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EQUATIONS FOR DISCHARGE CALCULATION IN COMPOUND CHANNELS HAVING HOMOGENEOUS ROUGHNESS*

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Abstract– Although different methods for estimating the discharge capacity of compound channels have been developed, no entirely satisfactory method exists. However, a large body of experimental data has now been acquired covering small scale and large scale laboratory compound channels. In this paper, discharge characteristics in straight compound channels having homogeneous roughness are studied, and a method for discharge calculation in these channels is presented by analysing some experimental results from a United Kingdom flood channel facility (UK-FCF). The approach presented uses two correction coefficients, *a* and *b*, which are applied to the component mean velocities predicted by the traditional vertical division method in order to find more accurate values of the mean velocities in the main channel and floodplains. It has been found that *a* and *b* can be expressed in terms of two dimensionless parameters of the channel, coherence and the relative depth (ratio of the floodplain depth to the total depth). Although the procedure developed in this study is based on data from UK-FCF, it is simple and shows satisfactory results when compared to the recently developed method by Lambert and Myers, termed the weighted divided channel method, and when applied to the independent data set collected by others.

Keywords– Compound channel, coherence, weighted divided channel method, vertical interface method

1. INTRODUCTION

Discharge calculation in compound channels consisting of a main channel and floodplains has been a challenging topic in recent years. In compound channels, even if the floodplains and the main channel have the same roughness, use of the overall hydraulic radius as a parameter to characterise the geometric properties of the section does not lead to good results for calculating mean velocity and discharge by standard equations such as Manning's equation. The failure of this traditional method is due to the presence of a momentum transfer mechanism between the fast-flowing main channel and floodplains, which are characterised by lower depths and velocities. This momentum transfer causes reductions in velocity and discharge in the main channel, together with increases in the corresponding floodplain parameters.

Although two and three-dimensional approaches are receiving attention for discharge calculation in compound channels [1, 2], these are complex and inconvenient to use in practice. Therefore, discharge calculations for compound channels are based mainly on refined one-dimensional methods of analysis.

The main objective of this investigation is to introduce two correction coefficients, which can be applied to component discharges or velocities in compound channels with homogeneous roughness in order to find more accurate values of the main channel and floodplain discharge values. The dependency of these correction coefficients on geometric parameters of the channel, such as its coherence and relative depth, is also shown. Data from the United Kingdom Flood Channel facility (UK-FCF) [3] and data reported by Wormleaton *et al.* [4] are used for this study.

2. BACKGROUND

Different one-dimensional methods have been proposed for calculating discharge in compound channels, they include:

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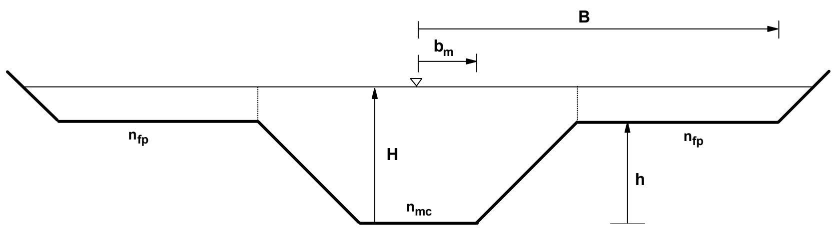
a) Interface methods

The interface methods propose that the easiest method of discharge calculation in compound channels involves dividing the channel cross section into simpler subsections through vertical, diagonal or horizontal interfaces such as those shown in Fig. 1. Then the independent discharge in each subsection can be calculated by using standard equations such as Manning's formula. Although interface methods are simple enough for both hand calculation and use in numerical models, they may not give good results for component discharges [5]. Further, in these traditional methods, there is no direct way to include interface shear stresses in calculations. Therefore, other attempts have been made to locate an interface with zero shear stress. In this regard, Yen and Ho [6] proposed empirical formulas for finding the inclination of a zero shear line which starts from the junction of the main channel and the floodplain. Another improvement in this direction was the method proposed by Wormleaton and Merret [7], termed the modified interface method. They introduced Φ-indices or coefficients which are applied to component discharges calculated by traditional interface methods. It must be pointed out that these methods need an empirical equation for evaluating the apparent shear stress in the interface.



Main Channel Width=2b_m=150 cm, Bankful Depth= h=15cm, Main Channel Side Slope=1(V): 1(H), Channel Bed Slope = 0.001027, Total Length of Channel = 49.67 m, n_{fp} = n_{mc} = 0.01

Fig. 1. Geometric configuration of UK-FCF [3]

b) Area method

In this method, an additional area is calculated which is included in floodplain or subtracted from the main channel to find an arbitrary zero-shear interface. This additional area can be calculated by applying the momentum balance equation in the direction of flow together with using an empirical equation, which gives an estimate of apparent shear stress along the vertical interface [8, 9].

c) Coherence method

The concept of coherence was introduced by Ackers [10, 11]. Coherence is defined as the ratio of the basic conveyance (treating the channel as a single unit) to that computed by summing the basic conveyances of the separate zones of the channel. The brief description of the coherence method given here is from Ref. [9]. Data analysis has shown that there are four distinct regions of flow behaviour for compound channels, and that these depend on the depth of floodplain flow. In the coherence method, the actual discharge may be computed by adjusting the basic discharge ($Q_{basic}=Q_{mc}+Q_{fp}$) to allow for the effect of momentum exchange between the main channel and its floodplains in each region of flow. Depending on the region of flow, this can be achieved via a discharge deficit ($Q=Q_{basic}-DISDEF$), or a discharge adjustment factor ($Q=Q_{basic} \times DISADF$). Ackers linked the discharge adjustment factors for each region to the channel coherence [10, 11].

d) Weighted divided channel method

This method was proposed by Lambert and Myers [12]. One alternative to traditional interface methods is the hypothesis that some division of the compound channel cross section is appropriate to account for the momentum transfer, and this division lies somewhere between the vertical and the horizontal division. Rather than trying to determine its location explicitly, the weighted divided channel method uses a weighting factor to allow a transition between the velocity given by using a vertical division and the velocity predicted by a horizontal division. The weighting factor varies between zero and unity and is applied to both the main channel and the floodplain areas to give improved mean velocity estimates for these areas.

Although appropriate weighting factors for rough and smooth compound channels have been proposed by Lambert and Myers [12], relating these factors to channel parameters may need further study.

3. THEORY OF THIS STUDY

There is a general belief among engineers that the easiest and most practical way of calculating normal discharge in compound channels is to divide the cross section into subsections by drawing vertical lines that start from the junction of the main channel and the floodplain as shown in Fig. 1. In this study, the velocities in the main channel and in the floodplains calculated by this division method are referred to as $V_{mc\text{-}VIM}$ and $V_{fp\text{-}VIM}$, respectively. Similarly, the component discharge values are referred to as $Q_{mc\text{-}VIM}$ and $Q_{fp\text{-}VIM}$. Manning's formula is used to determine these variables; and in applying the necessary procedure, the length of the assumed vertical interfaces is not considered in calculating the hydraulic radius of either the floodplain or the main channel. The following two equations are proposed for the estimation of more accurate values of the component velocities and discharges.

$$V_{mc} = aV_{mc-VIM} \tag{1}$$

$$V_{fp} = bV_{fp-VIM} \tag{2}$$

where *a* and *b* are correction coefficients to be used to improve the velocities and discharges calculated by the vertical interface method.

In a general sense, *a* and *b* depend on geometric and roughness characteristics of the main channel and the floodplain, the bed slope of the channel, and the main channel and floodplain Reynolds numbers. In this regard, a set of dimensionless parameters representing interaction effects between the main channel and the floodplain flow are found in [11].

Recently Myers and Lyness [13] analysed data from different scales of study and stated that the ratio of main channel to floodplain discharge in compound channels having homogeneous roughness is independent of the bed slope and scale and is a function of geometry only. It can be argued, therefore, that two main geometric parameters of the section, relative depth ((H-h)/H ratio in Figs. 1 and 2 and coherence, can be considered to be significant. Relative depth has been shown to be a dominant parameter by many researchers. Coherence is also considered as a lumped parameter that combines different geometric variables of a compound channel having homogeneous roughness. Therefore, Eqs. (3) and (4) were found to be appropriate forms for relating *a* and *b* in Eqs. (1) and (2) to coherence and the relative depth of the channel.

$$a = \alpha\left(\frac{(H-h)}{H}\right)^{\beta}(Coh)^{\gamma} \tag{3}$$

$$b = \eta\left(\frac{(H-h)}{H}\right)^{\lambda}(Coh)^{\theta} \tag{4}$$

where *H* is the total depth of flow, *h* is the bankful depth and *Coh* refers to the coherence of the channel. Other parameters in the equations are considered to be empirical parameters whose values are given in Section 5.

In this study, Manning' formula is used to define the coherence of the channel. Considering that coherence is defined as the ratio of the basic conveyance calculated by treating the channel as a single unit to that calculated by summing the basic conveyances of the separate zones, Eq. 5 can be used to calculate it.

$$Coh = \frac{\dfrac{A^{5/3}}{\left(\sum_{i=1}^{N} n_i^{3/2} P_i\right)^{2/3}}}{\sum_{i=1}^{N} \dfrac{1}{n_i} \dfrac{A_i^{5/3}}{P_i^{2/3}}} \tag{5}$$

where P_i and A_i are the wetted perimeter and the area of subsections, respectively. P and A are total wetted perimeter and area of the channel, and N is the number of separate subsections. Even if the main channel and floodplain exhibit the same roughness, the definition of coherence holds.

In Section 5 of this paper, the range of variability of *a* and *b* correction coefficients and their dependency on the coherence and relative depth of the channel are studied using experimental data from UK-FCF.

4. EXPERIMENTAL DATA

The experimental data used in this study are from two different and independent sources. The first set of data (parts a and b) is from UK-FCF, while the second set has been collected and introduced to the literature by Wormleaton *et al.* [4]. In this study, only data collected on homogeneously roughened channels were considered.

a) Experimental data from UK-FCF

Figure 1 shows the general configuration of the SERC flood channel facility together with other key dimensions of the channel related to this study. The data reported here are found in reference [3]. Other parameters which were variable during the experimental study are found in Table 1.

Table 1. Characteristics of different test series from UK–FCF used in this study (Part 1) [3]

Test series & number	(H-h)/H	M. channel discharge (L/s)	Single F. plain discharge (L/s)	B(cm)	F. plain side slope
(S1-1)	0.057	197.8	5.2	500	1(V) : 0(H)
(S1-2)	0.093	207.9	12.8		
(S1-3)	0.148	224.3	30.4		
(S1-4)	0.196	247.6	52.4		
(S1-5)	0.245	272.5	89.4		
(S1-6)	0.299	329.3	137.7		
(S2-1)	0.400	423.8	295.3	315	1(V) : 1(H)
(S2-2)	0.042	208.2	2.1		
(S2-3)	0.111	226.4	11		
(S2-4)	0.156	242.2	20		
(S2-5)	0.197	261.3	31.3		
(S2-6)	0.242	284.9	49		
(S2-7)	0.298	325.2	77.4		
(S2-8)	0.397	428.2	167.4		
(S2-9)	0.479	543.5	285.3		
(S3-1)	0.051	223	1	165	1(V) : 1(H)
(S3-2)	0.100	235.5	3.3		
(S3-3)	0.147	254.4	7.2		
(S3-4)	0.202	278.6	12		
(S3-5)	0.245	298.1	17.2		
(S3-6)	0.305	335.3	28		
(S3-7)	0.396	434.5	61.7		
(S3-8)	0.500	609.5	112.7		

Table 1 summarises the main characteristics of the different test series from UK-FCF which were used in this study. Detailed information about this data series was sent to the author by Myers [Personal communication with Professor Myers]. As noted in the table, the discharges of the main channel and floodplains are available separately for this data set. Individual discharges were obtained by integrating the point velocity readings. This data set is used here to study the range of variation in the *a* and *b* coefficients and to estimate the empirical parameters in Eqs. (3) and (4).

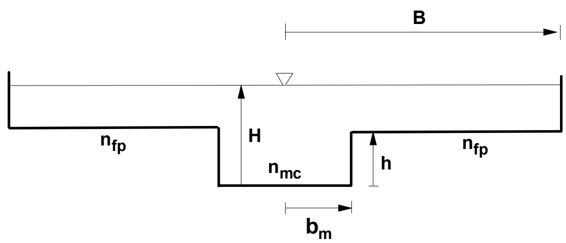
Table 2 presents data similar to those in Table 1. However, this table covers those data for which only total measured discharge is available. This data set is used to compare the results obtained in this study with those determined with the weighted divided channel method proposed by Lambert and Myers [12]. Therefore, detailed numerical values of the experiments have not been given in Table 2.

Table 2. Main characteristics of different test series from UK–FCF (part 2)

Test series	B (cm)	F. plain side slope	Number of results	(H-h)/H range
S1	500	1(V) : 0(H)	12	0.060-0.418
S2	315	1(V) : 1(H)	21	0.016-0.477
S3	165	1(V) : 1(H)	15	0.097-0.511

b) Experimental data reported in Ref. [4]

Wormleaton *et al.* [4] have reported some experimental data on a smooth homogeneous compound channel with the general configuration and characteristics shown in Fig. 2. As shown in Fig. 2, the shape and scale of this channel is different from that of UK-FCF.



*Channel Width = 2B = 121cm, Main Channel Width = 2*b_m *= 29cm, Bankful Depth = h = 12cm,*
Total Length of Channel = 10.75m, $n_{fp} = 0.011$, $n_{mc} = 0.01$

Fig. 2. General configuration of the compound channel used by Wormleaton *et al.* [4]

Table 3 summarises the main characteristics of the different tests conducted on this compound channel. As seen in the table, different values of the bed slope have been considered in the experimental work by Wormleaton *et al.* [4]. Also, only total discharge values have been reported in Table 1. This independent data set is used to compare the results obtained in this study with those of the weighted divided channel method.

5. DATA ANALYSIS AND RESULTS

As previously stated, the main objective of this investigation is to introduce two correction coefficients which can be applied to the component discharges or velocities in order to find more accurate values of the main channel and floodplain discharges. To achieve this, the data reported in Table 1 have been used to study these coefficients. Figure 3 shows the variability of the *a* and *b* coefficients in Eqs. (1) and (2) with relative depth. As shown in this figure, *b* varies between 1 and 1.2, while *a* is between 0.8 and 1.0. These values (a<1 and b>1) reveal the momentum transfer mechanism between the fast-flowing main channel and the floodplains. The data in Table 1 have also been used to determine Eqs. (3) and (4).

Table 3. Experimental data on a compound channel with homogeneous roughness [4]

Run number	(H-h)/H	Bed slope (Tens of thousands)	Total discharge (L/s)
(A1)	0.111	4.3	13.4
(A2)	0.143	4.3	16
(A3)	0.200	4.3	20.5
(A4)	0.250	4.3	26.0
(A5)	0.294	4.3	31.0
(A6)	0.333	4.3	37.0
(A7)	0.368	4.3	43.5
(A8)	0.111	9.4	17.2
(A9)	0.143	9.4	25.7
(A10)	0.172	9.4	29.2
(A11)	0.250	10.1	35.2
(A12)	0.143	18.0	31.0

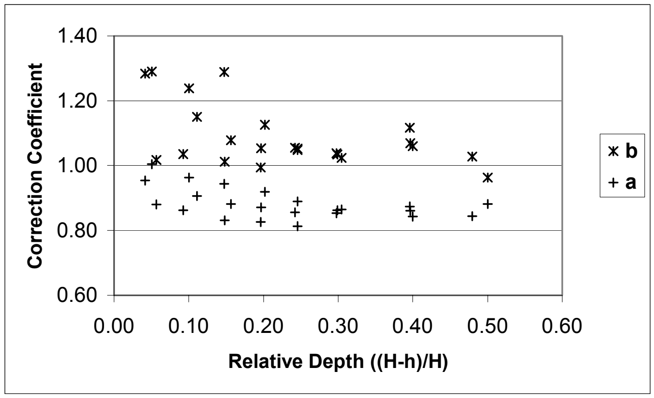


Fig. 3. Correction coefficients vs. relative depth for UK-FCF data (Table 1)

The results of applying non-linear regression techniques for finding the unknown parameters in Eqs. (3) and (4) are revealed in Eqs. (6) and (7), with the associated coefficients of determination (R^2). It is worth mentioning that the inclusion of the B/b_m ratio of the channel in these equations did not improve the results.

$$a = 0.782\left(\frac{(H-h)}{H}\right)^{-0.128}(Coh)^{0.353} \quad , \quad R^2 = 0.999 \tag{6}$$

$$b = 0.903\left(\frac{(H-h)}{H}\right)^{-0.197}(Coh)^{0.547} \quad , \quad R^2 = 0.998 \tag{7}$$

Table 4 shows the results of using different methods for finding component velocities for Table 1 data. Error criteria such as mean absolute relative error and maximum relative error have been used to quantify the differences.

Table 4. Mean absolute relative error (MARE) and maximum relative error (Max. RE) for different methods in predicting main channel and floodplain velocities (Table 1 data)

Method	Error type	Main channel	Flood plains
Vertical interface	MARE (%)	13.7	7.9
	Max. RE (%)	-23.0	+22.5
Weighted divided channel	MARE (%)	3.2	9.7
	Max. RE (%)	+8.6	+34.3
Improved vertical interface	MARE (%)	1.9	3.4
	Max. RE (%)	-6.8	+9.7

Table 4 shows that the improved vertical interface method performs better in predicting main channel and floodplain velocities. However, this better performance cannot be guaranteed because the same data were used to find the unknown parameters in Eqs. (1) and (2). In order to further evaluate the behavior of Eqs. (6) and (7) and the modified interface method, the various methods have been applied to data reported in Tables 2 and 3. Table 5 shows those comparative results; and it can be seen in this table that the improved interface method shows quite satisfactory results when compared to the modified weighted divided channel method.

Table 5. Mean absolute relative error (MARE) and maximum relative error (Max. RE) for different methods in predicting total discharge (Tables 2 and 3 data)

Method	Error type	Table 2 data	Table 3 data
Vertical interface	MARE (%)	9.1	9.6
	Max. RE (%)	+24.9	+36.2
Weighted divided channel	MARE (%)	4.3	7.0
	Max. RE (%)	+19.8	+29.1
Improved vertical interface	MARE (%)	2.1	7.1
	Max. RE (%)	+9.6	+27.9

Figure 4 shows the variation in calculated *a* and *b* coefficients with relative depth obtained for the Wormleaton *et al.* data [4]. Values in Fig. 4 follow the same pattern as those in Fig. 3. Figure 5 presents the ratio of the calculated discharge to the measured discharge for the different methods; and it is evident that the improved interface and weighted divided channel methods show acceptable (and close) behavior, while the vertical interface method always overestimates the total discharge. Although this comparison has been made in terms of total discharge, it is expected that the methods are able to estimate the component discharges reasonably well considering that the total discharges have been calculated by adjusting component discharge velocities. Another point that is observable in Fig. 5 is that all methods show a significant increase in flow ratios as the depth ratio is reduced to near unity. This may be due to the inability of the methods to take into account large interfacial shear stresses, or possibly improper selection of Manning's roughness coefficient in small floodplain depths.

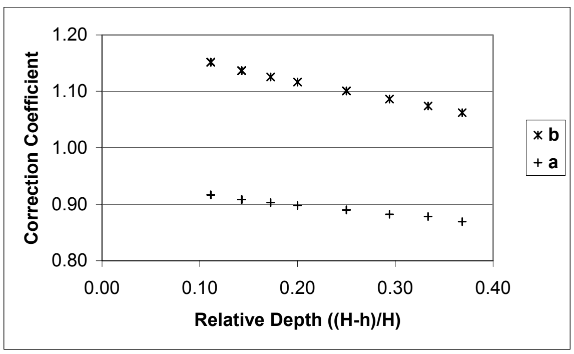


Fig. 4. Correction coefficients *vs*. relative depth for Wormleaton *et al.* data [4]

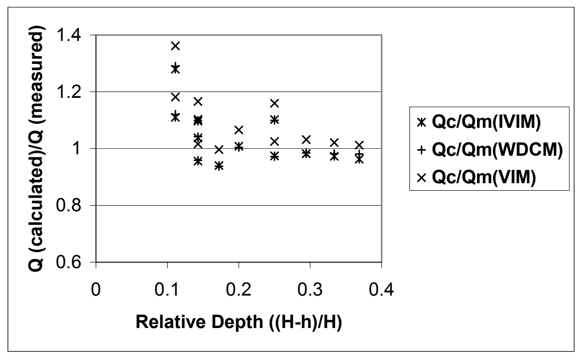


Fig. 5. Q (calculated)/Q (measured) *vs*. relative depth for different methods

6. ILLUSTRATIVE EXAMPLE

To illustrate, the proposed method is used to calculate the total discharge of a channel which conveys flood water from an urban watershed in the East of Iran. Figure 6 shows a schematic diagram of the channel together with its key dimensions. Manning's roughness coefficient of the channel is estimated to be 0.022, and the channel has a bottom slope of 0.012.

Two traditional methods are first used to calculate the normal discharge capacity of the channel at bankful level. The first method considers the channel as a single unit, the second method uses the interface method and divides the channel cross-section into subsections by vertical interfaces as shown in Figure 6. The first method yields a result of 97.43 m^3/s, while the second method results in 107.04 m^3/s for total discharge. The two methods show about a 10% difference. The smaller result (given by the first method) may be due to the inclusion of the effect of the wetted perimeter of the floodplain on the overall hydraulic radius of the channel. The second method totally ignores the interaction between the main channel and the floodplain and results in a higher discharge value. There is some confusion about finding the correct value of discharge in such channels. However, it can be argued that the discharge of the channel is greater than 97.43 m^3/s and less than 107.04 m^3/s.

To use the proposed method, an average value of 0.339 was estimated for relative depth. The value of 0.339 was calculated by considering that H is constant (2.8 m) and that for this specific shape, the floodplain depth varies linearly from 1.6 m to 0.3 m. The proposed method results in 99.56 m^3/s for total discharge. This value is close to the results found by the first method discussed above. It can be concluded that the proposed method not only gives more reasonable results for total discharge, but also corrects the component discharge values calculated by the vertical interface method. The values of 97.43 m^3/s and 99.56 m^3/s are close enough to indicate that the channel has the tendency to act as a single unit. A brief listing of some of the calculations involved in the methods is given below:

Single floodplain area = 3.829 m^2
Single floodplain wetted perimeter = 4.534 m
Floodplain hydraulic radius = 0.844 m
Single floodplain discharge = $Q_{fp\text{-}VIM}$ *= 17.03* m^3/s
Main channel area = 9.416 m^2
Main channel wetted perimeter = 4.849 m
Main channel hydraulic radius = 1.942 m
Main channel discharge = $Q_{mc\text{-}VIM}$ *= 72.98* m^3/s
Total discharge by vertical interface method = 107.04 m^3/s

Total area of the channel = 17.074 m^2
Total wetted perimeter of the channel = 13.917 m
Total discharge by taking the channel as a single unit = 97.43 m^3/s

$$Coh = \frac{\dfrac{17.074^{5/3}}{(13.917)^{2/3}}}{\dfrac{9.416^{5/3}}{4.849^{2/3}} + \dfrac{3.829^{5/3}}{4.534^{2/3}} + \dfrac{3.829^{5/3}}{4.534^{2/3}}} = 0.910$$

Relative depth = 0.339
a = 0.869
b = 1.061
Total floodplain discharge = Q_{fp} *= 1.061(17.03+17.03) = 36.14* m^3/s
Main channel discharge = Q_{mc} *= 0.869(72.98) = 63.42* m^3/s
Total discharge of the channel by the proposed method = 99.56 m^3/s

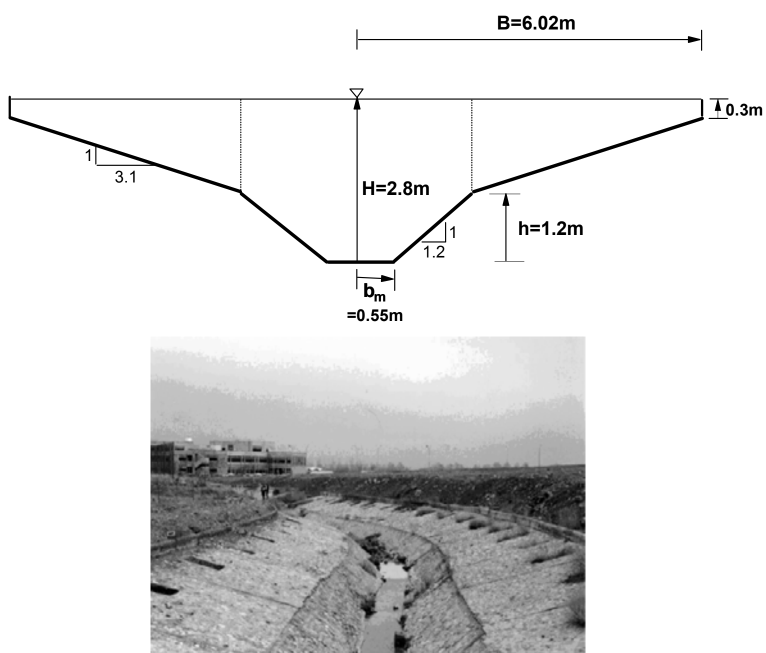


Fig. 6. Dimensions and characteristics of the channel discussed as illustrative example

7. CONCLUSIONS

This investigation was a study of flow characteristics in compound channels having homogeneous roughness in order to estimate component discharges in a simple way. The following conclusions can be made:

1. Data analysis conducted in this study shows that simple correction coefficients considered in Eqs. (1) and (2) are able to estimate both total and component discharges reasonably well. The main advantages of this method are that it is simple and only a modification to the conventional methods.
2. Data analysis showed that the correction coefficients (a and b in Eqs. (1) and (2)) mainly depend on the relative depth and coherence of the channel. Inclusion of the coherence of the channel in the empirical equations developed for other methods available in the literature may increase the accuracy and generality of these equations.
3. Review of the proposed equations and procedure shows that the ratio of the calculated main channel discharge to calculated floodplain discharge is a function of the channel geometry only. This is in agreement with the results reported by Myers and Lyness [13].
4. Although Eqs. (6) and (7) have been developed based on data from UK-FCF, they showed reasonable performance when applied to the Wormleaton *et al.* data [4, 7] that were collected on a channel with a different shape, bed slope and scale from UK-FCF.

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