AUSTRIAN JOURNAL OF STATISTICS
Volume 38 (2009), Number 2, 121–130

Bootstrap Statistical Inference for the Variance Based on Fuzzy Data

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Abstract: The bootstrap is a simple and straightforward method for calculating approximated biases, standard deviations, confidence intervals, testing statistical hypotheses, and so forth, in almost any nonparametric estimation problem. In this paper we describe a bootstrap method for variance that is designed directly for hypothesis testing in case of fuzzy data based on Yao-Wu signed distance.

Zusammenfassung: Der Bootstrap ist eine einfache und geradlinige Methode um in fast jedem nichtparametrischen Schätzproblem geschätzte Biases, Standardabweichungen, Konfidenzintervalle zu berechnen, wie auch statistische Hypothesen zu testen und so weiter. In diesem Aufsatz beschreiben wir eine Bootstrapmethode für die Varianz, welche unmittelbar für Hypothesentests im Falle von unscharfen Daten basierend auf Yao-Wu vorzeichenbehafteter Distanzen ausgelegt ist.

Keywords: Fuzzy Canonical Number, Yao-Wu Signed Distance, Confidence Interval, Testing Hypotheses, Degree of Acceptance, Degree of Rejection.

1 Introduction

Statistical analysis in traditional form is based on crispness of data, random variables, point estimations, hypotheses, and so on. There are many different situations in which such concepts are imprecise. On the other hand, the theory of fuzzy sets is a well known tool for the formulation and the analysis of imprecise and subjective concepts. Therefore, confidence intervals and testing hypotheses with fuzzy data can be important. Methods for statistical inference (confidence intervals and hypothesis tests) in fuzzy environments are developed in different approaches.

Filzmoser and Viertl (2004) present a test based on fuzzy values by introducing the fuzzy p-value. Torabi, Behboodian, and Taheri (2006) try to develop a new approach for testing fuzzy hypotheses when the available data are fuzzy, too. They state and prove a generalized Neyman-Pearson Lemma for such problems. Some methods of statistical inference with fuzzy data are reviewed by Viertl (2006). Buckley (2005, 2006) studies the problems of statistical inference in the fuzzy environment. Thompson and Geyer (2007) proposed the fuzzy p-value in latent variable problems. Taheri and Arefi (2008) exhibit an approach to test fuzzy hypotheses based on fuzzy test statistics.

The bootstrap using fuzzy data is developed in different approaches. Montenegro, Colubi, Casals, and Gil (2004) present asymptotic one-sample procedures. The asymptotic development of Körner (2000) concerns general fuzzy random variables (taking values in the space of compact convex fuzzy sets of a finite-dimensional Euclidean space). In

Gonzalez-Rodriguez, Montenegro, Colubi, and Gil (2006) it is shown that the one-sample method of testing the mean of a fuzzy random variable can be extended to general ones (more precisely, to those whose range is not necessarily finite and whose values are fuzzy subsets of a finite-dimensional Euclidean space).

In this paper we construct a new method for bootstrap testing hypotheses in a fuzzy environment which is completely different from those mentioned before. For this purpose we organize the matter in the following way: In Section 2 we describe some basic concepts of canonical fuzzy numbers and the Yao and Wu (2000) signed distance. In Section 3 we come up with crisp and fuzzy bootstrap confidence intervals for the variance. In Section 4 we summarize the testing of crisp and fuzzy hypotheses.

2 Preliminaries

In this section we study canonical fuzzy numbers and the Yao-Wu singed distance.

2.1 Canonical Fuzzy Numbers

Let X be the universal space, then a fuzzy subset $\widetilde{x}$ of X is defined by its membership function $\mu_{\widetilde{x}} : X \to [0,1]$. We denote by $\widetilde{x}_\alpha = \{x : \mu_{\widetilde{x}}(x) \geq \alpha\}$ the α-cut set of $\widetilde{x}$ and $\widetilde{x}_0$ is the closure of the set $\{x : \mu_{\widetilde{x}}(x) > 0\}$, and

(1) $\widetilde{x}$ is called a normal fuzzy set, if there exists a $x \in X$ such that $\mu_{\widetilde{x}}(x) = 1$,

(2) $\widetilde{x}$ is called a convex fuzzy set, if $\mu_{\widetilde{x}}(\lambda x + (1-\lambda)y) \geq \min(\mu_{\widetilde{x}}(x), \mu_{\widetilde{x}}(y))$ for all $\lambda \in [0,1]$,

(3) the fuzzy set $\widetilde{x}$ is called a fuzzy number, if $\widetilde{x}$ is a normal convex fuzzy set and its α-cut sets are bounded $\forall \alpha \neq 0$,

(4) $\widetilde{x}$ is called a closed fuzzy number, if $\widetilde{x}$ is a fuzzy number and its membership function $\mu_{\widetilde{x}}$ is upper semicontinuous,

(5) $\widetilde{x}$ is called a bounded fuzzy number, if $\widetilde{x}$ is a fuzzy number and its membership function $\mu_{\widetilde{x}}$ has compact support.

If $\widetilde{x}$ is a closed and bounded fuzzy number with $x^L_\alpha = \inf\{x : x \in \widetilde{x}_\alpha\}$ and $x^U_\alpha = \sup\{x : x \in \widetilde{x}_\alpha\}$ and its membership function is strictly increasing on the interval $[x^L_\alpha, x^L_1]$ and strictly decreasing on the interval $[x^U_1, x^U_\alpha]$ for any $\alpha \in [0,1]$, then $\widetilde{x}$ is called canonical fuzzy number.

Let "$\odot$" be a binary operation $\oplus$ or $\ominus$ between two canonical fuzzy numbers $\widetilde{a}$ and $\widetilde{b}$. The membership function of $\widetilde{a} \odot \widetilde{b}$ is defined by

$$\mu_{\widetilde{a}\odot\widetilde{b}}(z) = \sup_{x \circ y = z} \min\{\mu_{\widetilde{a}}(x), \mu_{\widetilde{b}}(y)\}\,, \qquad \forall z \in \mathbb{R}\,,$$

for $\odot \in \{\oplus, \ominus\}$ and $\circ \in \{+, -\}$.

In the following let $\odot_{int}$ denote a binary operation $\oplus_{int}$ or $\ominus_{int}$ between two closed intervals $\widetilde{a}_\alpha = [a^L_\alpha, a^U_\alpha]$ and $\widetilde{b}_\alpha = [b^L_\alpha, b^U_\alpha]$. Then $\widetilde{a}_\alpha \odot_{int} \widetilde{b}_\alpha$ is defined as

$$\widetilde{a}_\alpha \odot_{int} \widetilde{b}_\alpha = \{z \in \mathbb{R} : z = x \circ y,\ x \in \widetilde{a}_\alpha,\ y \in \widetilde{b}_\alpha\}\,.$$

If $\widetilde{a}$ and $\widetilde{b}$ are two closed fuzzy numbers, then $\widetilde{a} \oplus \widetilde{b}$ and $\widetilde{a} \ominus \widetilde{b}$ are also closed fuzzy numbers. Furthermore, we have

$$\left(\widetilde{a} \oplus \widetilde{b}\right)_\alpha = \widetilde{a}_\alpha \oplus_{int} \widetilde{b}_\alpha = \left[a_\alpha^L + b_\alpha^L, a_\alpha^U + b_\alpha^U\right] ,$$
$$\left(\widetilde{a} \ominus \widetilde{b}\right)_\alpha = \widetilde{a}_\alpha \ominus_{int} \widetilde{b}_\alpha = \left[a_\alpha^L - b_\alpha^U, a_\alpha^U - b_\alpha^L\right] .$$

2.2 Yao-Wu Signed Distance

Now we define a signed distance between fuzzy numbers which is used later. Several ranking methods have been proposed so far by Cheng (1998), Modarres and Sadi-Nezhad (2001), and Nojavan and Ghazanfari (2006). In this paper we use another ranking system for canonical fuzzy numbers, which is very realistic and is defined by Yao and Wu (2000) as the following:

Definition 1: For each $a, b \in \mathbb{R}$ define the signed distance d^* of a and b by $d^*(a, b) = a - b$. Thus, we have the following way to define the rank of any two numbers on $\mathbb{R}$. For each $a, b \in \mathbb{R}$

$$\begin{array}{ccccc} d^*(a,b) > 0 & \Leftrightarrow & d^*(a,0) > d^*(b,0) & \Leftrightarrow & a > b\,, \\ d^*(a,b) < 0 & \Leftrightarrow & d^*(a,0) < d^*(b,0) & \Leftrightarrow & a < b\,, \\ d^*(a,b) = 0 & \Leftrightarrow & d^*(a,0) = d^*(b,0) & \Leftrightarrow & a = b\,. \end{array}$$

Definition 2: For each $\widetilde{a}, \widetilde{b}$ (arbitrary canonical fuzzy numbers), define the signed distance of $\widetilde{a}$ and $\widetilde{b}$ as

$$d(\widetilde{a}, \widetilde{b}) = \int_0^1 (M_\alpha(\widetilde{a}) - M_\alpha(\widetilde{b}))\, d\alpha = \int_0^1 d^*(M_\alpha(\widetilde{a}), M_\alpha(\widetilde{b}))\, d\alpha\,,$$

where $M_\alpha(\widetilde{a})$ and $M_\alpha(\widetilde{b})$ equal $(a_{\underset{\sim}{\alpha}}^L + a_\alpha^U)/2$ and $(b_\alpha^L + b_\alpha^U)/2$, respectively. Furthermore, $d(\widetilde{a}, \widetilde{b})$ means the distance of $\widetilde{a}$ to $\widetilde{b}$.

Definition 3: (Yao and Wu, 2000) For each $\widetilde{a}, \widetilde{b}$ (arbitrary canonical fuzzy numbers) define the rankings $\prec$, $\succ$, and $\approx$ of $\widetilde{a}$ and $\widetilde{b}$ by

$$\begin{array}{ccccc} d(\widetilde{a},\widetilde{b}) > 0 & \Leftrightarrow & (\widetilde{a},0) > d(\widetilde{b},0) & \Leftrightarrow & \widetilde{a} \succ \widetilde{b}\,, \\ d(\widetilde{a},\widetilde{b}) < 0 & \Leftrightarrow & d(\widetilde{a},0) < d(\widetilde{b},0) & \Leftrightarrow & \widetilde{a} \prec \widetilde{b}\,, \\ d(\widetilde{a},\widetilde{b}) = 0 & \Leftrightarrow & d(\widetilde{a},0) = d(\widetilde{b},0) & \Leftrightarrow & \widetilde{a} \approx \widetilde{b}\,. \end{array}$$

3 Bootstrap Confidence Interval for Variances

In this section we introduce a way to get bootstrap crisp and fuzzy confidence intervals based on fuzzy data. Through the use of the bootstrap based on fuzzy observations we obtain accurate intervals without having to make use of the normal theory. This procedure estimates the χ^2-distribution directly from the fuzzy data. Here is the bootstrap method in more detail.

3.1 Crisp Confidence Interval

Suppose that we have a canonical fuzzy random sample $\widetilde{\mathbf{x}} = (\widetilde{x}_1, \ldots, \widetilde{x}_n)$. We generate B bootstrap fuzzy random samples $\widetilde{\mathbf{x}}^{*^1}, \ldots, \widetilde{\mathbf{x}}^{*^B}$ (i.e., each $\widetilde{\mathbf{x}}^{*^b}$ is a fuzzy sample of size n randomly drawn with replacement from $\widetilde{\mathbf{x}}$) and for each we compute

$$\chi^{2*^b} = \frac{(n-1)s^{2*^b}_{(\widetilde{\mathbf{x}})}}{s^2_{(\widetilde{\mathbf{x}})}}, \qquad b = 1, \ldots, B,$$

where

$$s^{2*^b}_{(\widetilde{\mathbf{x}})} = \frac{1}{n-1}\sum_{i=1}^{n} d^2(\widetilde{x}_i^{*^b}, \widetilde{\overline{\mathbf{x}}}^*), \quad \widetilde{\overline{\mathbf{x}}}^* = \frac{1}{n}\sum_{i=1}^{n} \widetilde{x}_i^{*^b}, \quad s^2_{(\widetilde{\mathbf{x}})} = \frac{1}{n-1}\sum_{i=1}^{n} d^2(\widetilde{x}_i, \widetilde{\overline{\mathbf{x}}}),$$

and d is the Yao-Wu signed distance. The γth percentile of χ^{2*^b} is estimated by the value $\widehat{t}^\gamma$ such that

$$\frac{\#\{\chi^{2*^b} \le \widehat{t}^\gamma\}}{B} = \gamma.$$

Finally, the crisp bootstrap confidence interval using fuzzy data is

$$\Pi^* = \left[\frac{(n-1)s^2_{(\widetilde{\mathbf{x}})}}{\widehat{t}^{1-\gamma}}, \frac{(n-1)s^2_{(\widetilde{\mathbf{x}})}}{\widehat{t}^{\gamma}}\right].$$

If $B\gamma$ is not an integer, the following procedure can be used. Assuming $\gamma \le 1/2$, let $k = [(B+1)\gamma]$ be the largest integer less or equal $(B+1)\gamma$. Then we define the empirical γ and $1-\gamma$ quantiles by the kth and $(B+1-k)$th largest values of Z^{*^b}, respectively.

Example 1: Suppose that we have taken a fuzzy random sample of size $n = 12$ from a population and that we have observed the triangular fuzzy data of Table 1.

Table 1: Fuzzy random sample of size $n = 12$ from a population

	Observation		Observation		Observation
1	(33, 35, 36)	5	(60, 63, 66)	9	(100, 103, 105)
2	(80, 82, 84)	6	(70, 70, 72)	10	(54, 56, 58)
3	(85, 87, 87)	7	(70, 73, 76)	11	(40, 40, 42)
4	(90, 90, 90)	8	(65, 70, 73)	12	(94, 96, 99)

If $B = 10000$, the estimates of the 5% and 95% percentiles are the 500th and 9500th largest of all χ^{2*^b} values. The last line of Table 2 shows the percentiles of χ^{2*^b} for the variance computed using 10000 bootstrap samples.

The bootstrap confidence interval ($\gamma = 0.05$ or 90%) using fuzzy data is

$$\Pi^* = \left(\frac{11 \cdot 444.922}{15.27}, \frac{11 \cdot 444.922}{4.523}\right) = (320.5, 1082.1).$$

Figure 1 shows the distribution of χ^{2*^b} computed using 10000 bootstrap samples.

Table 2: Percentiles of the χ^2_7 and χ^2_{11} and the bootstrap distribution of χ^{2*^b}

Percentile	0.005	0.01	0.025	0.05	0.95	0.975	0.99	0.995
χ^2_7	0.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278
χ^2_{11}	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757
Bootstrap	2.699	3.077	3.850	4.523	15.270	16.889	18.290	21.349

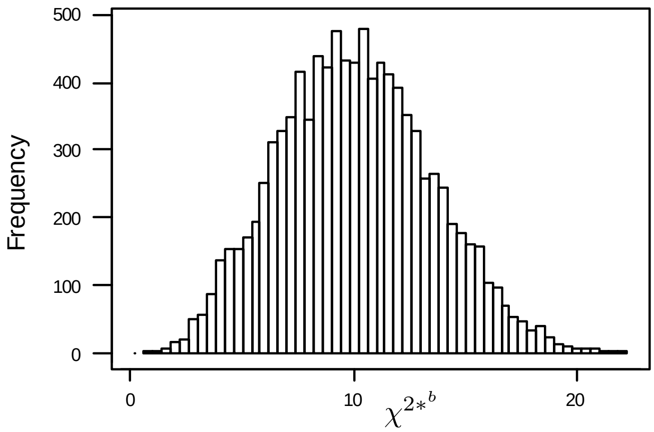

Figure 1: Bootstrap distribution based on $B = 10000$ generated χ^{2*^b} values

3.2 Fuzzy Confidence Interval

We generate B bootstrap fuzzy random samples $\widetilde{\mathbf{x}}^{*^1}, \ldots, \widetilde{\mathbf{x}}^{*^B}$. Then the α-cuts of the bootstrap confidence interval using fuzzy data are

$$\Pi^{**}_\alpha = \left\{ \left(\frac{\sum_{i=1}^n (x_i - \overline{\mathbf{x}})^2}{\widehat{t}^{1-\gamma}}, \frac{\sum_{i=1}^n (x_i - \overline{\mathbf{x}})^2}{\widehat{t}^{\gamma}} \right) : \ x_i \in \widetilde{x}_{i\alpha}, \ i = 1, \ldots, n \right\},$$

whenever its membership function is given by

$$\mu_{\Pi^{**}}(y) = \sup_{0 \le \alpha \le 1} \alpha \ I_{\Pi^{**}_\alpha}(y) \, .$$

Example 2: Consider the sample in Table 1. Now the α-cuts of the bootstrap confidence interval ($\gamma = 0.05$ or 90%) using fuzzy data are

$$\Pi^{**}_\alpha = \left\{ \left(\frac{\sum_{i=1}^{12} (x_i - \overline{\mathbf{x}})^2}{15.27}, \frac{\sum_{i=1}^{12} (x_i - \overline{\mathbf{x}})^2}{4.523} \right) : \ x_i \in \widetilde{x}_{i\alpha}, \ i = 1, \ldots, n \right\}.$$

For some α values we get the α-cuts as given in Table 3.

4 Bootstrap Hypotheses Tests of the Variance

We now introduce a way to get bootstrap tests for crisp and fuzzy hypotheses based on fuzzy data.

Table 3: α-cuts leading to respective confidence intervals (CIs)

α	0	0.1	0.2	0.3	0.4	0.5
CI	(317.7, 1115.8)	(318.1, 1112.7)	(318.4, 1109.6)	(318.8, 1106.6)	(319.2, 1103.2)	(319.7, 1101.0)
α	0.6	0.7	0.8	0.9	1	
CI	(320.2, 1098.4)	(320.7, 1095.8)	(321.3, 1093.4)	(321.9, 1091.1)	(322.5, 1088.9)	

4.1 Crisp Method and Crisp Hypotheses

Based on fuzzy observations $\widetilde{\mathbf{x}} = (\widetilde{x}_1, \ldots, \widetilde{x}_n)$ we consider an approach to test the following hypotheses:

- H_0: $\sigma = \sigma_0$ $\qquad H_1$: $\sigma \neq \sigma_0$,
- H_0: $\sigma \geq \sigma_0$ $\qquad H_1$: $\sigma < \sigma_0$,
- H_0: $\sigma \leq \sigma_0$ $\qquad H_1$: $\sigma > \sigma_0$.

Decision rule: We know that Π^* is a crisp confidence interval, thus

- if $\sigma_0^2 \in \Pi^*$, we accept H_0,
- if $\sigma_0^2 \notin \Pi^*$, we reject H_0.

Example 3: Consider the sample in Table 1. Suppose we are interested in a bootstrap test for the hypotheses

$$H_0\colon \sigma = 27 \qquad H_1\colon \sigma \neq 27\,.$$

Since we have $729 \in [320.5, 1082.1]$, we accept H_0.

4.2 Fuzzy Method and Crisp Hypotheses

According to the hypotheses in Subsection 4.1, we consider the problem in the following way:
Decision rule:

- if $\mu_{\Pi^{**}}(\sigma_0^2) < 1/2$, we reject H_0 with degree of rejection (DoR) $1 - \mu_{\Pi^{**}}(\sigma_0^2)$,
- if $\mu_{\Pi^{**}}(\sigma_0^2) > 1/2$, we accept H_0 with degree of acceptance (DoA) $\mu_{\Pi^{**}}(\sigma_0^2)$,
- if $\mu_{\Pi^{**}}(\sigma_0^2) = 1/2$, we accept H_0 or H_1.

Example 4: Consider the sample in Table 1. Suppose we are interested in a bootstrap test for the hypotheses

$$H_0\colon \sigma = 17.9 \qquad H_1\colon \sigma \neq 17.9\,.$$

We use the ability of the package Maple 7 and verify that $\mu_{\Pi^{**}}(17.9) = 0.684$ and that $1 - \mu_{\Pi^*}(17.9) = 0.316$. Thus, we accept H_0 with DoA 0.684.

4.3 Fuzzy Method and Fuzzy Hypotheses

We define some models as fuzzy sets of real numbers for modelling the extended versions of the simple, the one-, and the two-sided ordinary (crisp) hypotheses to the fuzzy ones.

Testing statistical hypotheses is a main topic in statistical inference. Typically, a statistical hypothesis is an assertion about the probability distribution of random variables. Traditionally, all statisticians assume that the hypothesis (for which we want to provide a test) are well-defined. Sometimes, this limitation force the statistician to make decision procedures in an unrealistic manner. This is because in realistic problems, we may come across with non-precise (fuzzy) hypotheses. For example, suppose that θ is the proportion of a population with a disease. We take a random sample and study this sample in order to have some idea about θ. In crisp hypotheses testing one uses hypotheses of the form H_0: $\theta = 0.2$ versus H_1: $\theta \neq 0.2$ or H_0: $\theta \leq 0.2$ versus H_0: $\theta > 0.2$, and so on. However, we sometimes like to test more realistic hypotheses. In this example, more realistic expressions about θ would be considered as small, very small, large, approximately 0.2, and so on. Therefore, a more realistic formulation of the hypotheses might be H_0: θ is small versus H_1: θ is not small. We call such expressions fuzzy hypotheses.

Definition 4: Let θ_0 be a real known number. A hypothesis of the form

- "H: θ is approximately θ_0" is called a fuzzy simple hypothesis.
- "H: θ is not approximately θ_0" is called a fuzzy two-sided hypothesis.
- "H: θ is essentially smaller than θ_0" is called a fuzzy left one-sided hypothesis.
- "H: θ is essentially larger than θ_0" is called a fuzzy right one-sided hypothesis.

We denote the above definitions by

H_0: θ is approx. θ_0 vs. H_1: θ is not approx. θ_0, or H_0: θ is $\widetilde{H}_0$ vs. H_1: θ is $\widetilde{H}_1$,
H_0: θ is approx. θ_0 vs. H_1: θ is certainly larger than θ_0, or H_0: θ is $\widetilde{H}_{0L}$ vs. H_1: θ is $\widetilde{H}_1$,
H_0: θ is approx. θ_0 vs. H_1: θ is certainly smaller than θ_0, or H_0: θ is $\widetilde{H}_{0R}$ vs. H_1: θ is $\widetilde{H}_1$.

These fuzzy hypotheses are shown in Figures 2 to 4.

Consider the problem of testing the fuzzy hypotheses $\widetilde{H}_0$ versus $\widetilde{H}_1$ based on a fuzzy random sample. $\widetilde{H}_{0L}$ versus $\widetilde{H}_1$ and $\widetilde{H}_{0R}$ versus $\widetilde{H}_1$ are similar to $\widetilde{H}_0$ versus $\widetilde{H}_1$.

Assumptions: Let

- C_T be the total area under $\widetilde{H}_0$,
- C_A be the area of the intersection between $\widetilde{H}_0$ and $\mu_{\Pi^{**}}$,
- C_R be the area of the intersection between $\widetilde{H}_0$ and $1 - \mu_{\Pi^{**}}$.

We know that $\widetilde{H}_0$ and Π^{**} are canonical fuzzy numbers, thus the areas C_T, C_A and C_R are finite.

Decision rule:

- if $C_T = C_A$ and $C_R = \emptyset$, then we accept H_0.
- if $C_T = C_R$ and $C_A = \emptyset$, then we reject H_0.
- if $C_A/C_R > 1$, then we accept H_0 with DoA C_A/C_T.
- if $C_A/C_R < 1$, then we reject H_0 with DoR C_R/C_T.
- if $C_A/C_R = 1$, then we accept H_0 and H_1.

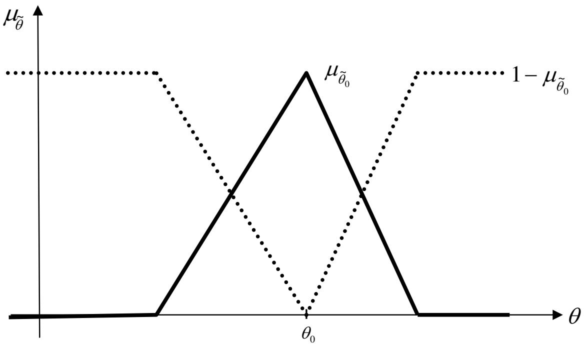

Figure 2: Fuzzy hypothesis $\widetilde{H}_0$ versus $\widetilde{H}_1$

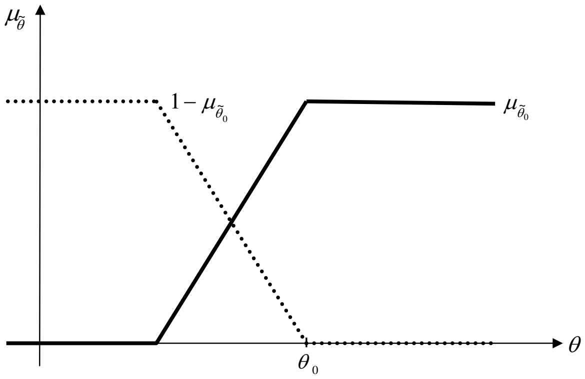

Figure 3: Fuzzy hypothesis $\widetilde{H}_{0L}$ versus $\widetilde{H}_1$

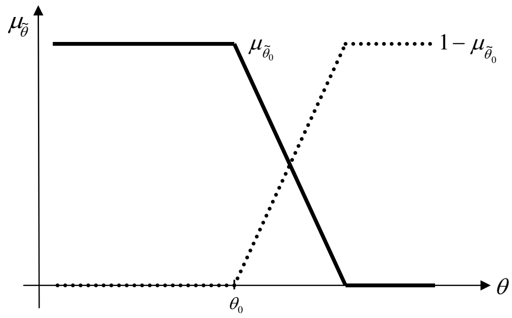

Figure 4: Fuzzy hypothesis $\widetilde{H}_{0R}$ versus $\widetilde{H}_1$

If $C_T = C_A$ and $C_R = \emptyset$, then the fuzzy number $\widetilde{\theta}_0$ lies in the fuzzy confidence interval Π^{**} and the null hypothesis $\widetilde{H}_0$ is certainly accepted with DoA $C_A/C_T = 1$. On the other hand, if C_A decreases then the value C_R/C_T increases and we certainly reject $\widetilde{H}_0$ with DoR $C_R/C_T = 1$ when $C_T = C_R$ and $C_A = \emptyset$. In other words, the value $\widetilde{\theta}_0$ lies in the fuzzy confidence interval $1 - \Pi^{**}$.

Taking greater membership functions 0.7 or 0.8 for $\widetilde{\theta}_0$ and Π^{**} we could reach more accurate values of C_T, C_A, and C_R. In summary, the above procedure is an applicable tool in fuzzy statistical inferential schemes. In the end of paper, we exhibit a decision making method which for $\alpha = 1$ is the same as under classical procedures.

Example 5: Consider the sample in Table 1. Now suppose that we want to test the fuzzy hypotheses

$$H_0\text{: } \sigma \text{ is } (25, 30, 55) \qquad H_1\text{: } \sigma \text{ is not } (25, 30, 35)\,.$$

Here, H_0 suggests that σ is approximatively 30, and H_1 suggests that σ is away from 30. Hence, based on the ability of Maple 7 we have $C_A/C_R = 1.97 > 1$. Thus, we accept H_0 with DoA $C_A/C_T = 0.865$. Figure 5 shows the distribution of the membership function $\mu_{\Pi^{**}}$ and fuzzy hypotheses $\widetilde{H}_0$ versus $\widetilde{H}_1$. Figure 6 shows essentially the same but the plot is based on a larger membership function of 0.42. Hence, we have $C_A/C_R = 13 > 1$. Thus, we accept H_0 with DoA $C_A/C_T = 0.914$.

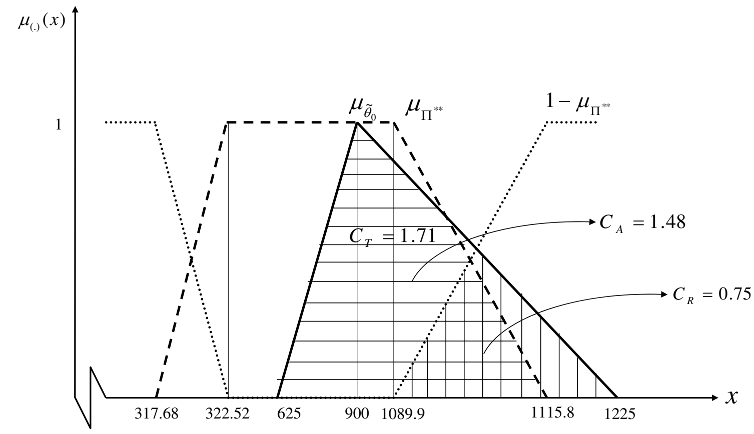

Figure 5: Membership function $\mu_{\Pi^{**}}$ and fuzzy hypotheses H_0 versus H_1

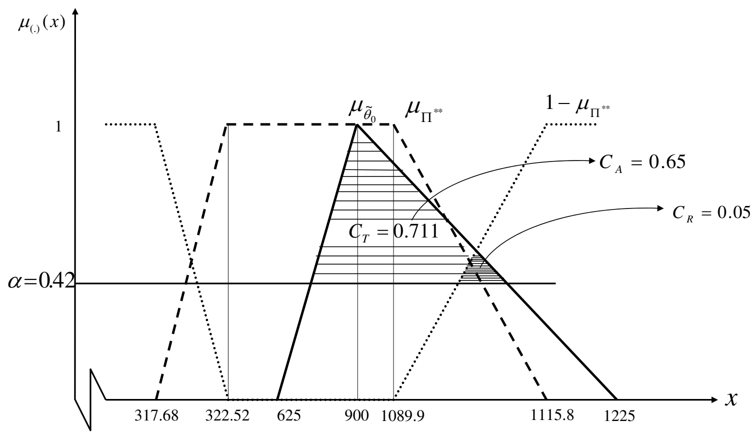

Figure 6: Membership function $\mu_{\Pi^{**}}$ and fuzzy hypotheses H_0 versus H_1 with $\alpha = 0.42$

5 Conclusions

The new approach for bootstrap statistical inference for the variance based on fuzzy data has the following issues:

1. It is established upon the notion of crisp and fuzzy confidence intervals (note that, in classical testing hypotheses, there is a relationship between interval estimation and testing hypothesis).
2. By introducing the concepts of DoA and DoR, it enables us to test fuzzy hypotheses in a rather natural way.

This procedure is based on the relationship between interval estimation and hypothesis tests in fuzzy environments. Extension of the proposed method to test the variance, correlation, and parameters in linear regression models is a potential area for future work.

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