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DISCRETE AND CONTINUOUS RANDOM VARIABLES WITH FUZZY PARAMETER

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Abstract

It is interesting to construct a method in order to verify a crisp mass (density) probability function and its mean and variance with fuzzy parameters. We look at some applications of these situations. In this paper, we arrive at this goal by some discrete and continuous random variables with fuzzy parameters.

1. Introduction

Statistical analysis, in traditional form, is based on crispness of data, random variable, point estimation, hypotheses, parameter and so on. As there are many different situations in which the above mentioned concepts are imprecise, the discrete and continuous random variables with fuzzy parameters approaches

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frequently are used in statistical inference. On the other hand, the theory of fuzzy sets is a well known tool for formulation and analysis of imprecise and subjective concepts. Therefore the random variables with fuzzy parameters can be important.

Arnold [1, 2] presented an approach to test fuzzily formulated hypotheses, in which he considered fuzzy constraints on the type I and II errors. Buckley [3, 4] exhibit an approach to fuzzy probability and fuzzy statistics, respectively.

In this paper, we construct a new method for discrete and continuous random variables with fuzzy parameters which is completely different from those mentioned above. For this purpose, we organize the matter in the following way: in Section 2, we describe some basic concepts of canonical fuzzy numbers, fuzzy hypotheses and mass (density) probability function with fuzzy parameter. In Section 3, we come up with discrete random variables and their applications. Section 4 provides continuous random variables and their applications. A brief conclusion is provided in Section 5.

2. Preliminaries

In this section, we describe canonical numbers, fuzzy parameter and mass (density) probability function with fuzzy parameter.

Let $(\Omega, \mathcal{F}, P)$ be a probability space, a random variable (r.v.) X is a measurable function from $(\Omega, \mathcal{F}, P)$ to $(\mathcal{X}, \mathcal{B}, P_X)$, where P_X is the probability measure induced by X and is called the *distribution* of the r.v. X, i.e.,

$$P_X(A) = P(X \in A) = \int_{X \in A} dP \qquad \forall A \in \mathcal{B}.$$

If P_X is dominated by a σ-finite measure v, i.e., $P_X << v$, then by the Radon-Nikodym theorem (Billingsley 1995), we have

$$P_X(A) = \int_{X \in A} f(x|\theta) dv(x) \qquad \forall A \in \mathcal{B},$$

where $f(x|\theta)$ is the Radon-Nikodym derivative of P_X with respect to v and is called the *probability density function* of X with respect to v. In a statistical context, the measure v is usually a "counting measure" or a "Lebesgue measure", hence $P_X(A)$ is $\sum_{x \in A} P(X = x|\theta)$ or $\int_A f(x|\theta) dx$, respectively.

2.1. Canonical fuzzy numbers

Let $\Theta = \{\theta | f(x|\theta) > 0\}$ be the "support" or "sample space" of Θ. Then a fuzzy subset $\tilde{\theta}$ of S_X is defined by its membership function $\mu_{\tilde{\theta}} : \Theta \to [0, 1]$. We denote by $\tilde{\theta}_\alpha = \{\theta : \mu_{\tilde{\theta}}(x) \geq \alpha\}$ the α-cut set of $\tilde{\theta}$ and $\tilde{\theta}_0$ is the closure of the set $\{\theta : \mu_{\tilde{\theta}}(x) > 0\}$, and

(1) $\tilde{\theta}$ is called a *normal fuzzy set* if there exists $\theta \in \Theta$ such that $\mu_{\tilde{\theta}}(x) = 1$;

(2) $\tilde{\theta}$ is called a *convex fuzzy set* if $\mu_{\tilde{\theta}}(\lambda x + (1 - \lambda) y) \geq \min(\mu_{\tilde{\theta}}(x), \mu_{\tilde{\theta}}(y))$ for all $\lambda \in [0, 1]$;

(3) $\tilde{\theta}$ is called a *fuzzy number* if $\tilde{\theta}$ is a normal convex fuzzy set and its α-cut sets, are bounded $\forall \alpha \neq 0$;

(4) $\tilde{\theta}$ is called a *closed fuzzy number* if $\tilde{\theta}$ is a fuzzy number and its membership function $\mu_{\tilde{\theta}}$ is upper semicontinuous;

(5) $\tilde{\theta}$ is called a *bounded fuzzy number* if $\tilde{\theta}$ is a fuzzy number and the support of its membership function $\mu_{\tilde{\theta}}$ is compact.

If $\tilde{\theta}$ is a closed and bounded fuzzy number with $\theta_\alpha^L = \inf\{\theta : \theta \in \theta_\alpha\}$ and $\theta_\alpha^U = \sup\{\theta : \theta \in \theta_\alpha\}$ and its membership function be strictly increasing on the interval $[\theta_\alpha^L, \theta_1^L]$ and strictly decreasing on the interval $[\theta_1^U, \theta_\alpha^U]$, then $\tilde{\theta}$ is called a *canonical fuzzy number*.

The fuzzy canonical numbers (such as triangular or trapezoidal fuzzy numbers) are very realistic in fuzzy set theory, so we use these numbers for our goal.

2.2. Fuzzy parameter

Probability and central tendencies are main branch of statistical inference. Typically, a statistical hypothesis (parameter) is an assertion about the probability distribution of one or more random variable(s). Traditionally, all statisticians assume the hypothesis for which we wish to provide a test is well-defined. This limitation, sometimes, forces the statistician to make decision procedure in an unrealistic manner. This is because in realistic problems, we may come across non-precise (fuzzy) hypothesis. For example, more realistic expressions about θ would be considered as: “small”, “very small”, “large”, “approximately 0.2”, and so on.

We define some models, as fuzzy sets of real numbers, for fuzzy parameters.

Definition 2.1. Let θ_0 be a real number and known. Then

(i) θ is approximately θ_0.

(ii) θ is essentially smaller than θ_0.

(iii) θ is essentially larger than θ_0.

We denote the above definitions by

The above areas are shown in Figures 1,2 and 3.

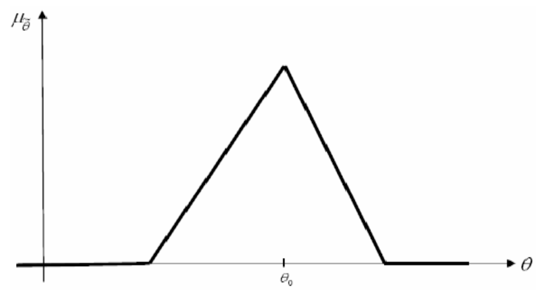

Figure 1. The fuzzy parameter (i).

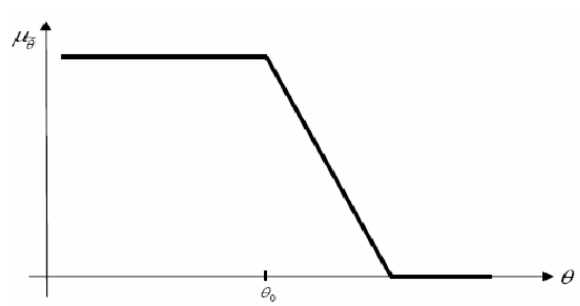

Figure 2. The fuzzy parameter (ii).

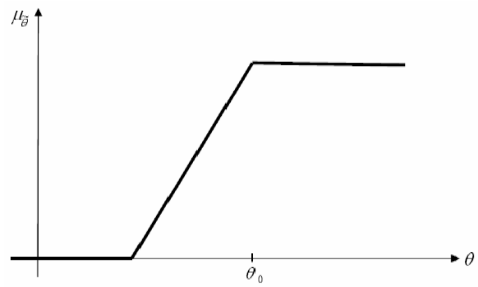

Figure 3. The fuzzy parameter (iii).

2.3. Mass (density) probability function

Let $S_X = \{x \in R : f(x \mid \tilde{\theta}) > 0\}$ be the "support" or "sample" space of X and let X be a r.v. and

$$f(x \mid \tilde{\theta}) = \frac{\int_0^1 \int_{\theta \in \tilde{\theta}_\alpha} H(\theta) f(x \mid \theta) d\theta d\alpha}{\int_0^1 \int_{\theta \in \tilde{\theta}_\alpha} H(\theta) d\theta d\alpha},$$

where $H(\theta) = \mu_{\tilde{\theta}}(\theta)$.

We call the density $f(x \mid \tilde{\theta})$ as the fuzzy probability mass (density) function of X. We note that $f(x \mid \tilde{\theta}) \geq 0$ and

$$\sum_{x_i \in S_X} f(x \mid \tilde{\theta}) = \sum_{x_i \in S_X} \frac{\int_0^1 \int_{\theta \in \tilde{\theta}_\alpha} H(\theta) f(x_i \mid \theta) d\theta d\alpha}{\int_0^1 \int_{\theta \in \tilde{\theta}_\alpha} H(\theta) d\theta d\alpha}$$

$$= \frac{\int_0^1 \int_{\theta \in \tilde{\theta}_\alpha} H(\theta) \sum_{x_i \in S_X} f(x_i \mid \theta) d\theta d\alpha}{\int_0^1 \int_{\theta \in \tilde{\theta}_\alpha} H(\theta) d\theta d\alpha}$$

$$= 1.$$

3. Discrete Random Variables and Applications

We start with the Binomial. The crisp Binomial probability mass function, usually written $B(n, \theta)$, where n is the number of independent experiments and θ is the probability of a "success" in each experiment. We assume that θ is not known exactly and is to canonical fuzzy parameter.

The crisp Poisson probability mass function has one parameter, usually written $P(\lambda)$, which we also assume is not known exactly. Hence we substitute canonical fuzzy number $\tilde{\lambda}$ for λ to obtain the Poisson probability mass function. We look at some applications of these discrete probability mass function in the end of section.

3.1. Binomial mass function

Let E be a non-empty proper subset of X. If θ is uncertain and we substitute the canonical fuzzy number $\tilde{\theta}$ for θ, then under our restricted algebra we obtain

$$P_{\tilde{\theta}}(E) = \sum_{x \in E} f(x \mid \tilde{\theta}),$$

where

$$f(x \mid \tilde{\theta}) = \frac{\int_0^1 \int_{\theta \in \tilde{\theta}_\alpha} H(\theta) \binom{n}{x} \theta^x (1-\theta)^{n-x} \, d\theta d\alpha}{\int_0^1 \int_{\theta \in \tilde{\theta}_\alpha} H(\theta) \, d\theta d\alpha}.$$

The mean and the variance of the Binomial distribution are calculated as following:

$$\begin{aligned}
\mu_B &= E_{\tilde{\theta}}(X) \\
&= \frac{\sum_{x=0}^{n} \int_0^1 \int_{\theta \in \tilde{\theta}_\alpha} x H(\theta) \binom{n}{x} \theta^x (1-\theta)^{n-x} \, d\theta d\alpha}{\int_0^1 \int_{\theta \in \tilde{\theta}_\alpha} H(\theta) \, d\theta d\alpha} \\
&= \frac{n \int_0^1 \int_{\theta \in \tilde{\theta}_\alpha} \theta H(\theta) \, d\theta d\alpha}{\int_0^1 \int_{\theta \in \tilde{\theta}_\alpha} H(\theta) \, d\theta d\alpha}
\end{aligned}$$

and

$$\sigma_B^2 = E_{\tilde{\theta}}(X^2) - \mu_B^2.$$

We use of the ability of package "Maple 6" to deal with complicated fuzzy parameter structure for the following examples.

Example 3.1. Let X be a r.v. from the $B(3, \tilde{\theta})$ population and $\tilde{\theta}$ is our triangular fuzzy parameter, then we have the following table:

Table 1. The probability, mean and variance in Example 3.1

$\tilde{\theta}$	$E = \{1\}$	$E = \{2\}$	$E = \{1, 2\}$	μ_B	σ_B^2
$\left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}\right)$	0.354	0.307	0.729	1.47	0.741
$\left(0, \frac{1}{2}, \frac{5}{6}\right)$	0.301	0.322	0.717	1.43	0.731
$\left(\frac{1}{6}, \frac{1}{2}, 0\right)$	0.322	0.322	0.717	1.25	0.725
$\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{6}\right)$	0.235	0.311	0.643	2.251	0.662
$\left(0, \frac{2}{3}, \frac{5}{6}\right)$	0.209	0.447	0.615	2.122	0.641
$\left(\frac{1}{2}, \frac{2}{3}, 0\right)$	0.211	0.447	0.62	2.111	0.632
$\left(0, \frac{1}{3}, \frac{1}{2}\right)$	0.437	0.264	0.6	1.126	0.582

3.2. Poisson mass function

Let X be a random variable having Poisson probability mass function. Now substitute canonical fuzzy parameter $\tilde{\lambda}$ for λ. For any subset $E \subseteq S_X$, we have

$$P_{\tilde{\lambda}} = \frac{\sum_{x \in E} \int_0^1 \int_{\theta \in \tilde{\lambda}_\alpha} H(\lambda) \frac{\exp\{-\lambda\}\lambda^x}{x!} d\lambda d\alpha}{\int_0^1 \int_{\theta \in \tilde{\lambda}_\alpha} H(\theta) d\lambda d\alpha},$$

where $H(\lambda) = \mu_{\tilde{\lambda}}(\lambda)$.

The mean and the variance of the Poisson distribution are calculated as following:

$$\mu_P = E_{\tilde{\lambda}}(X)$$

$$= \frac{\sum_{x=0}^{\infty} \int_0^1 \int_{\lambda \in \tilde{\lambda}_\alpha} x H(\lambda) \frac{\exp\{-\lambda\}\lambda^x}{x!} d\lambda d\alpha}{\int_0^1 \int_{\lambda \in \tilde{\lambda}_\alpha} H(\theta) d\lambda d\alpha}$$

$$= \frac{\int_0^1 \int_{\theta \in \tilde{\lambda}_\alpha} \lambda H(\lambda) d\lambda d\alpha}{\int_0^1 \int_{\theta \in \tilde{\lambda}_\alpha} H(\lambda) d\lambda d\alpha}$$

and

$$\sigma_P^2 = E_{\tilde{\lambda}}(X^2) - \mu_P^2.$$

Example 3.2. Let X be a r.v. from the $P(\tilde{\lambda})$ population and $\tilde{\lambda}$ is our triangular fuzzy parameter, then we have the following table:

Table 2. The Probability, mean and variance in Example 3.2

$\tilde{\lambda}$	$E = \{0, 1, 2\}$	μ_P	σ_P^2
$(0.5, 1, 1.5)$	0.9	1.05	1.05
$(1, 2, 3)$	0.618	2.11	2.11
$(3.5, 4, 5)$	0.228	4.08	4.08
$(5, 5.5, 7)$	0.403	5.57	5.57
$(7, 7, 8)$	0.012	7	7
$(7, 7.5, 8)$	0.015	7.54	7.54
$(8, 8, 9)$	0.012	8.1	8.1

3.3. Application

Let X be a r.v. having the $B(n, \theta)$. From crisp theory (Shao [3]), we know that if n is large and θ is small, then we can use the Poisson to approximate value of the

Binomial. For non-negative integers a and b, let $P([a, b])$ be the probability that $a \le X \le b$. Then using the Binomial, we have

$$P_\theta([a, b]) = \sum_{x=a}^{b} \binom{n}{x} \theta^x (1-\theta)^{n-x}.$$

Using the Poisson, with $\lambda = n\theta$, we calculate

$$P_\theta([a, b]) \approx \sum_{x=a}^{b} \frac{\exp\{-\lambda\}\lambda^x}{x!}.$$

Now, which to the fuzzy case. Let $\tilde{\theta}$ be small, which means that $\theta \in \tilde{\theta}_0$ are sufficient small. Then

$$P_{\tilde{\theta}}([a, b]) \approx \frac{\sum_{x=a}^{b} \int_0^1 \int_{\lambda \in n\tilde{\theta}_\alpha} H(\lambda) \frac{\exp\{-\lambda\}\lambda^x}{x!} d\lambda d\alpha}{\int_0^1 \int_{\theta \in \tilde{\theta}_\alpha} H(\theta) d\lambda d\alpha}.$$

Example 3.3. Suppose that we have $[a, b] = \{0, 1, 2\}$, triangular fuzzy number $\tilde{\theta} = \left(0, \frac{1}{3}, \frac{1}{2}\right)$ and $\tilde{\theta} = (0.01, 0.02, 0.03)$. We show the $P_{\tilde{\theta}}([a, b])$ in Table 3.

Table 3. The approximation of Binomial by Poisson

n	$\tilde{\theta}$	$P_{\tilde{\theta}}([a, b])$	$P_{\tilde{\lambda}}([a, b])$	$\tilde{\theta}$	$P_{\tilde{\theta}}([a, b])$	$P_{\tilde{\lambda}}([a, b])$
5	$\left(0, \frac{1}{3}, \frac{1}{2}\right)$	0.765	0.711	$\left(\frac{1}{100}, \frac{2}{100}, \frac{3}{100}\right)$	1	0.92
10	$\left(0, \frac{1}{3}, \frac{1}{2}\right)$	0.323	0.3	$\left(\frac{1}{100}, \frac{2}{100}, \frac{3}{100}\right)$	0.999	0.968
15	$\left(0, \frac{1}{3}, \frac{1}{2}\right)$	0.118	0.127	$\left(\frac{1}{100}, \frac{2}{100}, \frac{3}{100}\right)$	0.995	0.962
20	$\left(0, \frac{1}{3}, \frac{1}{2}\right)$	0.041	0.043	$\left(\frac{1}{100}, \frac{2}{100}, \frac{3}{100}\right)$	0.99	0.979

30	$\left(0, \frac{1}{3}, \frac{1}{2}\right)$	0.003	0.001	$\left(\frac{1}{100}, \frac{2}{100}, \frac{3}{100}\right)$	0.91	0.873
40	$\left(0, \frac{1}{3}, \frac{1}{2}\right)$	0	0	$\left(\frac{1}{100}, \frac{2}{100}, \frac{3}{100}\right)$	0.853	0.843
50	$\left(0, \frac{1}{3}, \frac{1}{2}\right)$	0	0	$\left(\frac{1}{100}, \frac{2}{100}, \frac{3}{100}\right)$	0.811	0.799
70	$\left(0, \frac{1}{3}, \frac{1}{2}\right)$	0	0	$\left(\frac{1}{100}, \frac{2}{100}, \frac{3}{100}\right)$	0.721	0.719
90	$\left(0, \frac{1}{3}, \frac{1}{2}\right)$	0	0	$\left(\frac{1}{100}, \frac{2}{100}, \frac{3}{100}\right)$	0.695	0.691

4. Continuous Random Variables and Applications

We consider the fuzzy Negative exponential in Subsection 4.1, followed by Normal in Subsection 4.2. In each case of a density function we first discuss how they are used to compute probabilities and then we find their mean and variance. We note that the parameter in the above density functions are a canonical fuzzy numbers.

We will denote the negative exponential probability density as $E(\lambda)$. The Normal is $N(\mu, \sigma^2)$.

4.1. Negative exponential density function

The Negative exponential $E(\lambda)$ has density $f(x|\lambda) = \lambda \exp\{-\lambda x\}$ for $x > 0$ and $f(x|\lambda) = 0$ otherwise, where $\lambda > 0$. The mean and variance of $E(\lambda)$ is $\frac{1}{\lambda}$ and $\frac{1}{\lambda}$, respectively. Now consider $E(\tilde{\lambda})$ for canonical fuzzy number $\tilde{\lambda}$. We compute $P_{\tilde{\lambda}}(E)$ for any subset E of $\mathcal{R}$ as following:

$$P_{\tilde{\lambda}}(E) = \frac{\int_{x \in E} \int_0^1 \int_{\lambda \in \tilde{\lambda}_\alpha} H(\lambda) \lambda \exp\{-\lambda x\} d\lambda d\alpha dx}{\int_0^1 \int_{\lambda \in \tilde{\lambda}_\alpha} H(\lambda) d\lambda d\alpha},$$

where $H(\lambda)$ is the membership function $\tilde{\lambda}$.

The mean and the variance of the Negative exponential distribution are calculated as following:

$$\mu_{Na} = E_{\tilde{\lambda}}(X)$$

$$= \frac{\int_0^{\infty}\int_0^1\int_{\lambda\in\tilde{\lambda}_\alpha} xH(\lambda)\lambda\exp\{-\lambda x\}d\lambda d\alpha}{\int_0^1\int_{\lambda\in\tilde{\lambda}_\alpha} H(\theta)d\lambda d\alpha}dx$$

$$= \frac{\int_0^1\int_{\lambda\in\tilde{\lambda}_\alpha} \frac{1}{\lambda}H(\lambda)d\lambda d\alpha}{\int_0^1\int_{\lambda\in\tilde{\lambda}_\alpha} H(\lambda)d\lambda d\alpha}$$

and

$$\sigma^2_{N_a} = E_{\tilde{\lambda}}(X^2) - \mu^2_{Na}.$$

4.2. Normal density function

The normal density $N(\mu, \sigma^2)$ has density function $f(x|\mu, \sigma^2)$, $x \in \mathcal{R}$, mean μ and variance σ^2. So consider the fuzzy Normal $N(\tilde{\mu}, \tilde{\sigma}^2)$ for fuzzy canonical numbers $\tilde{\mu}$ and $\tilde{\sigma}^2$. We wish to compute the fuzzy probability of obtaining a value in the subset E of $\mathcal{R}$. We have

$$P_{(\tilde{\mu}, \tilde{\sigma}^2)}(E)$$

$$= \frac{\int_{x\in E}\int_0^1\int_{\mu\in\tilde{\mu}_\alpha}\int_{\sigma^2\in\tilde{\sigma}^2_\alpha} H_1(\mu)H_2(\sigma^2)\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}d\sigma^2 d\mu d\alpha dx}{\int_0^1\int_{\mu\in\tilde{\mu}_\alpha}\int_{\sigma^2\in\tilde{\sigma}^2_\alpha} H_1(\mu)H_2(\sigma^2)d\sigma^2 d\mu d\alpha},$$

where $H_1(\mu)$ and $H_2(\sigma^2)$ are the membership functions $\tilde{\mu}$ and $\tilde{\sigma}^2$, respectively.

The mean and the variance of the Normal distribution are calculated as following:

$$\mu_{N0} = E_{(\tilde{\mu},\, \tilde{\sigma}^2)}(X)$$

$$= \frac{\int_{x \in E} \int_0^1 \int_{\mu \in \tilde{\mu}_\alpha} \int_{\sigma^2 \in \tilde{\sigma}^2_\alpha} x H_1(\mu) H_2(\sigma^2) \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\} d\sigma^2 d\mu d\alpha dx}{\int_0^1 \int_{\mu \in \tilde{\mu}_\alpha} \int_{\sigma^2 \in \tilde{\sigma}^2_\alpha} H_1(\mu) H_2(\sigma^2) d\sigma^2 d\mu d\alpha}$$

$$= \frac{\int_{x \in E} \int_0^1 \int_{\mu \in \tilde{\mu}_\alpha} \int_{\sigma^2 \in \tilde{\sigma}^2_\alpha} \mu H_1(\mu) H_2(\sigma^2) d\sigma^2 d\mu d\alpha dx}{\int_0^1 \int_{\mu \in \tilde{\mu}_\alpha} \int_{\sigma^2 \in \tilde{\sigma}^2_\alpha} H_1(\mu) H_2(\sigma^2) d\sigma^2 d\mu d\alpha}$$

and

$$\sigma^2_{N_0} = E_{(\tilde{\mu},\, \tilde{\sigma}^2)}(X^2) - \mu^2_{N_0}.$$

4.3. Applications

In this Subsection, we look at some applications of the Normal, the Binomial, the Poisson and the Negative exponential.

4.3.1. Normal approximation to binomial

Suppose we have m independent repetitions of experiment. If $P(r)$ is the probability of r successes in the m experiments, then

$$P(r) = \binom{m}{r} \theta^r (1-\theta)^{m-r},$$

for $r = 0, 1, \ldots, m,$ gives the Binomial distribution. We write $B(m, \theta)$ for the crisp Binomial and $B(m, \tilde{\theta})$ for the Binomial. In these experiments let us assume that θ is not known precisely and it needs to be estimated, or obtained from expert opinion. So θ is uncertain and we substitute $\tilde{\theta}$ for θ.

We use the ability of package "*Maple* 6" to deal with complicated fuzzy parameter structure for the following examples.

Example 4.1. Let $m = 100$ and $\tilde{\theta} = (0.5, 0.6, 0.7)$. For the Normal approximation to the Binomial to be reasonably accurate one usually assumes that

(Shao [6]) $m\theta > 5$ and $m(1 - \theta) > 5.$ For the fuzzy Normal approximation to the Binomial with fuzzy parameter to be reasonably good, we assume that $m\tilde{\theta} > 5$ and $m(1 - \tilde{\theta}) > 5,$ which is true in this example.

Now we need to compute the mean and the variance of this Binomial. We obtain

$$P([40,\ 60]) \approx \int_{z_1}^{z_2} f(x \mid 0,\ 1) = 0.58,$$

where $z_1 = \dfrac{(39 - \mu_B)}{\sigma_B}$ and $z_2 = \dfrac{(60 - \mu_B)}{\sigma_B}.$

4.3.2. Normal approximation to Poisson

The fuzzy Poisson was discussed in Subsection 3.2. We know, if λ is sufficiently large (Shao [6]), then we can approximate the crisp Poisson with the crisp Normal. Let $\lambda = 20$ and let $P([16,\ 21])$ be the probability that $16 \leq X \leq 21.$ Then

$$P([16,\ 21]) \approx \int_{z_1}^{z_2} f(x \mid 0,\ 1) = 0.415,$$

where

$$z_1 = \frac{(16 - \mu_P)}{\sigma_P} \quad \text{and} \quad z_2 = \frac{(21 - \mu_P)}{\sigma_P}.$$

4.3.3. Negative exponential

The crisp Negative exponential probability density function is related to the crisp Poisson probability mass function and the same is true in the fuzzy case. A machine has a standby unit available for immediate replacement upon failure. Assume that failures occur for these machines at a rate of λ per hour. Let X be a random variable which counts the number of failures during a period of T hours.

Assume that X has a Poisson probability mass function and the probability that $X = x,$ denoted by $P_T(x),$ is

$$P_T(x) = \frac{(\lambda T)^x \exp\{-\lambda T\}}{x!},$$

for $x = 0, 1, 2, \cdots$. Now let Y be the random variable whose value is the waiting time to the first failure. It is well-known that Y has the exponential probability density so that

$$P(Y > t) = \int_t^{\infty} \lambda \exp\{-\lambda x\} dx$$

which is the probability that the first failure occurs after t hours.

Now switch to the Poisson with $\tilde{\lambda} = (0.07, 0.1, 0.13)$ for λ and denote the fuzzy probability that the first failure occurs after 10 hours as $P_{\tilde{\lambda}}[10, \infty]$. Then we have

$$P_{\tilde{\lambda}}[10, \infty] = \frac{\int_{10}^{\infty}\int_0^1 \int_{\lambda \in \tilde{\lambda}_\alpha} H(\lambda)\lambda \exp\{-\lambda x\} d\lambda d\alpha dx}{\int_0^1 \int_{\lambda \in \tilde{\lambda}_\alpha} H(\lambda) d\lambda d\alpha} = \exp\{-0.12\}.$$

5. Conclusion

In this paper, we use a new probability mass (density) function based on fuzzy parameter. As for this paper is concerned it appears that the introduced probability mass (density) function is more simple and better than that of Buckley [3]. We can similarly apply this method for statistical inference about mean and variance for other distributions.

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