at different iterations of decomposed Newton–Raphson power flow. Power flow of contract 1 in line 1–2 at different iterations, which has maximum variation, is drawn in Fig. 4. Figs. 2–4 show non-linearity is taken into account in different iteration until linear solution converges non-linear solution. The share of different contracts in return value of different lines in percentage i.e.

$$R'^{(k)}_{ij}(h) = (R^{(k)}_{ij}(h)/R_{ij}).100$$

Table 6
The share different contracts in voltage angles of 4-bus test system computed using decomposed decoupled Newton–Raphson power flow

Bus no.	$\delta_i^{(0)}$ (°)	$\delta_i^{(1)}$ (°)	$\delta_i^{(2)}$ (°)	$\delta_i^{(3)}$ (°)	$\delta_i^{(4)}$ (°)
1	3.9567	3.6320	−5.4215	1.5245	4.2216
2	−14.8839	−6.2321	−0.4022	−4.2291	−4.0206
3	−2.5634	−2.3342	3.4620	−0.9833	−2.7079
4	0	0	0	0	0

Table 7
The share different contracts in line power flows of 4-bus test system computed using decomposed decoupled Newton–Raphson power flow

Line no.	P_{lineij} (pu)	$P^{(1)}_{lineij}$ (pu)	$P^{(2)}_{lineij}$ (pu)	$P^{(3)}_{lineij}$ (pu)	$P^{(4)}_{lineij}$ (pu)
1–2	3.7993	1.9872	−1.0040	1.1568	1.6592
1–3	0.8906	0.8122	−1.2064	0.3412	0.9436
1–4	1.3087	1.1990	−1.7871	0.5011	1.3956
2–3	−3.2007	−1.0128	−1.0040	−0.8432	−0.3408
3–4	−1.3093	−1.2004	1.7901	−0.5018	−1.3972

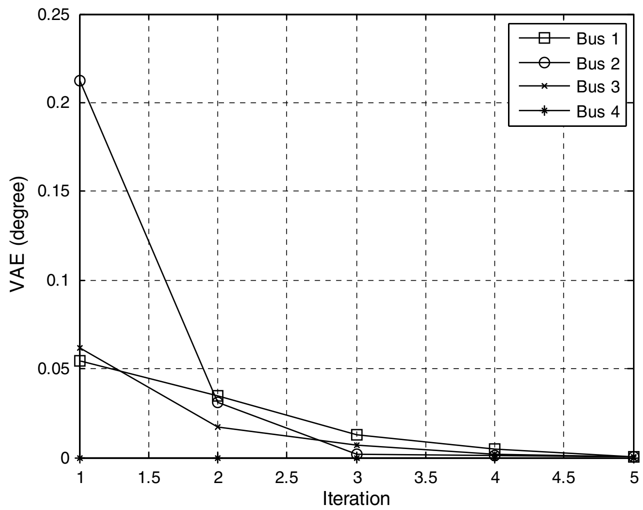

Fig. 2. Voltage angle errors at different iterations for 4-bus test system.

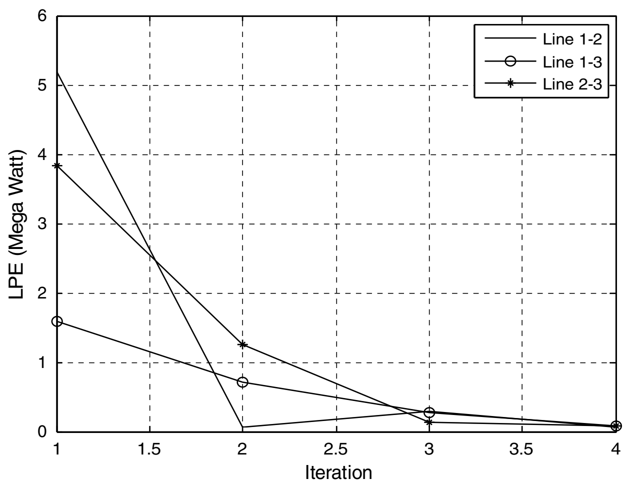

Fig. 3. Line power errors at different iterations for 4-bus test system.

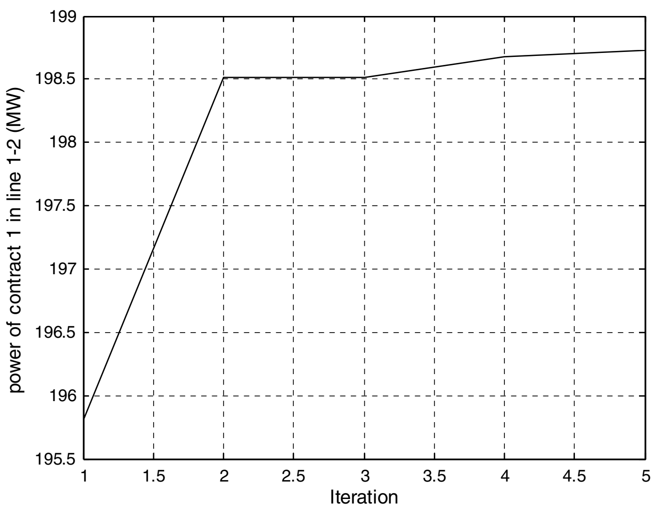

Fig. 4. Power flow of contract 1 in line 1–2 for 4-bus test system.

Table 8
The share of contracts in return value of lines in percentage

Line no.	$R'^{(1)}_{ij}(h)$	$R'^{(2)}_{ij}(h)$	$R'^{(3)}_{ij}(h)$	$R'^{(4)}_{ij}(h)$
1–2	41.37	0	24.08	34.55
1–3	38.73	0	16.27	45
1–4	38.73	0	16.19	45.08
2–3	31.64	31.37	26.34	10.65
3–4	38.73	0	16.19	45.08

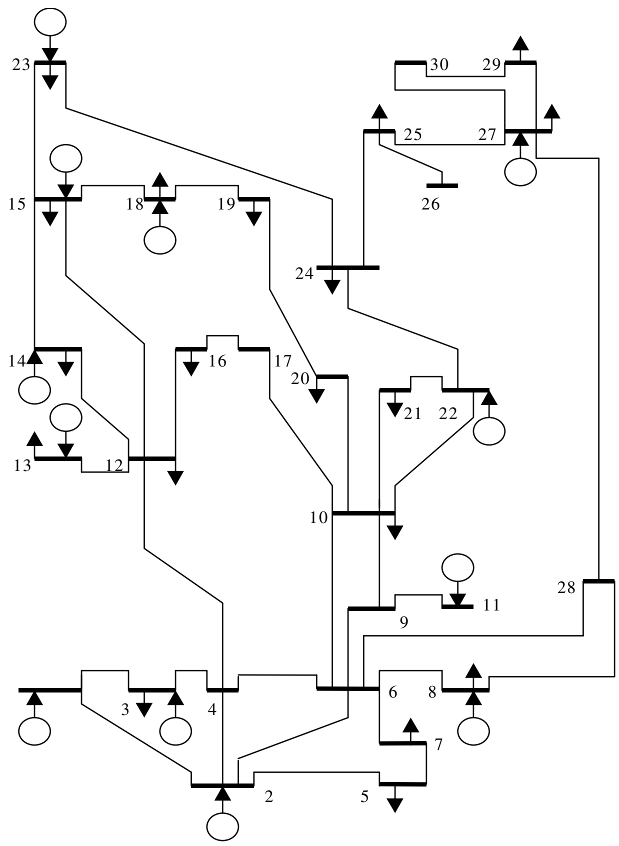

Fig. 5. Single line diagram of IEEE 30 buses test system.

is given in Table 8. As Table 7 shows, contract 2 creates counter power flow in lines 1–2, 1–3, 1–4, and 3–4 and hence only contracts 1 and 3 should pay for this lines base on their power flow shares in these lines. None of the contracts create counter power flow in line 2–3 and hence all contracts should pay for this line base on their power flow shares in this line. Table 8 confirms the results.

6.2. IEEE 30-Bus test system

In this subsection decomposed power flow is applied to IEEE 30-bus test system [23], which is shown in Fig. 5. Table 9 shows the active power contracts that are considered for IEEE 30-bus test system. Tables 10 and 11 show the voltage angles, line power flows, and the share of contracts in voltage angles and line power flows. Voltage angle error of different buses and line power error of different lines at final iteration of decomposed Newton–Raphson power flow are shown in these tables. Figs. 6 and 7 show voltage angle error for a few buses and line power flow error for a few lines at different iterations of decomposed decoupled Newton–Raphson

Table 9
Active power contracts of IEEE 30-bus test system in MW

	Generation	Consumption
1-Power pool contracts	400 MW at bus 1	200 MW at bus 7
		40 MW at bus 8
		100 MW at bus 21
		60 MW at bus 29
2-Scheduling coordinator 1	300 MW at bus 23	200 MW at bus 10
		100 MW at bus 25
3-Scheduling coordinator 2	160 MW at bus 13	200 MW at bus 3
	240 MW at bus 14	200 MW at bus 19
4-Scheduling coordinator 3	300 MW at bus 1	150 MW at bus 3
	300 MW at bus 2	150 MW at bus 5
		300 MW at bus 7

Table 10
The share of contracts in voltage angles of IEEE 30-bus test system computed using decomposed decoupled Newton–Raphson power flow

Bus no.	δ_i	$\delta_i^{(1)}$	$\delta_i^{(2)}$	$\delta_i^{(3)}$	$\delta_i^{(4)}$
1	1.0302	0.7036	0.1021	−0.1093	0.3332
2	0.7735	0.5287	0.1003	−0.0948	0.2387
3	0.3940	0.4318	0.1083	−0.1568	0.0101
4	0.4141	0.3768	0.1096	−0.0813	0.0083
5	0.1280	0.3237	0.0946	−0.0832	−0.2076
6	0.2782	0.3033	0.0889	−0.0717	−0.0429
7	−0.0572	0.2017	0.0912	−0.0764	−0.2743
8	0.2575	0.2832	0.0871	−0.0707	−0.0428
9	0.1854	0.2138	0.0452	−0.0393	−0.0349
10	0.1363	0.1665	0.0222	−0.0222	−0.0307
11	0.1854	0.2138	0.0452	−0.0393	−0.0349
12	0.8548	0.2608	0.2530	0.3520	−0.0117
13	1.0807	0.2608	0.2530	0.5760	−0.0117
14	1.3079	0.2459	0.3056	0.7699	−0.0140
15	0.8423	0.2246	0.3807	0.2536	−0.0173
16	0.5462	0.2203	0.1539	0.1913	−0.0199
17	0.2575	0.1824	0.0611	0.0409	−0.0275
18	0.4131	0.2065	0.2689	−0.0414	−0.0215
19	0.1027	0.1934	0.1882	−0.2550	−0.0245
20	0.1112	0.1866	0.1463	−0.1962	−0.0261
21	0.1361	0.1070	0.0651	−0.0064	−0.0303
22	0.1653	0.1172	0.0789	−0.0013	−0.0301
23	1.0076	0.1866	0.6724	0.1699	−0.0219
24	0.3742	0.1321	0.2197	0.0504	−0.0286
25	0.0259	0.0971	−0.0475	0.0091	−0.0334
26	0.0259	0.0971	−0.0475	0.0091	−0.0334
27	0.0189	0.0755	−0.0043	−0.0164	−0.0364
28	0.2452	0.2739	0.0782	−0.0654	−0.0421
29	−0.1615	−0.1049	−0.0043	−0.0164	−0.0364
30	0	0	0	0	0

power flow. Power flow of contract 4 in line 5–2 at different iterations, which has maximum variation, is drawn in Fig. 8. Figs. 6–8 show that voltage angle errors and line power errors vanish and non-linearity is more taken into account as number of iterations increases. The share of different contracts in return value of different lines is given in Table 12. The Table 11 shows that:

- Contract 1 creates counter power flow in line 10–22 and hence only contracts 2, 3, and 4 should pay for these lines base on their power flow shares.
- Contracts 1 and 4 create counter power flow in lines 3–4, 4–12, and 12–14 and hence only contracts 2 and 3 should pay for these lines base on their power flow shares.
- Contract 1, 3, and 4 create counter power flow in line 15–23 and hence only contracts 2 should pay for this line.
- Only contract 1 creates power flow in lines 27–29, 27–30, and 29–30 and hence only this contract should pay for these lines.
- None of contracts create counter power flow in lines 12–16, 16–17, 15–18, 18–19, 10–17, 21–22, 22–24, 23–24, and 24–25 and hence all contracts should pay for these lines based on their power flow shares.

Table 11
The share of contracts in line power flows of IEEE 30-bus test system computed using decomposed decoupled Newton–Raphson power flow

Line no.	P_{lineij}	$P^{(1)}_{lineij}$	$P^{(2)}_{lineij}$	$P^{(3)}_{lineij}$	$P^{(4)}_{lineij}$
1–2	3.9723	2.7074	0.0291	−0.2233	1.4592
1–3	3.0277	1.2926	−0.0291	0.2233	1.5408
2–4	1.8281	0.7711	−0.0471	−0.0684	1.1725
3–4	−0.4723	1.2926	−0.0291	−1.7767	0.0408
2–5	2.8712	0.9035	0.0247	−0.0504	1.9933
2–6	2.2730	1.0327	0.0514	−0.1045	1.2934
4–6	3.0222	1.6352	0.4591	−0.2113	1.1392
5–7	1.3712	0.9035	0.0247	−0.0504	0.4933
6–7	3.6288	1.0965	−0.0247	0.0504	2.5067
6–8	0.4559	0.4422	0.0404	−0.0239	−0.0028
6–9	0.4456	0.4297	0.2096	−0.1556	−0.0381
6–10	0.2543	0.2451	0.1195	−0.0886	−0.0217
9–11	0	−0.0000	−0.0000	−0.0000	−0.0000
9–10	0.4456	0.4297	0.2096	−0.1556	−0.0381
4–12	−1.6664	0.4285	−0.5352	−1.6339	0.0741
12–13	−1.6000	−0.0000	−0.0000	−1.6000	−0.0000
12–14	−1.3893	0.0451	−0.1590	−1.2824	0.0070
12–15	0.0762	0.2200	−0.7767	0.5988	0.0341
12–16	1.2467	0.1634	0.4004	0.6498	0.0330
14–15	1.0107	0.0451	−0.1590	1.1176	0.0070
16–17	1.2467	0.1634	0.4004	0.6498	0.0330
15–18	1.8998	0.0798	0.4927	1.3088	0.0185
18–19	1.8998	0.0798	0.4927	1.3088	0.0185
19–20	−0.1002	0.0798	0.4927	−0.6912	0.0185
10–20	0.1002	−0.0798	−0.4927	0.6912	−0.0185
10–17	−1.2467	−0.1634	−0.4004	−0.6498	−0.0330
10–21	0.0028	0.6523	−0.4714	−0.1731	−0.0051
10–22	−0.1564	0.2657	−0.3064	−0.1125	−0.0033
21–22	−0.9972	−0.3477	−0.4714	−0.1731	−0.0051
15–23	−0.8129	0.1853	−1.4284	0.4075	0.0227
22–24	−1.1537	−0.0820	−0.7777	−0.2856	−0.0084
23–24	2.1871	0.1853	1.5716	0.4075	0.0227
24–25	1.0334	0.1033	0.7939	0.1220	0.0143
25–26	0	−0.0000	−0.0000	−0.0000	−0.0000
25–27	0.0334	0.1033	−0.2061	0.1220	0.0143
28–27	0.5666	0.4967	0.2061	−0.1220	−0.0143
27–29	0.4307	0.4306	−0.0000	−0.0000	−0.0000
27–30	0.1691	0.1694	−0.0000	−0.0000	−0.0000
29–30	−0.1692	−0.1692	−0.0000	−0.0000	−0.0000
8–28	0.0559	0.0422	0.0404	−0.0239	−0.0028
6–28	0.5106	0.4544	0.1658	−0.0981	−0.0115

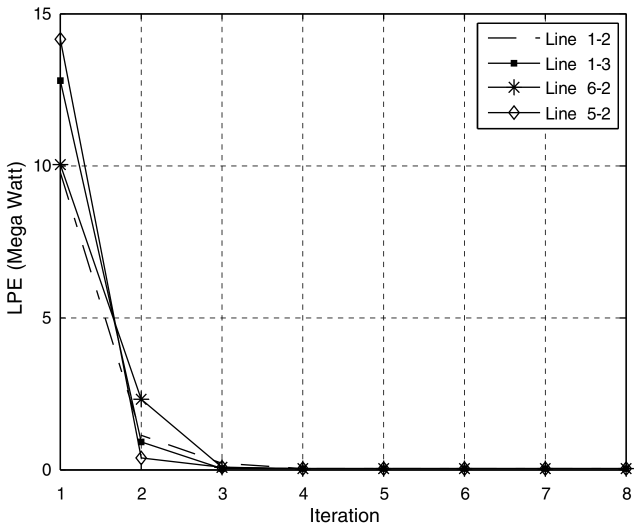

Fig. 7. LPEs for a few lines at different iterations, IEEE 30-bus test system.

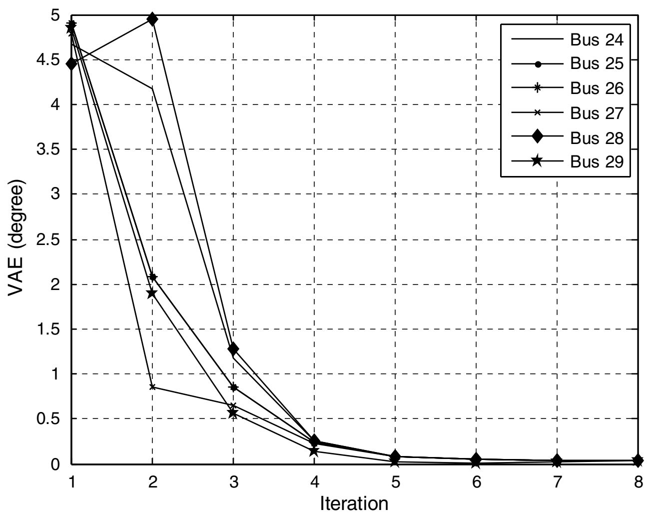

Fig. 6. VAEs for a few buses at different iterations, IEEE 30-bus test system.

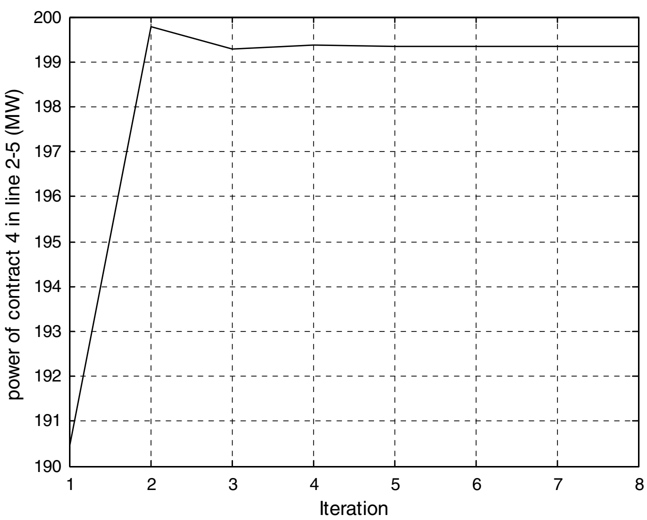

Fig. 8. Power flow of contract 4 in line 5–2, IEEE 30-bus test system.

Table 12
The share of different contracts in return value of different lines in percentage

Line no.	$R'^{(1)}_{ij}(h)$	$R'^{(2)}_{ij}(h)$	$R'^{(3)}_{ij}(h)$	$R'^{(4)}_{ij}(h)$
1–2	64.5287	0.6928	0	34.7785
1–3	42.2869	0	7.3059	50.4072
2–4	39.6745	0	0	60.3255
3–4	0	1.6097	98.3903	0
2–5	30.9265	0.8471	0	68.2264
2–6	43.4368	2.1629	0	54.4003
4–6	50.5714	14.1972	0	35.2314
5–7	63.5596	1.7410	0	34.6994
6–7	30.0110	0	1.3783	68.6107
6–8	91.6362	8.3638	0	0
6–9	67.2137	32.7863	0	0
6–10	67.2261	32.7739	0	0
9–11	0	0	0	0
9–10	67.2137	32.7863	0	0
4–12	0	24.6755	75.3245	0
12–13	0.0000	0.0000	100.0000	0.0000
12–14	0	11.0281	88.9719	0
12–15	25.7982	0	70.1997	4.0021
12–16	13.1082	32.1189	52.1243	2.6487
14–15	3.8542	0	95.5481	0.5978
16–17	13.1082	32.1189	52.1242	2.6487
15–18	4.2015	25.9363	68.8899	0.9723
18–19	4.2015	25.9363	68.8899	0.9723
19–20	0	0	100.0000	0
10–20	0	0	100.0000	0
10–17	13.1082	32.1189	52.1242	2.6487
10–21	100.0000	0	0	0
10–22	0	72.5723	26.6450	0.7827
21–22	34.8677	47.2669	17.3555	0.5099
15–23	0	100.0000	0	0
22–24	7.1051	67.4150	24.7528	0.7272
23–24	8.4724	71.8579	18.6339	1.0358
24–25	9.9987	76.8177	11.8033	1.3804
25–26	0	0	0	0
25–27	43.1306	0	50.9149	5.9544
28–27	70.6689	29.3311	0	0
27–29	100.0000	0	0	0
27–30	100.0000	0	0	0
29–30	100.0000	0.0000	0.0000	0.0000
8–28	51.1328	48.8672	0	0
6–28	73.2707	26.7293	0	0

- In this operating point, generation and load of buses 11 and 26 is zero. Bus 11 is connected only to bus 9 and bus 26 is connected only to bus 25, hence the power flow of lines 9–11 and 25–26 is zero and none of contracts pay for them.

Table 12 confirm abovementioned results.

7. Conclusion

In this paper a new transmission pricing method base on voltage angle decomposition is presented. In this method the contribution of each contract on voltage angels and consequently on line power flows is computed. The extent-of-use of contract k on capacity of lineij is equal to the contribution of contract k on the power flow of lineij. The extent-of- use contracts that create counter power flow in a line is considered zero to encourage the contracts that release transmission capacity. Numerical studies show that as number of iterations increases linear solutions tends to nonlinear solution, more non-linearity is taken into account, and voltage angle errors and line power errors vanish.

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