International Journal of Uncertainty,
Fuzziness and Knowledge-Based Systems
Vol. 18, No. 6 (2010) 819–841

DOI: 10.1142/S0218488510006817

MODELING (*r*, *Q*) POLICY IN A TWO-LEVEL SUPPLY CHAIN SYSTEM WITH FUZZY DEMAND

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Received 11 October 2008
Revised 13 June 2010

In this paper a two level supply chain system is studied, in which the final demand is assumed to be fuzzy with triangular membership function. The inventory control policy of (r,Q) is followed for this system and unsatisfied demand is assumed to be back ordered. The objective is to minimize the total cost of the system, including ordering, holding and shortage costs. The model happens to be a nonlinear programming. Considering the complexity arising from the model, we also develop a genetic algorithm to obtain a near-optimal solution. The method is illustrated through some numerical examples.

Keywords: Supply chain; inventory control; fuzzy set; genetic algorithm.

1. Introduction

Uncertainty is rooted in the nature of supply chain and caused by its expansive network. A supply chain consists of all stages involved, directly or indirectly, to fulfill a customer request. "The supply chain includes not only manufacturers and suppliers, but transporters, warehouses, retailers, and customers themselves.[1]" Obviously, in such an expansive network, managing uncertainty is a major challenge. "There are three different sources of uncertainty that plague supply chain networks: supplier performance (late delivery), manufacturing process (machine breakdown, transportation reliability) and customer demand (volume and mix).[2]"

To manage uncertainty in a supply chain system, different approaches such as stochastic process or fuzzy sets concepts can be applied. Expressing uncertain data as random variables is discussed in the literature previously; see Refs. 3, 4 and 5. However, in many cases stochastic process can not be used, due to the lack of sufficient historical

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data to derive distribution function of random variables. A probability distribution is usually derived from the past information. This requires a valid hypothesis that evidence collected are complete and unbiased, and that the stochastic mechanism generating the data recorded continues in force on an unchanged basis, see Ref. 6. If all these requirements are not satisfied, then the conventional probabilistic reasoning methods are not appropriate. As an example, in the case of launching a new product, no previous information exists and thus probabilistic models can not be developed. Therefore, in these situations another practical approach is to express uncertain data as fuzzy numbers. In other words, uncertain parameters can be specified based on managerial experience and subjective judgment only. It may be convenient to express these uncertainties by using various imprecise linguistic expressions. For example, an expert can estimate the average demand for a product as d_m, while stating that it is not less than d_t and not greater than d_u. Similarly, a supplier can be evaluated as *very reliable*, in terms of the percentage of a raw material orders that can deliver, or lead time is predicted to be most likely in the interval $[L_l, L_u]$.

In this paper, we apply fuzzy set approach to model a two-level supply chain system, where demand is uncertain and its probability distribution function is not known. The final demand is assumed to be fuzzy with triangular membership function. The triangular membership function is utilized because of its simplicity. The inventory control policy of each level is continuous review, (r,Q).

We review the models developed in the literature regarding inventory policies in supply chain networks with fuzzy demand, briefly. Petrovic *et al.*[7] developed fuzzy modeling and also simulated a supply chain in an uncertain environment. In his model, discrete fuzzy sets are used to represent various kinds of uncertainties of demand or uncertainties of external supplies. Inventories are reviewed at fixed periods and the policy of up-to level order is adopted. The objective is to determine the stock levels as well as order quantities for each inventory in a supply chain, during a finite time horizon in order to obtain an acceptable delivery performance at a reasonable total cost. Petrovic *et al.*[8] developed a serial supply chain model with fuzzy demand and lead time to reach an acceptable service level of the supply chain and at a reasonable total cost. In this model, each inventory in the supply chain is controlled based on a periodic review policy. SCSIM introduced by Petrovic,[6] analyzes supply chain behavior and performance in the presence of uncertainty. SCSIM treats a supply chain which includes raw materials, a number of in-process inventories and final-products as well as production facilities. Main sources of uncertainty inherent in the serial supply chain and its environment are identified, including customer demand, external supply of raw material and lead times. Two types of models are included in SCSIM: (1) supply chain fuzzy analytical models to determine the optimal order-up-to levels for all inventories in a fuzzy environment and (2) a supply chain simulation model to evaluate supply chain performance achieved over time by applying order-up-to levels recommended by the fuzzy models. Giannoccaro *et al.*[9] presented a methodology based on the concept of echelon stock and fuzzy set theory. Fuzzy set theory is used to model the uncertainty associated with both market demand and inventory costs properly. They also applied periodic-review control policy based on the echelon stock concept. According to the echelon periodic-review control policy, at each stage, the echelon stock is reviewed at constant time intervals and an order is issued to the upstream stage to raise the echelon stock up to a target level. Wang and Shu[2] developed a fuzzy decision methodology that provides an alternative framework to handle supply chain uncertainties and to determine supply chain inventory strategies. A fuzzy supply chain model based on possibility theory is developed to evaluate supply chain performances. Based on the proposed fuzzy supply chain model, a genetic algorithm approach is developed to determine the order-up-to levels of stock-keeping units in the supply chain to minimize the supply chain inventory cost subject to the restriction of fulfilling the target fill rate of the finished product. Xie *et al.*[10] considered a two-level supply chain consist of a production and a number of inventory units. Each facility applies periodic inventory control policy. It is supposed that the supply chain operates under uncertainty in customer demand, which is described by imprecise terms and modeled by fuzzy sets.

Although this research considers the inventory system of supply chain, we mention some similar works in single level inventory systems. Gen *et al.*[11] examine an inventory control problem, in which input data such as set up cost, holding and shortage costs are triangular fuzzy numbers. They consider continuous review policy and assume the reorder point is pre-determined. They determine the maximal ordering cycles which results in the minimal total cost. Chang *et al.*[12] consider a single level inventory problem with backorder, and constant demand. Due to the uncertainty of inputs, such as transportation time they consider backorder quantity as a triangular fuzzy number. They obtain an economic order quantity and an economic backorder quantity in fuzzy sense. Ouyang and Yao[13] investigate a continuous review inventory model in which order quantity and lead time are decision variables. They consider two fuzziness of annual demand, i.e., fuzzy number of annual demand and statistic-fuzzy number of annual demand and give an algorithm procedure to obtain the optimal ordering strategy for each case. Modarres and Pirayesh[14] study continuous review and periodic review inventory policies in which demand is a triangular fuzzy number. The unsatisfied demands are back ordered. The objective is to minimize the total cost of the system consists of ordering; holding and shortage costs.

Up to our knowledge, no paper is yet published which deals with continuous review inventory control policy (r,Q) in a centralized fuzzy supply chain. In our model, each firm in the supply chain applies (r,Q) policy.

This paper is organized as follows. In the next section, the problem is defined. Section 3 describes problem formulation. Section 4 is devoted to solution approach. Section 5 gives a numerical example. Finally, Sec. 6 presents the conclusions and suggestions for further researches in this area.

2. Problem

In this section the problem is introduced with more details.

2.1. *Model definition*

A two-level supply chain, such as in Fig. 1, is assumed. The customer provides his/her demand through firm *B*. while firm *B* replenishes through firm *A*, according to its own inventory control policy. Firm *A* also a business unit, follows its inventory control policy and replenishes its required inventory from an external supplier. This model is a typical one in a supply chain, called single-vendor single-buyer system (see Refs. 15, 16 and 17). Although in all these papers the demand is assumed to be deterministic.

The objective of this model is to determine an inventory control policy for both firms *A* and *B* so as to minimize the total system cost.

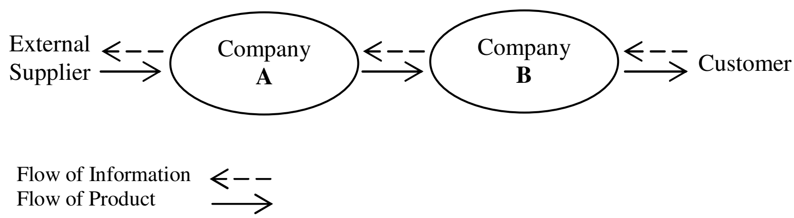

Fig. 1. A two-level supply chain.

2.1.1. *Inventory control policy for firms B and A*

Firm *B* follows the continuous rule of (r_B, Q_B); *i.e.,* whenever the inventory level decreases to r_B, then an amount of Q_B is ordered.

Since the orders issued by firm *B* are the demands entering firm *A*, it can be concluded that within any time interval between two successive orders of firm *B* the demand for firm *A* is a constant and deterministic value of Q_B. Hence, inventory level in firm *A* varies discretely, as shown in Fig. 2.

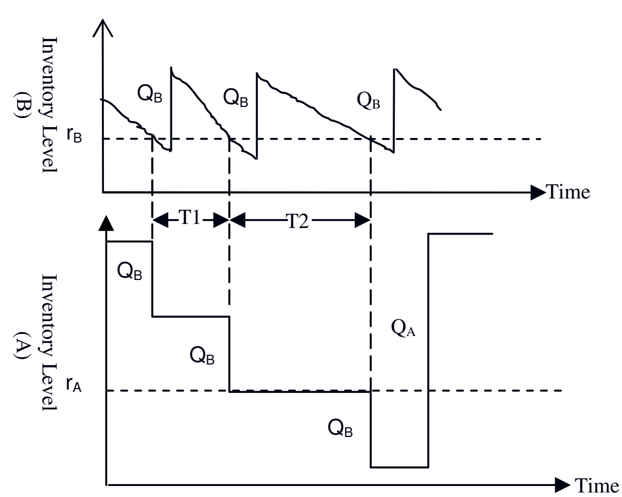

Fig. 2. Inventory level changes at firm A and B.

If firm *A*'s inventory is less than the demanded amount by firm *B* then the total demand (Q_B) is backordered. In other words, lot-splitting is not allowed. Thus, it can be concluded that $Q_A=K_1Q_B$ and $r_A=K_2Q_B$, where K_1 and K_2 are nonnegative integers.

2.2. *Assumptions*

The assumptions of the model are as follow:

(1) Demand entering firm *B* during the lead time is a fuzzy number and its membership function is as follows.

$$\mu_{\tilde{X}_B}(x_B)=\begin{cases} 0 & x_B < a_1 \ ; a_1 > 0 \\ \dfrac{x_B - a_1}{a_2 - a_1} & a_1 \le x_B \le a_2 \\ \dfrac{a_3 - x_B}{a_3 - a_2} & a_2 \le x_B \le a_3 \\ 0 & x_B > a_3 \end{cases}$$

$\tilde{X}_B$: Demand of firm *B* during the lead time, which is a fuzzy number

$\mu_{\tilde{X}_B}(x_B)$: Membership function of $\tilde{X}_B$

To analysis average annual cost at firm *B* we need to average annual demand in firm B, denoted by D_B. We assume that D_B is an input data which is defuzzified value of annual demand using centroid method.

(2) Customer demand is confined to a single product

(3) The transportation time of each order placed by firm *B* as well as the lead time for replenishing the orders of firm *A* from an external supplier is assumed to be deterministic and given

(4) Both firms inventory follow the control policy of (r,Q)

(5) Shortage for both firms is back ordered

(6) If the inventory of firm *A* is less than the amount requested by *B*, then the whole demand is back ordered; *i.e.,* there is no lot-splitting at firm *A*

(7) The external supplier has unlimited capacity

2.3. *Objective function*

The objective function is to minimize the total system cost which is sum of the ordering, holding and shortage cost of both firms, *A* and *B*.

2.4. *Decision variables*

Since both firms follow the inventory control policy of (r,Q), the reorder point (r) and quantity order (Q) of each firm have to be decision variables. By this policy, an inventory order of (Q) is placed when the inventory level declines to a predetermined amount of (r).

2.5. *Notation*

The notations used in the formulation are:

Parameters:

- A_A: Ordering cost in firm A,
- A_B: Ordering cost in firm B,
- h_A: Holding cost per unit per unit time at firm A,
- h_B: Holding cost per unit per unit time at firm B,
- π_A: Shortage cost per unit in firm A,
- π_B: Shortage cost per unit in firm B,
- L_A: Lead time for delivery to firm A,
- L_B: Lead time for delivery to firm B,
- D_A : Average annual demand in firm A,
- D_B: Average annual demand in firm B,
- $\bar{I}(Q_B, r_B)$: Average inventory level within each cycle at firm B,
- $\bar{Y}(Q_B, r_B)$: Mean inventory on-hand at firm B,
- λ_B: Average demand during lead time period in firm B,
- λ_A: Average demand during lead time period in firm A,
- b: Amount of shortage during lead time at firm B,
- b_A: Amount of shortage during lead time at firm A,
- $\mu_{\tilde{B}}(b)$: Membership function of b,
- T_B: Interval between two successive orders of firm B,
- $\bar{T}_B$: Average of T_B,

Decision Variables:

- Q_A: Order quantity of firm A,
- Q_B: Order quantity of firm B,
- r_A: Reorder point of firm A,
- r_B: Reorder point of firm B,

As a usual notation in the literature, a fuzzy number, say x, is represented by $\tilde{X}$ and its membership function as $\mu_{\tilde{X}}(x)$.

3. Problem Formulation

In this problem, the annual inventory cost for each firm is also determined. One should consider the fact that orders placed by firm B are demands for firm A.

3.1. *Inventory analysis at Firm B*

As mentioned before, Firm B follows the continuous rule of (r_B, Q_B). The total cost of this firm which includes holding cost, shortage cost and ordering cost are calculated as follows.

3.1.1. *Mean holding cost during an inter arrival time at firm B*

Average inventory level within each cycle, $\bar{I}(Q_B, r_B)$, depends on reorder point (r_B) and order quantity (Q_B). It is also known that Q_B/D_B is the mean inter arrival time. Thus, the average cost per this interval is,

$$h_B \frac{Q_B}{D_B} \bar{I}(Q_B, r_B)$$

During each cycle, inventory is at its minimum level just before the order receipt and at its maximum level just after that. Then, at these two points the average inventory level is $r_B - \lambda_B$ and $r_B - \lambda_B + Q_B$, respectively, where λ_B is the average demand during lead time period. If $\bar{Y}(Q_B, r_B)$ is the mean inventory on-hand (while there is no shortage) and demand occurs linearly, then,

$$\bar{Y}(Q_B, r_B) = \frac{Q_B}{2} + r_B - \lambda_B$$

On the other hand, it can be considered that $\bar{I}(Q_B, r_B)$ and $\bar{Y}(Q_B, r_B)$ are approximately equal, because the shortage period is short enough compared with the total time interval. Therefore, the average inventory level is as follows.

$$\bar{I}(Q_B, r_B) \cong \frac{Q_B}{2} + r_B - \lambda_B$$

During the lead time demand is fuzzy, by our assumption. Thus, λ_B is substituted by its defuzzified value. Defuzzifying $\tilde{X}_B$ by using centroid method results in,

$$def(\tilde{X}_B) = \frac{\int \mu_{\tilde{X}_B}(x_B) x_B d_{x_B}}{\int \mu_{\tilde{X}_B}(x_B) d_{x_B}} = \frac{\lambda_1}{\lambda_2}$$

where, λ_1 and λ_2 are calculated as follows.

$$\lambda_1 = \int \mu_{\tilde{X}_B}(x_B) x_B d_{x_B} = \int_{a_1}^{a_2} \frac{x_B - a_1}{a_2 - a_1} x_B dx_B + \int_{a_2}^{a_3} \frac{a_3 - x_B}{a_3 - a_2} x_B dx_B$$

$$= \frac{1}{a_2 - a_1}\left[\frac{1}{3}a_2^{\,3} - \frac{1}{2}a_2^{\,2} a_1 + \frac{1}{6}a_1^{\,3}\right] + \frac{1}{a_3 - a_2}\left[\frac{1}{6}a_3^{\,3} - \frac{1}{2}a_3 a_2^{\,2} + \frac{1}{3}a_2^{\,3}\right]$$

$$\lambda_2 = \int \mu_{\tilde{X}_B}(x_B) d_{x_B} = \int_{a_1}^{a_2} \frac{x_B - a_1}{a_2 - a_1} dx_B + \int_{a_2}^{a_3} \frac{a_3 - x_B}{a_3 - a_2} dx_B$$

$$= \frac{1}{a_2 - a_1} . \frac{(a_2 - a_1)^2}{2} + \frac{1}{a_3 - a_2} . \frac{(a_3 - a_2)^2}{2} = \frac{a_2 - a_1}{2} + \frac{a_3 - a_2}{2}$$

$$= \frac{a_3 - a_1}{2}$$

Thus, the average inventory level within each cycle is:

$\bar{I}(Q_B, r_B) = \frac{Q_B}{2} + r_B - \lambda_B$, where, $\lambda_B = \lambda_1 / \lambda_2$

3.1.2. *Shortage cost during an inter arrival time at firm B*

Shortage happens whenever demand is greater than reorder point during lead time. Let b be the amount of shortage during lead time, then,

$$b = x_B - r_B \ ; \qquad x_B > r_B$$

Therefore, b is also a fuzzy number. Its membership function for $a_1<r_B<a_2$ and $a_2<r_B<a_3$ is as follows.

Case 1: $a_1<r_B<a_2$

$$x_B = b + r_B ; \qquad x_B > r_B$$

$$\Rightarrow \mu_{\tilde{B}}(b) = \begin{cases} \dfrac{b + r_B - a_1}{a_2 - a_1} & r_B \le b + r_B \le a_2 \\ \dfrac{a_3 - b - r_B}{a_3 - a_2} & a_2 \le b + r_B \le a_3 \end{cases}$$

$$\Rightarrow \mu_{\tilde{B}}(b) = \begin{cases} \dfrac{b + r_B - a_1}{a_2 - a_1} & 0 \le b \le a_2 - r_B \\ \dfrac{a_3 - b - r_B}{a_3 - a_2} & a_2 - r_B \le b \le a_3 - r_B \end{cases}$$

Case 2: $a_2<r_B<a_3$

$$\mu_{\tilde{B}}(b) = \frac{a_3 - b - r_B}{a_3 - a_2} \qquad r_B \le b + r_B \le a_3$$

$$\Rightarrow \mu_{\tilde{B}}(b) = \frac{a_3 - b - r_B}{a_3 - a_2} \qquad 0 \le b \le a_3 - r_B$$

To calculate the shortage cost, b is defuzzified for both cases. In case 1:

$$\int \mu_{\tilde{B}}(b) b d_b = \int_0^{a_2 - r_B} \frac{b + r_B - a_1}{a_2 - a_1} b d_b + \int_{a_2 - r_B}^{a_3 - r_B} \frac{a_3 - b - r_B}{a_3 - a_2} b d_b$$

$$= \frac{1}{a_2 - a_1}\left[\frac{1}{3}(a_2 - r_B)^3 + \frac{1}{2}(r_B - a_1)(a_2 - r_B)^2\right]$$

$$+ \frac{1}{a_3 - a_2}\left[\frac{1}{2}(a_3 - r_B)^3 - \frac{1}{3}(a_3 - r_B)^3 - \frac{1}{2}(a_3 - r_B)(a_2 - r_B)^2 + \frac{1}{3}(a_2 - r_b)^3\right]$$

$$= f_1(r_B)$$

$$\int \mu_{\tilde{B}}(b)d_b = \int_0^{a_2-r_B} \frac{b + r_B - a_1}{a_2-a_1} d_b + \int_{a_2-r_B}^{a_3-r_B} \frac{a_3 - b - r_B}{a_3-a_2} d_b$$

$$= \frac{1}{a_2-a_1}\left[\frac{1}{2}(a_2-r_B)^2 + (r_B-a_1)(a_2-r_B)\right]$$

$$+ \frac{1}{a_3-a_2}\left[(a_3-r_B)^2 - \frac{1}{2}(a_3-r_B)^2 - (a_3-r_B)(a_2-r_B) + \frac{1}{2}(a_2-r_B)^2\right]$$

$$= f_2(r_B)$$

$$def(b) = \frac{f_1(r_B)}{f_2(r_B)} = f(r_B)$$

Defuzzified b in Case 2:

$$\int \mu_{\tilde{B}}(b) b d_b = \int_0^{a_3-r_B} \frac{a_3-b-r_B}{a_3-a_2} b d_b = \frac{(a_3-r_B)^3}{6(a_3-a_2)} = g_1(r_B)$$

$$\int \mu_{\tilde{B}}(b) d_b = \int_0^{a_3-r_B} \frac{a_3-b-r_B}{a_3-a_2} d_b = \frac{(a_3-r_B)^2}{2(a_3-a_2)} = g_2(r_B)$$

$$def(b) = \frac{g_1(r_B)}{g_2(r_B)} = \frac{\dfrac{(a_3-r_B)^3}{6(a_3-a_2)}}{\dfrac{(a_3-r_B)^2}{2(a_3-a_2)}} = \frac{a_3-r_B}{3} = g(r_B)$$

Therefore, the mean shortage cost during a given period is:

$$\begin{aligned}
&\pi_B f(r_B) y_B + \pi_B g(r_B)(1 - y_B) \\
&s.t \\
&a_1 y_B \le r_B \le a_2 y_B + M(1-y_B) \\
&a_2(1-y_B) \le r_B \le a_3(1-y_B) + M y_B \\
&y_B = 0 \ or \ 1
\end{aligned}$$

M is a sufficiently large positive number, and y_B is a binary variable.

3.1.3. *Annual inventory cost of firm B*

The total cost of each cycle, by considering the above calculations is as follows.

$$A_B + h_B \frac{Q_B}{D_B}\left[\frac{Q_B}{2} + r_B - \lambda_B\right] + \pi_B f(r_B) y_B + \pi_B g(r_B)(1 - y_B)$$
$$s.t$$
$$a_1 y_B \le r_B \le a_2 y_B + M(1 - y_B)$$
$$a_2(1 - y_B) \le r_B \le a_3(1 - y_B) + M y_B$$
$$y_B = 0 \ or \ 1$$

Since the average number of cycles during a year equals D_B/Q_B, then the average annual cost is,

$$K(Q_B, r_B) = \frac{D_B A_B}{Q_B} + h_B\left[\frac{Q_B}{2} + r_B - \lambda_B\right] + \frac{\pi_B D_B f(r_B)}{Q_B} y + \frac{\pi_B D_B g(r_B)}{Q_B}(1 - y_B)$$
$$s.t$$
$$a_1 y_B \le r_B \le a_2 y_B + M(1 - y_B)$$
$$a_2(1 - y_B) \le r_B \le a_3(1 - y_B) + M y_B$$
$$y_B = 0 \ or \ 1$$
$$Q_B, r_B \ge 0$$

3.2. *Annual inventory cost of Firm A*

As mentioned in Sec. 2.1.1 within any interval between two successive orders of firm B the demand for firm A is a constant and deterministic value of Q_B. On the other hand, since lot-splitting is not allowed, it can be concluded that $Q_A=K_1Q_B$ and $r_A=K_2Q_B$, where K_1 and K_2 are nonnegative integers.

Holding cost for firm A is approximated by assuming that average duration between two successive demands is identical for all intervals, and denoted by $\overline{T}_B$. To calculate inventory on hand, this value is approximated by net inventory value based on the assumption that shortage period duration is short enough in comparison with the whole planning period time.

Inventory level is at its maximum level just after receiving orders and at minimum point just before order is received, or r_A-λ_A+Q_A and r_A-λ_A, respectively, where λ_A represents the average demand for A during lead time.

To calculate λ_A, it is possible to assume that there exists a demand (equal to Q_B) during $\overline{T}_B$. Thus, the average number of demands during L_A is $\frac{L_A}{\overline{T}_B}$ and

$$\lambda_A = \frac{L_A}{\overline{T}_A} Q_B \tag{1}$$

Inventory holding costs during each cycle is calculated based on the fact that the inventory level reduces discretely from r_A-λ_A+Q_A to r_A-λ_A. Then,

$$[(r_A-\lambda_A+Q_A)+(r_A-\lambda_A+Q_A-Q_B)+...+(r_A-\lambda_A+Q_A-K_1Q_B)]\overline{T}_B.h_A$$
$$=[(K_1+1)(r_A-\lambda_A+Q_A)-(1+2+...+K_1)Q_B]\overline{T}_B.h_A$$
$$=(K_1+1)(r_A-\lambda_A+Q_A-\frac{K_1Q_B}{2}).\overline{T}_B.h_A$$
$$=(K_1+1)(r_A-\lambda_A+\frac{Q_A}{2}).\overline{T}_B.h_A \tag{2}$$

To calculate $\overline{T}_B$ we need to identify its function. T_B , which is an interval between two successive orders of firm B. We show T_B is a fuzzy number and it equals $\overline{T}_B$ when defuzzified.

Since Q_B is consumed in firm B during two successive inter-interval (T_B) and demand reduces linearly with respect to time (by the assumption), then $\tilde{T}_B=\frac{L_B}{\tilde{X}_B}Q_B$, where $\tilde{X}_B$ is the demand in firm B during L_B. Bearing in mind $\tilde{X}_B$ membership function, then,

$$\mu_{\tilde{T}_B}(t_B)=\begin{cases}\dfrac{a_3t_B-Q_BL_B}{t_B(a_3-a_2)} & \dfrac{Q_BL_B}{a_3}\le t_B\le\dfrac{Q_BL_B}{a_2}\\ \dfrac{Q_BL_B-a_1t_B}{t_B(a_2-a_1)} & \dfrac{Q_BL_B}{a_2}\le t_B\le\dfrac{Q_BL_B}{a_1}\end{cases} \tag{3}$$

$$def(\tilde{T}_B)=\frac{\int\mu_{\tilde{T}_B}(t_B)t_Bd_{t_B}}{\int\mu_{\tilde{T}_B}(t_B)d_{t_b}}$$

$$\int\mu_{\tilde{T}_B}(t_B)t_Bd_{t_B}=(Q_BL_B)^2\left[\frac{a_3-a_1}{2a_1a_2a_3}\right]$$

$$\int\mu_{\tilde{T}_B}(t_B)d_{t_b}=Q_BL_B\left[\frac{1}{a_3-a_2}\ln(\frac{a_2}{a_3})+\frac{1}{a_2-a_1}\ln(\frac{a_2}{a_1})\right]$$

$$def(\tilde{T}_B)=\frac{(Q_BL_B)^2\left[\frac{a_3-a_1}{2a_1a_2a_3}\right]}{Q_BL_B\left[\frac{1}{a_3-a_2}\ln(\frac{a_2}{a_3})+\frac{1}{a_2-a_1}\ln(\frac{a_2}{a_1})\right]}$$

$$=Q_BL_B\underbrace{\frac{\left[\frac{a_3-a_1}{2a_1a_2a_3}\right]}{\left[\frac{1}{a_3-a_2}\ln(\frac{a_2}{a_3})+\frac{1}{a_2-a_1}\ln(\frac{a_2}{a_1})\right]}}_{\theta}=Q_BL_B\theta \tag{4}$$

Relations (1) and (4) yield

$$\lambda_A = \frac{L_A}{\bar{T}_B} Q_B = \frac{L_A}{Q_B L_B \theta} Q_B = \frac{L_A}{L_B \theta} \tag{5}$$

By (2), (4), and (5) the holding costs for firm *A* is,

$$(K_1 + 1)(r_A - \frac{L_A}{L_B \theta} + \frac{Q_A}{2})(Q_B L_B \theta) h_A$$

Shortage occurs whenever demand is greater than order point quantity during lead time (x_A) and it is calculated as

$$b_A = x_A - r_A; \;\; x_A > r_A$$

To derive x_A one can assume that, during the interval of $\tilde{T}_B$ a demand of Q_B enters firm *A*. So, the demand during L_A is $\frac{L_A}{\tilde{T}_B}$ and denoted as *m*, hereon. Since $\tilde{T}_B$ is a fuzzy number, *m* is also a fuzzy number with the following membership function.

$$\mu_{\tilde{M}}(m) = \begin{cases} \dfrac{Q_B L_B m - a_1 L_A}{L_A (a_2 - a_1)} & \dfrac{a_1 L_A}{Q_B L_B} \le m \le \dfrac{a_2 L_A}{Q_B L_B} \\ \dfrac{a_3 L_A - Q_B L_B m}{L_A (a_3 - a_2)} & \dfrac{a_2 L_A}{Q_B L_B} \le m \le \dfrac{a_3 L_A}{Q_B L_B} \end{cases},$$

$$\begin{aligned} b_A &= x_A - r_A; & x_A &> r_A \\ &= m Q_B - K_2 Q_B; & m Q_B &> K_2 Q_B \\ &= Q_B \underbrace{(m - K_2)}_{n_b}; & m &> K_2 \end{aligned}$$

n_b is a fuzzy number, because it is a function of *m*. The membership function of n_b depends on whether $\frac{a_1 L_A}{Q_B L_B} \le K_2 \le \frac{a_2 L_A}{Q_B L_B}$ or $\frac{a_2 L_A}{Q_B L_B} \le K_2 \le \frac{a_3 L_A}{Q_B L_B}$. Therefore, both cases are studied separately.

Case 1:

$$\frac{a_1 L_A}{Q_B L_B} \le K_2 \le \frac{a_2 L_A}{Q_B L_B}$$

$$n_b = m - K_2; \qquad m > K_2$$

$$\Rightarrow m = n_b + K_2; \;\; m > K_2$$

$$\Rightarrow \mu_{\tilde{N}_b}(n_b) = \begin{cases} \dfrac{Q_B L_B (n_b + K_2) - a_1 L_A}{L_A (a_2 - a_1)} & K_2 \le n_b + K_2 \le \dfrac{a_2 L_A}{Q_B L_B} \\ \dfrac{a_3 L_A - Q_B L_B (n_b + K_2)}{L_A (a_3 - a_2)} & \dfrac{a_2 L_A}{Q_B L_B} \le n_b + K_2 \le \dfrac{a_3 L_A}{Q_B L_B} \end{cases}$$

$$\Rightarrow \mu_{\tilde{N}_b}(n_b)=\begin{cases}\dfrac{Q_BL_Bn_b+Q_BL_BK_2-a_1L_A}{L_A(a_2-a_1)} & 0\le n_b\le \dfrac{a_2L_A}{Q_BL_B}-K_2\\ \dfrac{a_3L_A-Q_BL_Bn_b-Q_BL_BK_2}{L_A(a_3-a_2)} & \dfrac{a_2L_A}{Q_BL_B}-K_2\le n_b\le \dfrac{a_3L_A}{Q_BL_B}-K_2\end{cases}$$

Case 2:

$$\frac{a_2L_A}{Q_BL_B}\le K_2\le \frac{a_3L_A}{Q_BL_B}$$

$$\mu_{\tilde{N}_b}(n_b)=\frac{a_3L_A - Q_BL_B(n_b + K_2)}{L_A(a_3-a_2)} \quad ; K_2\le n_b+K_2\le \frac{a_3L_A}{Q_BL_B}$$

$$\Rightarrow \mu_{N_b}=\frac{a_3L_A-Q_BL_Bn_b-Q_BL_BK_2}{L_A(a_3-a_2)} \quad ; 0\le n_b\le \frac{a_3L_A}{Q_BL_B}-K_2$$

To calculate the shortage costs, it is required to calculate defuzzified $\tilde{N}_b$ in both cases.

Defuzzified $\tilde{N}_b$ in Case 1 results in:

$$\int \mu_{N_b}(n_b)n_bd_{n_b}=\int_0^{\frac{a_2L_A}{Q_BL_B}-K_2}\frac{Q_BL_Bn_b+Q_BL_BK_2-a_1L_A}{L_A(a_2-a_1)}n_bd_{n_b}+\int_{\frac{a_2L_A}{Q_BL_B}-K_2}^{\frac{a_3L_A}{Q_BL_B}-K_2}\frac{a_3L_A-Q_BL_Bn_b-Q_BL_BK_2}{L_A(a_3-a_2)}n_bd_{n_b}$$
$$=\xi_1(K_2,Q_B)$$

$$\int \mu_{N_b}(n_b)d_{n_b}=\int_0^{\frac{a_2L_A}{Q_BL_B}-K_2}\frac{Q_BL_Bn_b+Q_BL_BK_2-a_1L_A}{L_A(a_2-a_1)}d_{n_b}+\int_{\frac{a_2L_A}{Q_BL_B}-K_2}^{\frac{a_3L_A}{Q_BL_B}-K_2}\frac{a_3L_A-Q_BL_Bn_b-Q_BL_BK_2}{L_A(a_3-a_2)}.d_{n_b}$$
$$=\xi_2(K_2,Q_B)$$

$$def(\tilde{N}_b)=\frac{\xi_1(K_2,Q_B)}{\xi_2(K_2,Q_B)}=\xi(K_2,Q_B)$$

Hence, the defuzzified shortage value is $def(b_A)=\xi(K_2,Q_B)Q_B$. Defuzzifying $\tilde{N}_b$ in Case 2 results in:

$$\int \mu_{\tilde{N}_b}(n_b)n_bd_{n_b}=\int_0^{\frac{a_3L_A}{Q_BL_B}-K_2}\frac{a_3L_A-Q_BL_Bn_b-Q_BL_BK_2}{L_A(a_3-a_2)}.n_bd_{n_b}=\gamma_1(K_2,Q_B)$$

$$\int \mu_{\tilde{N}_b}(n_b).d_{n_b}=\int_0^{\frac{a_3L_A}{Q_BL_B}-K_2}\frac{a_3L_A-Q_BL_Bn_b-Q_BL_BK_2}{L_A(a_3-a_2)}.d_{n_b}=\gamma_2(K_2,Q_B),$$

$$def(\tilde{N}_b)=\frac{\gamma_1(K_2,Q_B)}{\gamma_2(K_1,Q_B)}=\gamma(K_2,Q_B)$$

In this case, the defuzzified shortage value is $def(b_A)=\gamma(k_2,Q_B).Q_B$.

To calculate the shortage cost within an interval, π_A should be multiplied by the defuzzified shortage. Since there are two cases, a binary variable is introduced to model it. Hence, the shortage cost within an interval is as follows.

$$\pi_A \xi(K_2,Q_B).Q_B.y_A + \pi_A \gamma(K_2,Q_B).Q_B(1-y_B)$$

$$s.t$$

$$\frac{a_1 L_A}{Q_B L_B} y_A \le K_2 \le \frac{a_2 L_A}{Q_B L_B} y_A + M(1-y_A)$$

$$\frac{a_2 L_A}{Q_B L_B}(1-y_A) \le K_2 \le \frac{a_3 L_A}{Q_B L_B}(1-y_A) + My_A$$

$$y_A = 0 \ or \ 1$$

(M is a very large and positive number)

Firm A's inventory cost during each cycle is:

$$A_A + (K_1+1)(r_A - \frac{L_A}{L_B\theta} + \frac{Q_A}{2})(Q_B L_B \theta)h + \pi_A \xi(K_2,Q_B).Q_B.y_A$$
$$+\pi_A \gamma(K_2,Q_B).Q_B(1-y_B)$$

The average number of annual intervals is $\frac{D_A}{Q_A}$. Thus, firm A's average annual cost is:

$$K(Q_A,r_A) = \frac{D_A A_A}{Q_A} + \frac{D_A}{Q_A}(K_1+1)(r_A - \frac{L_A}{L_B\theta} + \frac{Q_A}{2})(Q_B L_B \theta)h + \frac{D_A}{Q_A}\pi_A \xi(K_2,Q_B).Q_B.y_A$$
$$+\frac{D_A}{Q_A}\pi_A \gamma(K_2,Q_B).Q_B(1-y_B)$$

$$s.t$$

$$\frac{a_1 L_A}{Q_B L_B} y_A \le K_2 \le \frac{a_2 L_A}{Q_B L_B} y_A + M(1-y_A)$$

$$\frac{a_2 L_A}{Q_B L_B}(1-y_A) \le K_2 \le \frac{a_3 L_A}{Q_B L_B}(1-y_A) + My_A$$

$$Q_A = K_1 Q_B$$

$$r_A = K_2 Q_B$$

$$y_A = 0 \ or \ 1$$

$$Q_A, r_A \ge 0$$

$$K_1, K_2 : Integer$$

We need to point out that D_A, the average annual demand for firm A is equal to that of firm B, *i.e.* $D_A = D_B$.

3.3. *Objective function*

In this model, we intend to minimize total system cost. So, the sum of both firms cost function yields the objective function. There are a binary variable in each average annual cost of firms, y_A and y_B. Depending on the values of these variables four situations can be made for the objective function. Thus, the objective function can be written as the following four sub problems. One of these sub problems which has the minimum total cost is the overall optimal solution.

The decision variables are r_A, Q_A, r_B and Q_B. Since $Q_A=K_1.Q_B$ and $r_A=K_2.Q_B$, we substitute $K_1.Q_B$ and $K_2.Q_B$ for Q_A and r_A , respectively. Thus, the objective function is presented in respect of r_B, Q_B, K_1 and K_2.

$$
\left.
\begin{array}{l}
\min K(Q_B, r_B, K_1, K_2) = \dfrac{D_B A_B}{Q_B} + h_B\left[\dfrac{Q_B}{2} + r_B - \lambda_B\right] + \dfrac{\pi_B D_B f(r_B)}{Q_B} \\
+\dfrac{D_A A_A}{K_1 Q_B} + \dfrac{D_A}{(K_1 Q_B)}(K_1+1)(K_2 Q_B - \dfrac{L_A}{L_B\theta} + \dfrac{K_1 Q_B}{2})(Q_B L_B \theta) h_A \\
+\dfrac{D_A}{(K_1 Q_B)}\pi_A \xi(K_2, Q_B).Q_B. \\
s.t \\
a_1 \le r_B \le a_2 \\
\dfrac{a_1 L_A}{Q_B L_B} \le K_2 \le \dfrac{a_2 L_A}{Q_B L_B} \\
Q_B, Q_A \ge 0 \\
K_1, K_2 : Integer
\end{array}
\right\} \text{Sub problem1}
$$

$$
\left.
\begin{array}{l}
\min K(Q_B, r_B, K_1, K_2) = \dfrac{D_B A_B}{Q_B} + h_B\left[\dfrac{Q_B}{2} + r_B - \lambda_B\right] + \dfrac{\pi_B D_B g(r_B)}{Q_B} \\
+\dfrac{D_A A_A}{K_1 Q_B} + \dfrac{D_A}{(K_1 Q_B)}(K_1+1)(K_2 Q_B - \dfrac{L_A}{L_B\theta} + \dfrac{K_1 Q_B}{2})(Q_B L_B \theta) h_A \\
+\dfrac{D_A}{(K_1 Q_B)}\pi_A \xi(K_2, Q_B).Q_B. \\
s.t \\
a_2 \le r_B \le a_3 \\
\dfrac{a_1 L_A}{Q_B L_B} \le K_2 \le \dfrac{a_2 L_A}{Q_B L_B} \\
Q_B, Q_A \ge 0 \\
K_1, K_2 : Integer
\end{array}
\right\} \text{Sub problem2}
$$

$$\left.\begin{aligned}
&\min K(Q_B, r_B, K_1, K_2) = \frac{D_B A_B}{Q_B} + h_B\left[\frac{Q_B}{2} + r_B - \lambda_B\right] + \frac{\pi_B D_B f(r_B)}{Q_B}\\
&+\frac{D_A A_A}{K_1 Q_B} + \frac{D_A}{(K_1 Q_B)}(K_1+1)(K_2 Q_B - \frac{L_A}{L_B\theta} + \frac{K_1 Q_B}{2})(Q_B L_B \theta) h_A\\
&+\frac{D_A}{(K_1 Q_B)}\pi_A \gamma(K_2, Q_B).Q_B\\
&s.t\\
&a_1 : r_B : a_2\\
&\frac{a_2 L_A}{Q_B L_B} \le K_2 \le \frac{a_3 L_A}{Q_B L_B}\\
&Q_B, Q_A \ge 0\\
&K_1, K_2 : Integer
\end{aligned}\right\}\text{Sub problem3}$$

$$\left.\begin{aligned}
&\min K(Q_B, r_B, K_1, K_2) = \frac{D_B A_B}{Q_B} + h_B\left[\frac{Q_B}{2} + r_B - \lambda_B\right] + \frac{\pi_B D_B g(r_B)}{Q_B}\\
&+\frac{D_A A_A}{K_1 Q_B} + \frac{D_A}{(K_1 Q_B)}(K_1+1)(K_2 Q_B - \frac{L_A}{L_B\theta} + \frac{K_1 Q_B}{2})(Q_B L_B \theta) h_A\\
&+\frac{D_A}{(K_1 Q_B)}\pi_A \gamma(K_2, Q_B).Q_B\\
&s.t\\
&a_2 \le r_B \le a_3\\
&\frac{a_2 L_A}{Q_B L_B} \le K_2 \le \frac{a_3 L_A}{Q_B L_B}\\
&Q_B, Q_A \ge 0\\
&K_1, K_2 : Integer
\end{aligned}\right\}\text{Sub problem4}$$

4. Solution Approach

The model is obviously so complicated that an optimal solution can not be obtained by any analytical approach. To solve this model, it is required to apply a metaheuristic method. Hence, we develop a genetic algorithm to solve this problem.

Genetic algorithm (GA) is a stochastic global search which operates on a population of solutions applying the principle of survival of the fittest solutions. GA is a procedure to search the population in parallel and its search direction is influenced with the objective function. The algorithm is independent of the complexity of the considered performance index. It suffices to specify the objective function and finite bounds of optimized variables.[18] For more details regarding genetic algorithm, the reader is referred to Refs. 19, 20 and 21.

The GA used in this research paper is similar to the ones used in standard literature, see Ref. 20. This is known as the simple genetic algorithm. The principles of genetic algorithm can be written as follows:

Step 1. Randomly initialize a population.
Step 2. Compute the fitness of each individual of the population.
Step 3. Generate new population by using genetic operators.
Step 4. Compute the fitness of each individual in the new generation.
Step 5. If the convergence condition is satisfied, stop, otherwise return to Step 3.

The main elements of GA of our proposed algorithm are described next.

4.1. *String representation*

Each individual (chromosome) in the population, which is a candidate solution, corresponds to a specific value of decision variables. The structure of the chromosomes is shown in Fig. 3. The chromosomes consist of substrings whose number is equal to the number of decision variables. The decision variables are r_B, Q_B, K_1 and K_2. In GAs, the search process operates on the encoded decision variables rather than the decision variables themselves. How to translate the decision variables to a GA chromosome is a major issue. The binary, integer, real-valued, messy and tree structure representations are the most important and widely used by many genetic and evolutionary algorithms.[22]

Goldberg[20] proposed principle of minimal alphabets for encodings, that is, the alphabet of the encoding should be as small as possible while still allowing a natural representation of solutions. The principle of minimal alphabets advises us to use bit string representation. Also, binary string is simple to implement.[23] Hence, Binary encodings are the most commonly used and nature-inspired representations for genetic algorithms, see Ref. 20.

The binary encoding has the effect that genotypes of some phenotypical neighbors are completely different. The gray encoding was designed to overcome this problem, see Ref. 24. In the gray encoding, every neighbor of a phenotype is also a neighbor of the corresponding genotype. Hence, in this paper each variable is encoded as a binary representation in type of gray encoding.

Depending on the desired precision, the encoding is performed using the different gene lengths. Here, the gene length of Q_B , r_B , K_1 and K_2 are 10, 10, 4 and 4 respectively.

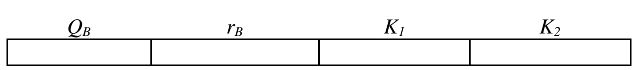

Fig. 3. GA chromosome structure.

4.2. *Objective and fitness functions*

Fitness function (FF) is the evaluation function used to determine the degree of appropriateness of each solution candidate in the solution domain. The fitness values are derived from objective function, through a ranking function. The objective function values of the individuals are first sorted in an ascending order. A fitness value is then assigned to each individual, depending on its position in the sorted population. The assigned fitness value is calculated as follows:

$$FF(n) = FF_{\max} - (2FF_{\max} - 2) \times \left(\frac{N_{ind} - n}{N_{ind} - 1} \right)$$

where, n is the position of the chromosome, N_{ind} is the number of chromosomes in the population, and FF_{max} is the maximum assigned fitness value. In this study, FF_{max} is assumed to be 2.

4.3. *GA operators*

Three basic genetic operators are employed to generate the new population from the initial population of chromosomes: selection, crossover, and mutation. The operators are used to make genetic evolution during solving process. Before proceeding to introduce the genetic operators, it should be noted that while creating a new generation, some of the fit individuals are propagated through to successive generation. The fraction of the population that is replaced in each cycle is named as generation gap.

4.3.1. *Selection*

Selection selects the chromosomes in the population based on their fitness for reproduction. The fitter the individual the more it is chosen for reproduction. Roulette wheel selection method (see Ref. 20) used in our algorithm, is among the most popular selection methods. In this selection method, the probability of the individual n to be selected is

$$P(n) = \frac{FF(n)}{\sum_{i=1}^{N_{ind}} FF(i)}$$

The fitness function values (FFs) should be nonnegative.

4.3.2. *Crossover*

Crossover is responsible for data exchange between distinct solutions. A uniform crossover is employed in this algorithm. In this type of crossover, at first, a random mask string whose length is the length of chromosome is generated. The bits of this string determine the parent whose corresponding bit will supply the offspring. This process is illustrated in Fig. 4 (this figure shows just part of strings). The offspring 1 is generated by taking the bits from parent 1 if the corresponding mask bit is 1 and the bits from parent 2 if the corresponding mask bit is 0. Offspring 2 is created using the inverse of the mask string.

Parent 1:	1	0	0	1	1	0	1	1	0	1
Parent 2:	0	0	1	1	1	1	0	0	0	1
Mask:	1	1	0	0	1	1	0	0	0	0
Offspring 1:	1	0	1	1	1	0	0	0	0	1
Offspring 2:	0	0	0	1	1	1	1	1	0	1

Fig. 4. Example of uniform crossover.

4.3.3. *Mutation*

Mutation is responsible for the injection of new information and ensures that the probability of searching a particular subspace of the solution domain is never zero. New chromosome is created by a random change of every bit of the old one from 0 to 1 or vice-versa, as is shown in Fig. 5.

Original string:	1	0	0	1	1	0	0	1	0	1
Mutated string:	1	0	0	0	1	0	0	1	0	1
				▲ Mutation						

Fig. 5. Example of mutation operator.

4.4. ***Values for the parameters of the GA***

In this paper we set the parameters based on the general conclusions which there are in this area. In the following, we state these conclusions and determine values for the parameters of the GA.

4.4.1. *Population size*

The population size affects performance of GAs. GAs generally do poorly with very small populations.[25] In other word, if a larger population size is used, it has higher probability to obtain better results. But, it takes much computation time and reduces the efficiency of the algorithm.[26] The population size used in earlier researches usually is in range of 10 to 200 (see Refs. 25–29). Since we are not sensitive to computation time so we use a large population size equal to 200.

4.4.2. *Crossover rate*

The crossover rate controls the frequency with which the crossover operator is applied. The higher the crossover rate, the more quickly new structures are introduced into the population. If the crossover rate is too high, high-performance structures are discarded faster than selection can produce improvements. If the crossover rate is too low, the search may stagnate due to the lower exploration rate.[25] The suitable ranges of crossover rate have been suggested in some articles, see Refs. 25 and 27. Based on the suggested values and some trials, the suitable range of crossover rate is 0.7–0.9, see Ref. 26. In this paper crossover rate is set to 0.7.

4.4.3. *Mutation rate*

Mutation increases the variability of the population. A low level of mutation serves to prevent any given bit position from remaining forever converged to a single value in- the entire population. A high level of mutation yields an essentially random search.[25] De Jong[27] states that the mutation rate as 1/pop_size seems to be about the best setting. Thus, in this paper we set the mutation rate equal to 1/pop_size which is 0.005.

4.5. *Termination criteria*

Generally, specifying a convergence criterion for a genetic algorithm is difficult. A common criterion that is used in this GA is to terminate genetic algorithm after a predetermined number of iterations. In this study the algorithm is terminated after 150 iterations.

5. Numerical Example

In this section we give a numerical example and run the proposed genetic algorithm for 10 times. The minimum total cost of these runs is considered as the optimal solution.

The data for the designed numerical example:
$a_1=10$, $a_2=13$, $a_3=25$, $A_A=10$, $A_B=5$, $h_A=2$, $h_B=6$, $\pi_A=4$, $\pi_B=4$, $L_A=1/12$, $L_B=1/48$, $D_B=360$

Table 1. Results of genetic algorithm for the example.

Run No.	Minimum Cost				Minimum Total System Cost
	Sub problem1	Sub problem2	Sub problem3	Sub problem4	
1	418.18	303.18	493.29	360.32	303.18
2	418.18	303.18	484.01	357.93	303.18
3	418.18	302.18	493.29	357.93	302.18
4	418.18	302.18	493.29	357.93	302.18
5	418.18	303.18	484.01	359.01	303.18
6	419.02	302.18	484.01	360.32	302.18
7	418.18	302.18	484.01	357.93	302.18
8	418.18	302.18	484.01	357.93	302.18
9	418.18	303.18	484.01	360.32	303.18
10	418.18	302.18	493.29	360.32	302.18

From Table 1 it is concluded that the proposed genetic algorithm converges to a near optimal solution. For this example the minimum cost is 302.18 and the solution associated with this cost is: $Q_B = 26.27$, $r_B = 25$, $K_1 = 3$ and $K_2 = 2$.

5.1. *Sensitivity analysis*

To investigate the impact of different parameters of firm *B* on the total system cost, sensitivity analysis is performed for a_1, a_2 and a_3, by starting from the data of this example. Table 2 shows how the total system cost changes with respect to a_1 which represents the minimum demand in firm *B* during a lead time. Obviously, the larger a_1, the less total system cost is resulted. Similarly, the effect of changing a_3, the maximum demand in firm *B* during a lead time, on the total system cost is shown in Table 3. In this case, the total system cost increases when a_3 is getting larger. It can be concluded that the larger variation (difference between a_1 and a_3) results in higher cost which is reasonable.

Finally, Table 4 represents the sensitivity of a_2, the demand in firm *B* during a lead time with membership of one. It is implied that the best value for a_2 is 16, if $a_1 = 10$ and $a_3 = 25$. In other words, the most appropriate shape for the fuzzy triangle is to have equal sides.

Table 2. Results of sensitivity.

Analysis on a_1

a_1	Total System Cost
1	581.01
2	452.67
3	400.64
4	371.05
5	351.22
6	335.08
7	322.44
8	312.09
9	306.58
10	302.18

Table 3. Results of sensitivity.

Analysis on a_3

a_3	Total System Cost
16	271.82
17	274.70
18	278.07
19	282.90
20	285.49
21	288.72
22	292.02
23	295.36
24	298.75
25	302.18

Table 4. Results of sensitivity.

Analysis on a_2

a_2	Total System Cost
11	308.05
12	304.87
13	302.18
14	298.92
15	295.74
16	294.08
17	294.35
18	295.46
19	297.26
20	297.36

6. Conclusions and Suggestions for Further Research

This paper deals with continuous review inventory control policy (r,Q) in a two level fuzzy supply chain. Fuzzy numbers are applied to present customer demand. The objective is to derive and minimize the total cost of two levels including ordering, holding and shortage cost. Considering the complexity arising from the model, we developed a genetic algorithm to obtain the optimal solution. The model developed in this paper involves three fields of supply chains, inventory control, and fuzzy sets. Each area is potentially expansible for further research. For example; in the field of supply chains one can extend this model to a single-vendor multi-buyer system. In the field of inventory control the comparison of effectiveness of our model with the common models which assume stochastic demand is a subject for future research. In the field of fuzzy sets one might consider different shapes of membership function; e.g. trapezoidal, bell-shaped, etc. More sensitivity analysis and effect of different parameters of the model on the optimal solution are interesting topics for further research.

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