UNSTEADY MIXED CONVECTION WITH HEATING EFFECTS IN A HORIZONTAL ANNULUS WITH CONSTANT ROTATING OUTER CYLINDER AND CONSTANT MAGNETIC FIELD

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Abstract:

In this study the problem of mixed convection of a fluid in the fully developed region between two horizontally concentric cylinders with infinite lengths which are held at different uniform temperatures is investigated numerically. The forced flow is induced by the cold rotating outer cylinder in slowly constant angular velocity with its axis at the center of annulus. Buoyanci effect also considered with Boussinesq approximation. Investigations are made for various combinations of non-dimensional group numbers, Reynolds (Re), Rayleigh (Ra), Hartmann (Ha) and Eckert (Eck). We will show how the increase of Ra, Re and Pr values affect the streamline and isotherms and also Nusselt distributions on the inner and outer cylinders. Another purpose of this study is to investigate effects of applying radially constant magnetic field as well as heat generation due to viscous dissipation terms on the characteristics of heat and fluid flow. It is also found that the flow oscillations in the annulus can be suppressed effectively by imposing an external MHD force in radial direction. Moreover, it is shown that viscous dissipation decreases heat transfer rate on the inner cylinder and increases this phenomenon on the outer one.

Keywords: Mixed convection, constant rotation, constant magnetic field

1- INTRODUCTION:

Recently, the laminar in either vertical or horizontal concentric annulus has received attentions by various investigators because of its wide applications such as heat exchangers, cooling systems in electrical devices, solar collectors, cooling of turbine rotors and cooling of high speed gas bearings. Up to date, most works for mixed-convection problems in rotating systems have been performed for the flows in vertical cylindrical annuli [1-3]. Relatively few studies, however, have been made for the flows in horizontal annuli. A few authors [4-6] have studied mixed-convective flows within a horizontal annulus with a heated rotating inner cylinder. The flow patterns in rotating cylinders categorized into three basic types according to the number of eddies by J.S.Yoo [7]. Abu-Hijleh and Heilen [8], has proposed a correlation for the average Nusselt number as a function of Reynolds number and buoyancy parameter from a rotating cylinder. Mixed heat and fluid flow in rotating annuli has been investigated numerically by Lee [9]. It is shown that the effects of viscous dissipation enhance the effects of buoyancy and vice versa in an inclined channel by Barletta and Zanchini [10]. Also it is shown with using electromagnetic fields, heat transfer and fluid flow over a non-isothermal horizontal cylinder in porous medium can be controlled in [11]. Non-uniform circumferential heating in horizontal concentric annuli [12] and categorization of the flow regimes according to the number of eddies established on the Ra-Re plane for various numbers [13] are another researches in this subject. A study of double-diffusive mixed convection of a binary fluid within a two-dimensional horizontal annulus with cooled rotating outer cylinder has been investigated by Al-Amiri and Khanafer [14]. A study to provide an inverse approach for estimating the viscosity of fluid and thermal behavior of the concentric cylinders has been presented in [15]. Mixed convection with a temperature-dependent viscosity and its effects on the dimensionless pressure and velocity is another study that has been investigated by Zanchini [16]. In the newest study, Barletta et al. [17], investigated combined forced and free flow of an electrically conducting fluid in a vertical annular porous medium surrounding a straight cylindrical electric cable with radially varying magnetic field.

The objective of the present study is to investigate numerically the problem of mixed convection of a fluid in the fully developed region between two horizontally concentric cylinders with infinite lengths which are held at different uniform temperatures. The forced flow is induced by the cold rotating outer cylinder in slowly constant angular velocity with its axis at the center of annulus. Buoyanci effect also considered with Boussinesq approximation. Investigations are made for various combinations of non-dimensional group numbers, Reynolds (Re), Rayleigh (Ra), Hartmann (Ha) and Eckert (Eck). We will show that how increasing of Ra, Re and Pr values affect the streamline and isotherms and also Nu distributions on the inner and outer cylinders. Another purpose of this study is to investigate effects of applying radially constant magnetic field as well as heat generation due to viscous dissipation terms on the characteristics of heat and fluid flow. It is also found that the flow oscillations in the annulus can be suppressed effectively by imposing an external MHD force in radial direction. Moreover, it is shown that viscous dissipation decreases heat transfer rate on the inner cylinder and increases this quantity on the outer one.

NOMENCLATURES:

B_0 Constant magnetic field
D Characteristic length of annulus, (R_o-R_i)
Eck Eckert number
g Gravity acceleration (m/s^2)
Ha Hartmann number
$Nu_{Cond.}$ Nusselt number of pure conduction state
Nu_i , Nu_o Local Nusselt number at the inner and outer cylinder
$\overline{Nu_i}$ Mean Nusselt number at the inner cylinder
$\overline{Nu_o}$ Mean Nusselt number at the outer cylinder
P Hydrodynamic pressure
P^* Pressure
$\Pr$ Prandtl number
Q Net circulation of fluid in the direction of cylinder's rotation, $|\Psi_2 - \Psi_1|$
r Dimensionless radial coordinate
r_i Dimensionless radii of the inner cylinder
r_o Dimensionless radii of the outer cylinder
r^* Radial coordinate
R_i , R_o Radius of the inner and outer cylinders, m
Ra Rayleigh number
Re Reynolds number
T Temperature
T_i, T_o Temperature of the inner and outer cylinders
T_m Reference temperature
t^* Time, s
V_r, V_ϕ Dimensionless radial and circumferential velocity components
V_r^*, V_ϕ^* Radial and circumferential velocity components, m/s

Greeks

β Volumetric thermal expansion coefficient (K^{-1})
θ Dimensionless temperature
ϕ Circumferential coordinate
Φ Viscous dissipation
Ψ_1, Ψ_2 Values of the stream function at the inner and outer cylinder, respectively.
Ω Angular velocity of outer cylinder, s^{-1}
ν Kinematic viscosity, m^2s^{-1}
σ Electrical conductivity of fluid
σ_o Ratio of the inner cylinder diameter to the gap width, $\sigma_o=2R_i/(R_o-R_i)$
ρ Fluid density, kg/m^3

2- PROBLEM FORMULATION:

The problem is formulated in a polar coordinate system, as in Fig.1. The non-dimensional full Navier-Stokes and energy equations for mixed convection in the annulus between horizontal concentric cylinders with heated rotating outer cylinder with constant velocity are:

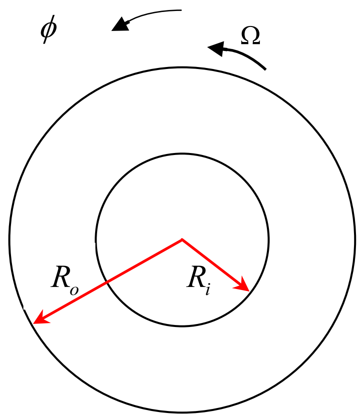


Fig.1- Geometry of the cylinders

Continuity equation:

$$\frac{\partial(rV_r)}{r\partial r}+\frac{\partial V_\phi}{r\partial\phi}=0 \tag{1}$$

r- Momentum:

$$\frac{\partial V_r}{\partial t}+\frac{\partial(rV_rV_r)}{r\partial r}+\frac{\partial(V_rV_\phi)}{r\partial\phi}-\frac{V_\phi^{\;2}}{r}=\frac{-\partial P}{\partial r}+\frac{1}{\mathrm{Re}}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_r}{\partial r}\right)+\frac{\partial}{r\partial\phi}\left(\frac{\partial V_r}{r\partial\phi}\right)-\frac{V_r}{r^2}-\frac{2}{r^2}\frac{\partial V_\phi}{\partial\phi}\right]$$
$$+\frac{Ra}{\mathrm{Pr}\,\mathrm{Re}^2}\theta\cos\phi-V_r\frac{Ha^2}{\mathrm{Re}} \tag{2}$$

φ- Momentum:

$$\frac{\partial V_\phi}{\partial t}+\frac{\partial(rV_rV_\phi)}{r\partial r}+\frac{\partial(V_\phi V_\phi)}{r\partial\phi}+\frac{V_\phi V_r}{r}=\frac{-1}{r}\frac{\partial P}{\partial\phi}+\frac{1}{\mathrm{Re}}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_\phi}{\partial r}\right)+\frac{\partial}{r\partial\phi}\left(\frac{\partial V_\phi}{r\partial\phi}\right)-\frac{V_\phi}{r^2}+\frac{2}{r^2}\frac{\partial V_r}{\partial\phi}\right] \tag{3}$$
$$-\frac{Ra}{\mathrm{Pr}\,\mathrm{Re}^2}\theta\sin\phi$$

Energy Equation:

$$\frac{\partial\theta}{\partial t}+\frac{\partial(rV_r\theta)}{r\partial r}+\frac{\partial(V_\phi\theta)}{r\partial\phi}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)+\frac{\partial}{r\partial\phi}\left(\frac{\partial\theta}{r\partial\phi}\right)\right]+\frac{Ek}{\mathrm{Re}}\Phi \tag{4}$$

where,

$$\Phi=2\left[\left(\frac{\partial V_r}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial V_\phi}{\partial\phi}+\frac{V_r}{r}\right)^2\right]+\left(\frac{\partial V_\phi}{\partial r}-\frac{V_\phi}{r}+\frac{1}{r}\frac{\partial V_r}{\partial\phi}\right)^2$$

Where the non-dimensional parameters are:

$t=\frac{t^*}{D/(R_o\Omega)}$	$V_r=\frac{V_r^*}{R_o\Omega}$	$V_\phi=\frac{V_\phi^*}{R_o\Omega}$
$r=\frac{r^*}{D}$	$P=\frac{P^*}{\rho(R_o^2\Omega^2)}$	$\theta=\frac{T-T_o}{T_i-T_o}$
$\mathrm{Re}=\frac{R_o\Omega D}{\nu}$	$\mathrm{Pr}=\frac{\nu}{\alpha}$	$Ra=\frac{g\beta(T_i-T_o)D^3}{\nu\alpha}$
$Ek=\frac{R_o^2\Omega^2}{c_p(T_i-T_o)}$	$Ha=B_0D\sqrt{\frac{\sigma}{\rho\nu}}$	

(5)

The dimensionless Nusselt number in pure conduction in the absence of fluid motion is:

$$Nu_{cond}=\frac{1}{ln\left(\frac{r_o}{r_i}\right)} \tag{6}$$

The local Nusselt number is defined as the actual heat flux divided by $Nu_{Cond.}$:

$$Nu_i(\phi)=-\left(r\frac{\partial\theta}{\partial r}\right)/Nu_{Cond.} \quad \text{at } r=r_i \tag{7}$$

$$Nu_o(\phi)=-\left(r\frac{\partial\theta}{\partial r}\right)/Nu_{Cond.} \quad \text{at } r=r_o \tag{8}$$

and the mean Nusselt numbers on the inner and outer cylinders are given by:

$$\overline{Nu_i}=\frac{1}{2\pi}\int_0^{2\pi}Nu_i(\phi)\,d\phi \tag{9}$$

$$\overline{Nu_o}=\frac{1}{2\pi}\int_0^{2\pi}Nu_o(\phi)\,d\phi \tag{10}$$

Also, the net circulation of fluid in the direction of cylinder's rotation is defined by:

$$Q=|\Psi_2-\Psi_1| \tag{11}$$

3- NUMERICAL PROCEDURES

Finite volume scheme consisting of tri-diagonal matrix algorithm (TDMA) is used to solve these equations by SIMPLE algorithm. Convergence criteria for the numerical solution of the above equations are obtained by comparing the residue of two repeated sequences of each of the momentum and energy equations in which | Residual (n+1) - Residual (n) | $< 5x10^{-9}$. Mesh independency is shown in the following Table where IM and JM are the number of nodes in ϕ-and $r-$directions for specific values of non-dimensional **groups of Re=100, Pr=0.7, Eck=0.0002, Ra=10000.0 and Ha=20.0**:

Table1. Mesh Independency

IM	JM	Nu_i	Nu_o	Q
18	5	1.7064	1.7280	0.100921
32	9	1.7526	1.7613	0.109995
56	17	1.6838	1.6842	0.105361
100	***31***	***1.6747***	***1.6757***	***0.103831***
180	55	1.6747	1.6759	0.103835

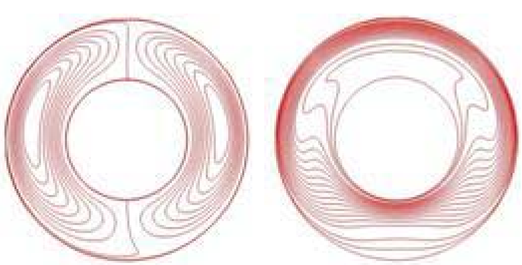
4- PRESENTATION OF RESULTS

A wide range of pertinent parameters such as Reynolds number, Rayleigh number, Prandtl number, Hartmann number and Eckert number are considered in this study in the range of $1\leq Re\leq 200$, $0\leq Ra\leq 50000$, $1\leq Pr \leq 10$, $0\leq Ha \leq 200$, $0.001\leq Eck \leq 0.1$ and the ratio of the inner cylinder diameter to the annulus gap width is taken between $0.5\leq \sigma_o \leq 5$.

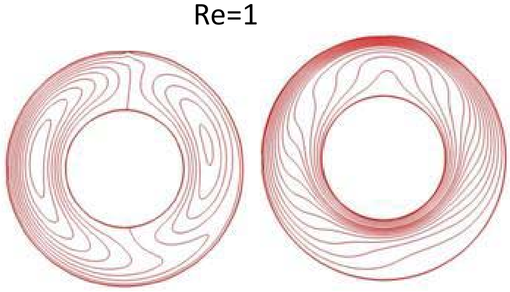
The effect of Reynolds number on the streamlines and isotherms is shown in Fig. 2 for a stationary inner cylinder where other parameters are constant. For small Reynolds number the flow field within the annulus is characterized by two symmetric kidney-shaped eddies with respect to the vertical axis. This pattern in the fluid flow is mainly due to the influence of the buoyancy force. As the Reynolds number increases due to the rotation of the outer cylinder, three different flow patterns are illustrated in Fig. 2 which are characterized by two eddies, one eddy and no eddy depending on the speed of the outer cylinder. The flow field induced by pure buoyancy force at very small Reynolds number consists of two kidney-shaped eddies which are symmetric with respect to the vertical axis $\phi = 0$. When Reynolds number is increased slightly, the symmetry of flow patterns breaks down and speed of the eddy in the region of $\pi\leq\phi\leq 2\pi$ is reduced. As a result of strong induced flow by the rotating outer cylinder, the eddy in the right hand side of the annulus disappears at Re=100. This effect becomes more pronounced at higher Reynolds numbers (Re=200). The isotherms presented in Fig. 2 also show the above characteristics well. As it's shown, the isotherms when Re=1 is very similar to the case of natural convection because of high aspect ratio between Ra to Re^2 but with increase of the speed of rotation, the effect of natural convection disappears gradually. Also it can be found that at Re=200, there is no oscillation of flow within the annulus, therefore isotherms have completely circle shapes with the same center similar to forced flow pattern.

The effects of the Prandtl number on the streamlines and isotherms for Re=100.0, Eck=0.0007, Ra=10000.0 and Ha=20.0 are shown in Fig. 3. By using the above constant values and with considering the fixed value for the Ra number, there is noticeable aspect ratio between Grashof number and Re^2 for lower Pr numbers. Therefore, buoyancy-induced flow plays more important role on the flow patterns and this causes the existence of two-eddy flow pattern for Pr=0.5. As the value of Pr number increases, the aspect ratio between Grashof number and Re^2 decreases with the Ra number remaining at the fixed value and this tends to enhance the influence of force-induced flow. The above reasons along with forced flow being opposed by buoyancy-induced flow in the right hand side of the annulus cause reduction of the strength of the eddy in that region and disappearance of the eddy at Pr=2, eventually. The eddy remained in the left hand side within the annulus is disappeared with increasing the value of Pr number, too. In similar way, the effects of high values of Pr number on the stratifying the temperature field in the radial direction can be simply observed in this Figure. It is interesting to note that the thermal plumes near the upper portion of the inner cylinder are tilted to the direction of cylinder's rotation at Pr=2.

Streamlines Isotherm

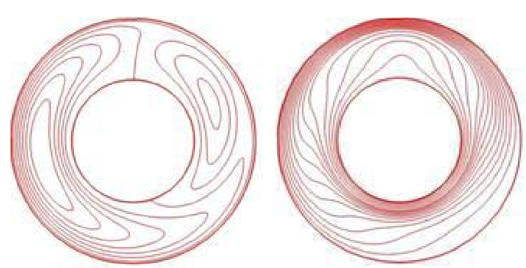


Re=1

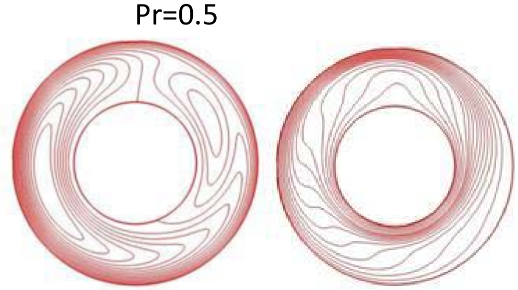


Re=20

Streamlines Isotherms



Pr=0.5



Pr=1

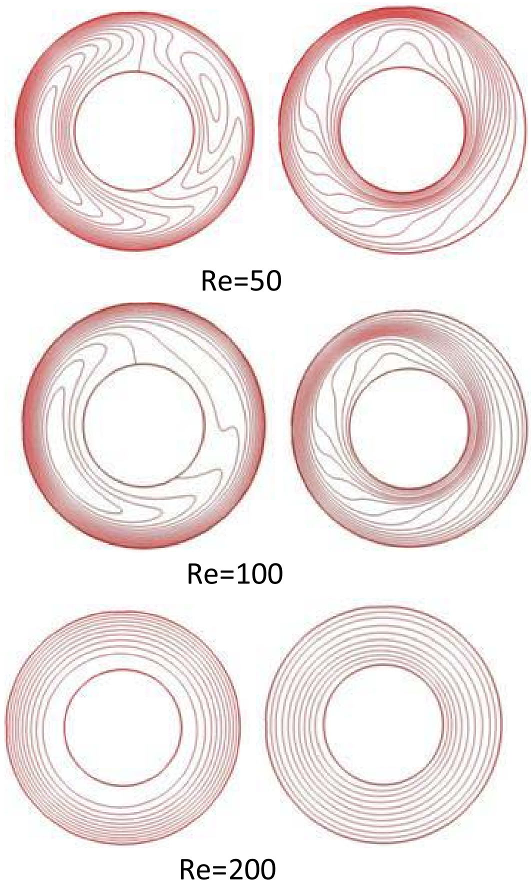


Fig. 2. Effect of Reynoldz number on the streamlines and Isotherms (Pr=2 ,Eck=0.0007,Ra=10000.0 ,Ha=20.0)

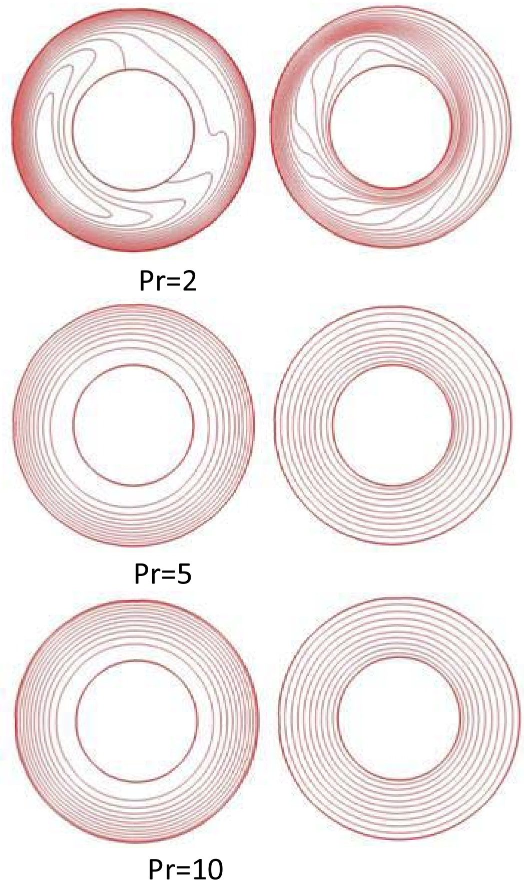


Fig. 3. Effect of Prandtl number on the streamlines and Isotherms (Re=100.0,Eck=0.0007,Ra=10000.0 ,Ha=20.0)

To see the effects of Ra number on the local heat fluxes, the distributions of local Nusselt numbers at the inner and outer cylinders for Re=50, Pr=1, Eck=0.001 and Ha=20 are shown in Figs 4 and 5. As it has been illustrated, with increase of the value of Ra number, local heat transfer rate increases specially at the points of maximum local Nusselt number at both of the inner and outer surfaces when ϕ is about 180 degrees for the inner cylinder and is about 0.0 degrees for the outer cylinder. On the other hand, there's no prominent change at the minimum points of local heat fluxes with varying the values of Ra number at the both of cylinders. It must be noticed that when Ra=0.0, there is no effect of natural convection within the concentric annulus. In this case, the local values of Nusselt number in both of the surfaces are about one.

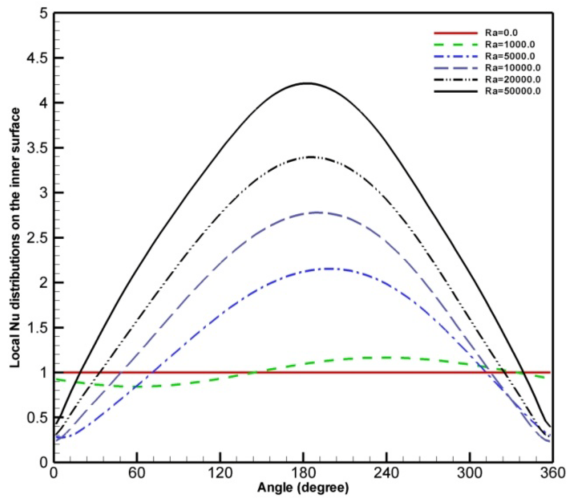


Fig.4. Effect of Ra number on the local Nu number distributions at the inner cylinder (Re=50, Pr=1 , Eck=0.001, Ha=20.0)

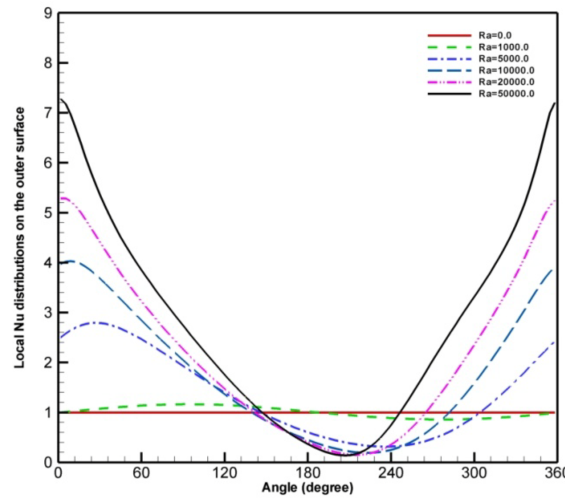


Fig. 5. Effect of Ra number on the local Nu number distributions at the outer cylinder (Re=50, Pr=1 , Eck=0.001, Ha=20.0)

The effects of viscous dissipation terms on the rate of heat transfer are proposed in Figs. 6 and 7. With increasing the value of Eckert number, the influences of heat generation due to the viscous dissipations in the concentric annulus are enhanced, too. It is so clear that with enhancing the amount of heat generation within the annulus, the average temperature of the fluid at the circumferential area sections is increased. According to non-dimensional procedure, the dimensionless temperatures of the inner and outer cylinders are held at 1.0 and 0.0, respectively. It is obvious that with increasing the amount of average temperature at every circumferential area sections, the rate of heat transfer between the fluid and the inner cylinder is decreased because of reducing the temperature differences between them and on the contrary with this, the heat transfer rate between the fluid and the outer cylinder is increased because of enhancing the temperature differences between them. The Figs. 6 and 7, which are obtained with variation of Eckert number from 0.001 to 0.1, confirm the above explanations well. Also it can be easily found from Fig. 6 that the fluctuations of local Nu distributions are suppressed by increasing the value of Eckert number gradually.

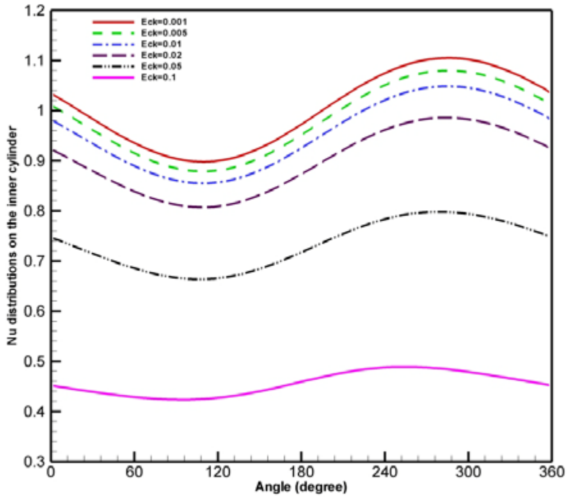


Fig.6. Effect of Eck number on the local Nu number distributions at the inner cylinder (Re=100, Pr=20 , Ra=10000.0, Ha=0.0)

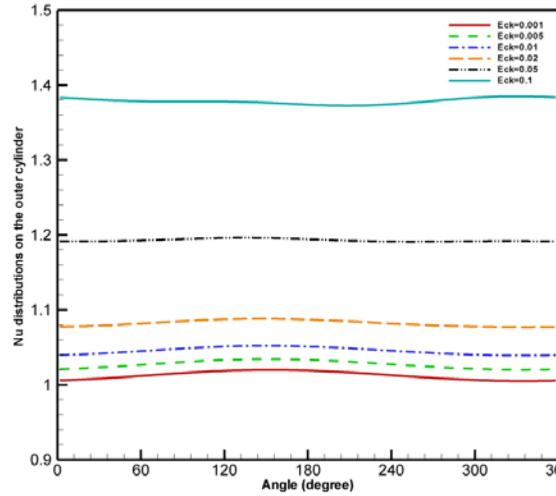


Fig. 7. Effect of Eck number on the local Nu number distributions at the outer cylinder (Re=100, Pr=20 , Ra=10000.0, Ha=0.0)

Figs. 8 and 9 show the effects of imposing the constant magnetic field on the flow and temperature distributions for Re=100, Pr=1, Eck=0.0 and Ra=10000 at ϕ=0. In the absence of magnetic field (Ha=0.0), the circumferential velocity and temperature distributions include high-gradient slope, especially near the cold cylinder. It is interesting to note that as the strength of the magnetic field increases (Ha=50.0), the variation of temperature and flow field become more uniform than before. As Hartmann number is further increased (Ha=100), these distributions are nearly conduction like and this is due to the suppression of convection by the magnetic field. Also, it can be seen that the flow field becomes completely stable and conduction like at Ha=200.

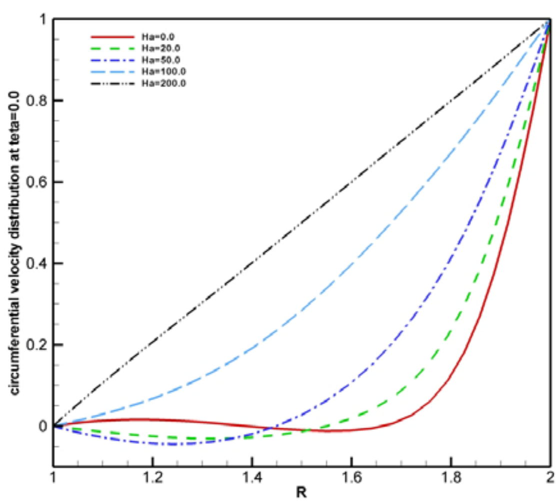


Fig.8. Effect of Ha number on the circumferential velocity distribution at ϕ=0.0 (Re=100, Pr=1 , Ra=10000.0, Eck=0.0)

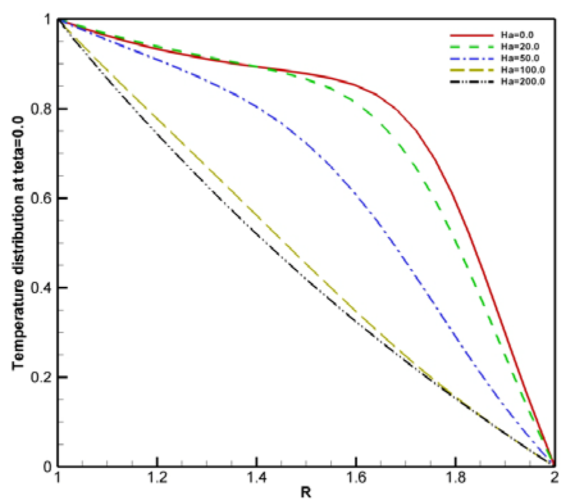


Fig.9. Effect of Ha number on the temperature distribution at ϕ=0.0 (Re=100, Pr=1 , Ra=10000.0, Eck=0.0)

Up to this point, the case of σ_o=2 has been presented. The streamline and isotherms for the other geometric configurations of annuli are shown in the figure below. As it's illustrated, the variation of geometric parameter of the annulus σ_o alters the fluid flow and the temperature patterns within the annulus. As the ratio of the inner cylinder diameter to the gap width increases (σ_o=5.0), the fluid has a less space to move within the annulus which results a significant decrease in the heat transfer rate. But for σ_o=0.5, the convective fluid has more space to move around and the fluid motion and heat transfer characteristics tend to be more convective like.

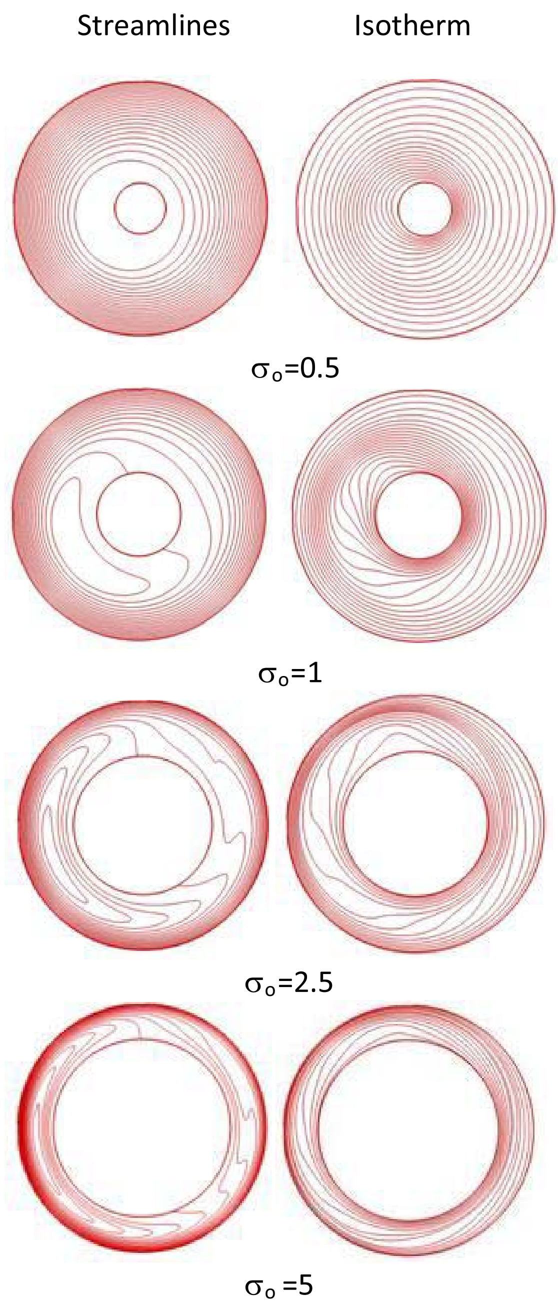


Fig.10. Effect of the inner cylinder diameter to the gap

width on the streamlines and isotherms
(Re=100, Pr=2, Eck=0.01, Ra=10000, Ha=20)

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