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TOROIDAL SOLUTIONS IN HOŘAVA GRAVITY

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Recently a new four-dimensional nonrelativistic renormalizable theory of gravity was proposed by Hořava. This gravity reduces to Einstein gravity at large distances. In this paper we present different toroidal solutions to the equations of motion using the new action for gravity. Our solutions describe the near horizon geometry with slow rotating parameter.

Keywords: Hořava gravity; toroidal solutions.

1. Introduction

A new four-dimensional nonrelativistic renormalizable theory of gravity was recently proposed by Hořava.[1] It is believed that this theory is a UV completion for the Einstein theory of gravitation. Recently lots of effort have been dedicated to understand this theory.[2–33] In Ref. 2 the solutions with spherical symmetry has been found. It also presents equations of motion for Hořava gravity. The topological black hole solutions has been found in Ref. 16. In this paper, in Sec. 2, we review briefly the static toroidal solution, which is a special solution found in Ref. 16. In Sec. 3 we try to find the rotational solutions. We use the equations of motion presented in Ref. 2 and show that there are different possible solutions to the equations of motion.

We start from the four-dimensional metric written in the ADM formalism[34]

$$ds_4^2 = -N^2\,dt^2 + g_{ij}(dx^i - N^i\,dt)(dx^j - N^j\,dt)\,. \tag{1.1}$$

The Einstein–Hilbert action in this formalism is given by

$$S_{\rm EH} = \frac{1}{16\pi G}\int d^4x\sqrt{g}N(K_{ij}K^{ij} - K^2 + R - 2\Lambda)\,, \tag{1.2}$$

where G is the four-dimensional Newton's constant and K_{ij} is the second fundamental form and is defined by

$$K_{ij} = \frac{1}{2N}(\partial_t g_{ij} - \nabla_i N_j - \nabla_j N_i)\,. \tag{1.3}$$

The action proposed by Hořava is a nonrelativistic renormalizable gravitational theory and is given by[1]

$$S = \int dt\, d^3x \sqrt{g} N \left\{ \frac{2}{\kappa^2} (K_{ij}K^{ij} - \lambda K^2) + \frac{\kappa^2 \mu^2 (\Lambda_W R - 3\Lambda_W^2)}{8(1-3\lambda)} \right.$$
$$\left. + \frac{\kappa^2\mu^2(1-4\lambda)}{32(1-3\lambda)} R^2 - \frac{\kappa^2\mu^2}{8} R_{ij}R^{ij} + \frac{\kappa^2\mu}{2w^2} \epsilon^{ijk} R_{i\ell} \nabla_j R^\ell_k - \frac{\kappa^2}{2w^4} C_{ij}C^{ij} \right\}, \quad (1.4)$$

where λ, κ, μ, w and Λ_W are constant parameters, and C_{ij} is the Cotton tensor, defined by

$$C^{ij} = \epsilon^{ik\ell} \nabla_k \left(R^j{}_\ell - \frac{1}{4} R \delta^j_\ell \right) = \epsilon^{ik\ell} \nabla_k R^j{}_\ell - \frac{1}{4} \epsilon^{ikj} \partial_k R \,. \quad (1.5)$$

Using the relation

$$\epsilon^{ijk} R_{i\ell} \nabla_j R^\ell{}_k = R_{i\ell} \left[C^{i\ell} - \frac{1}{4} \epsilon^{ij\ell} \partial_j R \right] = R_{i\ell} C^{i\ell} \,, \quad (1.6)$$

one can rewrite the action (1.4) as

$$S = \int dt\, d^3x (\mathcal{L}_0 + \mathcal{L}_1) \,,$$
$$\mathcal{L}_0 = \sqrt{g} N \left\{ \frac{2}{\kappa^2} (K_{ij}K^{ij} - \lambda K^2) + \frac{\kappa^2\mu^2(\Lambda_W R - 3\Lambda_W^2)}{8(1-3\lambda)} \right\}, \quad (1.7)$$
$$\mathcal{L}_1 = \sqrt{g} N \left\{ \frac{\kappa^2\mu^2(1-4\lambda)}{32(1-3\lambda)} R^2 - \frac{\kappa^2}{2w^4} \left(C_{ij} - \frac{\mu w^2}{2} R_{ij} \right) \left(C^{ij} - \frac{\mu w^2}{2} R^{ij} \right) \right\}.$$

By comparing $\mathcal{L}_0$ with the general theory of relativity in the ADM formalism, one can read the speed of light, the Newton's constant and the cosmological constant as

$$c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1-3\lambda}} \,, \quad G = \frac{\kappa^2}{32\pi c} \,, \quad \Lambda = \frac{3}{2} \Lambda_W \,. \quad (1.8)$$

Additionally, demanding that $\mathcal{L}_0$ gives the usual four-dimensional Einstein–Hilbert Lagrangian (general covariance), one finds that $\lambda = 1$.

2. Toroidal Static Solution

The topological black hole solution has been found in Ref. 16. We are interested to the special case of toroidal symmetric solutions in this paper. So in this section we review the special solution found in Ref. 16 with toroidal symmetry. We start from the ansatz

$$ds^2 = -N(r)^2 dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + d\phi^2) \quad (2.1)$$

and insert it into the total Lagrangian $\mathcal{L}_0 + \mathcal{L}_1$. Due to the special form of the ansatz, the Cotton tensor is zero.

The Lagrangian with general value for λ is given by

$$\mathcal{L}_0 + \mathcal{L}_1 = \frac{3\mu^2\kappa^2 N}{8(-1+3\lambda)r^2 f^{\frac{1}{2}}}\left(\frac{(1-\lambda)r^2}{6}f'^2 + \frac{2}{3}r(\Lambda_W r^2 + \lambda f)f' + \frac{1}{3}(1-2\lambda)f^2 + \frac{2}{3}\Lambda_W r^2 f + \Lambda_W^2 r^4\right), \tag{2.2}$$

where a prime denotes the derivative with respect to r. The solution to the equations of motion is[16]

$$f(r) = -Mr^n - \Lambda_W r^2\,, \quad N^2(r) = f(r)(Cr)^{2(1-2n)}\,, \quad n = \frac{2\lambda - \sqrt{-2+6\lambda}}{-1+\lambda}\,, \tag{2.3}$$

where M and C are the constants of integrations.

The above solution has two real roots for $M > 0$ and $\Lambda_W < 0$ at $r_- = 0$ and $r_+ = \left(-\frac{M}{\Lambda_W}\right)^{\frac{1}{2-n}}$. The scalar curvature is given by $\mathcal{R} = 2(3\Lambda_W + M(n+1)r^{n-2})$, and because when $\lambda \to +\infty$, $n \to 2$, therefore we always have a curvature singularity at $r = 0$. When $\Lambda_W > 0$, $r = 0$ is a naked singularity.

3. Rotating Solutions

In this section we try to find other solutions to the Hořava gravity by including the rotation. Because of the rotation, we do not have enough symmetry to apply the previous method (i.e. inserting the ansatz into the Lagrangian), instead we must solve the equations of motion directly. The equations of motion are very difficult to solve since they are up to six derivatives and the metric in the rotating solutions depend on the rotation coordinate as well as the radial coordinate. To overcome this difficulty we try to find the near horizon geometry of the rotating black holes. This will simplify the equations of motion since, as we will see in what follows, the functional form of the solutions with respect to the radial coordinate will be fixed, so it remains to find their dependence on the rotation coordinate.

3.1. *Extremality*

To find the radial behavior of the extremal solutions we start to find the extremality condition for the general solution found in (2.3). We first find the temperature of the solution (2.3). The temperature of this black hole can be computed by finding the surface gravity at the horizon. The result will be

$$\begin{aligned} T &= \frac{1}{2\pi}\left(2\Lambda_W(n-1)r_0^{-2n+2} + \left(\frac{3}{2}n - 1\right)Mr_0^{-n}\right) \\ &= \frac{\Lambda_W\left(\frac{n}{2}-1\right)}{2\pi}\left(-\frac{M}{\Lambda_W}\right)^{\frac{2(n-1)}{n-2}}\,, \end{aligned} \tag{3.1}$$

where the last equality comes from the fact that the location of the horizon is at $r_0 = r_+$. The extremality condition holds when the temperature is zero, and so we find the critical value of M for an extremal solution to be zero. In this way the geometry of the extremal solution is

$$ds^2 = r^{4(1-n)}dt^2 - \frac{dr^2}{\Lambda_W r^2} + r^2(d\theta^2 + d\phi^2)\,. \tag{3.2}$$

3.2. *Two derivative solutions*

Before we start to solve the equations of motion, we consider the special case where the equations of motion only contain up to two derivative terms. In this case we expect to find the known solutions for Einstein gravity. The solution to the equations of motion for Einstein gravity is given by[35]

$$ds^2 = -N^2dt^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}\,d\theta^2 + \frac{\Sigma^2}{\rho^2}(d\phi - \varpi dt)^2\,, \tag{3.3}$$

with

$$\begin{aligned} &\rho^2 = r^2 + a^2\theta^2\,, &&\Delta_\theta = 1 + \frac{a^2}{l^2}\theta^4\,, &&\Delta_r = a^2 - 2Mr + \frac{r^4}{l^2}\,, \\ &\Sigma^2 = r^4\Delta_\theta - a^2\theta^4\Delta_r\,, &&\varpi = \frac{\Delta_r\theta^2 + r^2\Delta_\theta}{\Sigma^2}a\,, &&N^2 = \frac{\rho^2\Delta_\theta\Delta_r}{\Sigma^2}\,, \end{aligned} \tag{3.4}$$

where a is the rotation parameter and in our notation $l^2 = -\frac{2}{\Lambda_W}$. We are interested to find the extremal solution and its near horizon geometry. The extremal condition holds when

$$M = \frac{1}{2}\frac{a^2l^2 + r_0^4}{r_0 l^2}\,, \quad r_0^2 = \frac{1}{\sqrt{3}}al\,, \tag{3.5}$$

where r_0 is the location of the horizon. To find the near horizon geometry we need to change our variables to some new dimensionless coordinates as follows:

$$r = r_0 + \frac{\epsilon}{y}a\,, \quad t = \frac{c_0}{\epsilon}\tau\,, \quad \phi = \hat{\phi} + \frac{\sqrt{3}c_0}{l\epsilon}\tau\,, \quad c_0^2 = \frac{r_0^2}{12}\,. \tag{3.6}$$

Sending $\epsilon \to 0$ one finds the following metric

$$\begin{aligned} ds^2 = \left(1 + \frac{a^2\theta^2}{r_0^2}\right)\Bigg(&-\frac{1}{2\sqrt{3}}\frac{a^3}{ly^2}d\tau^2 + \frac{l^2}{6y^2}dy^2 \\ &+ \frac{r_0^2}{1 + \frac{a^2}{l^2}\theta^4}d\theta^2 + r_0^2\frac{1 + \frac{a^2}{l^2}\theta^4}{\left(1 + \frac{a^2}{r_0^2}\theta^2\right)^2}\left(d\hat{\phi} + \frac{a}{ly}d\tau\right)^2\Bigg)\,. \end{aligned} \tag{3.7}$$

This is the near horizon geometry of the rotating black holes with toroidal symmetry. We expect that it satisfies the equations of motion up to two derivative terms. As a double check, we have inserted this solution into the equations of motion and found they satisfy these equations.

3.3. *Higher derivative solutions*

We are interested to find the effect of higher curvature terms. To find the solutions we follow the following steps.

(i) In the rotating solutions the Cotton tensor is not necessarily zero and this makes the problem difficult to solve. To find the rotating solution, we consider the slow rotation condition, i.e. $a \ll l$ as a parameter of perturbation and solve the equations of motion up to $\mathcal{O}(a)$.

(ii) To find the extremal rotating solution we use the tree-level solution (3.7) as a guide. We start from the ansatz

$$ds^2 = -\frac{A_1^2(\theta)}{y^2}d\tau^2 + \frac{A_2(\theta)}{y^2}\,dy^2 + A_3(\theta)d\theta^2 + A_4(\theta)\left(d\hat{\phi} + \frac{a}{ly}d\tau\right)^2 , \quad (3.8)$$

where y is the radial near horizon coordinates and the other functions in the metric are some general functions. This metric satisfies the equation of motion coming from variation of the Lagrangian with respect to N, the laps function. So we just need to insert this general ansatz into the other equations of motion coming from variation with respect to the shift functions N^i and the metric g^{ij}.

(iii) One may notice that we have a freedom for time scaling in the metric. We have fixed this by choosing the above proper off-diagonal term.

(iv) There is another freedom when one chooses the function $A_3(\theta)$. Because this is only a field redefinition, all different functions of θ will be equivalent by a change of coordinate on θ. To fix this freedom we assume the functional form

$$A_3(\theta) = r_0^2 \frac{1 + \frac{a^2}{r_0^2}\theta^2}{1 + \frac{a^2}{l^2}\theta^4} , \quad (3.9)$$

where we have chosen it in such a way that we can compare the new metric with the previous two derivative cases.

(v) To solve the equations of motion perturbatively in terms of the rotating parameter a, we choose polynomial functions with unknown constant coefficients as

$$\begin{aligned} A_1^2(\theta) &= s_1 a^3(1 + b_1 a\theta^2)\,, \\ A_2(\theta) &= s_2(1 + b_2 a\theta^2)\,, \\ A_4(\theta) &= s_4 a(1 + b_4 a\theta^2)\,, \end{aligned} \quad (3.10)$$

where we have used the fact that we have a symmetry under $(\theta \leftrightarrow -\theta)$.

(vi) Similar to (3.7) the regularity condition implies (see e.g. Ref. 36)

$$A_1(\theta)A_2^{\frac{1}{2}}(\theta) \to \text{const}\,, \quad \frac{A_3(\theta)}{A_4(\theta)} \to 1\,, \quad \theta \to 0\,, \quad (3.11)$$

which gives a simple constraint as $s_4 = \frac{r_0^2}{a}$.

(vii) Similar to the Einstein gravity solution, we assume that $r_0^2 = za$ with z as a function of the constants of the Hořava gravity. In fact, this is nothing but the normalization for $A_3(\theta)$.

By considering all these facts, we find the four algebraic equations (see App. A). As can be seen there are four equations and s_1, s_2, b_1, b_2, b_4 and $s_4 = z$ as unknown constants. There are no more constraints left since we have used all symmetries and boundary conditions

3.3.1. *w-independent solution*

One amazing observation of the equations shows that when $b_2 = b_4$ then the constants are independent of w. At this step even before solving the equations of motion one can verify that the Cotton tensor is zero for this ansatz. To find the solution to the equations of motion we follow the following steps. We begin by solving the first three equations and find the following values for s_1, b_1 and b_2:

$$
\begin{aligned}
b_1 &= \frac{-\frac{2}{3}(l^2 - 6s_2)}{\left(\lambda - \frac{1}{3}\right) l^2 z s_2((\lambda - 1)l^2 + 4s_2)} \\
&\quad \times \left((\lambda - 1)^2 l^4 - \frac{3}{2}(\lambda - 1)\left(z^2 - \frac{8}{3}s_2\right) l^2 + 12\left(\lambda - \frac{5}{6}\right) z^2 s_2 \right), \\
b_2 &= -\frac{2z(l^2 - 6s_2)}{l^2((\lambda - 1)l^2 + 4s_2)}, \\
s_1 &= \frac{1}{8}\frac{(l^2(\lambda - 1) + 4s_2)^2 z}{((\lambda - 1)l^4 + 8l^2 s_2 - 24s_2^2)s_2},
\end{aligned}
$$

and then we insert these values into the fourth equation, which gives the following equation for s_2:

$$
(\lambda - 1)l^6 + (-8\lambda + 12)s_2 l^4 - 48l^2 s_2^2 + 96s_2^3 = 0 \tag{3.12}
$$

which is independent of z. The above solution is a family of one parameter solutions (only depend on z) with zero Cotton tensor. In the special case of $\lambda = 1$ the above solution will simplifies to

$$
b_1 = b_2 = \frac{z}{l^2}\frac{1 \pm \sqrt{3}}{1 \pm \frac{\sqrt{3}}{3}}, \quad s_1 = \frac{z}{l^2(1 \mp \sqrt{3})}, \quad s_2 = \frac{1}{4}\left(1 \pm \frac{\sqrt{3}}{3}\right) l^2. \tag{3.13}
$$

As a special point on this family of solutions and as an example one may choose $b_1 = b_2 = b_4$. Again if we solve the equations of motion we will find the following values for a general value of λ after solving the first three equations (in Appendix):

$$
b_1 = b_2 = b_4 = -\frac{4}{3z},
$$

$$s_1 = \frac{9}{64}\frac{(-1+3\lambda)z^5(4l^2+9z^2)}{l^2\big((\lambda-1)l^2-\frac{3}{2}z^2\big)\big((\lambda-1)l^4-3l^2z^2-\frac{27}{8}z^4\big)}\,,$$

$$s_2 = -\frac{1}{2}\frac{l^2\big(2l^2(\lambda-1)-3z^2\big)}{4l^2+9z^2}\,.$$

Inserting the above values into the fourth equation gives an equation for z:

$$(\lambda-1)^2l^6-6\left(\lambda-\frac{3}{4}\right)l^4z^2-\frac{27}{4}(\lambda-1)l^2z^4+\frac{81}{32}z^6=0\,. \tag{3.14}$$

This equation shows that the location of the horizon depends on λ and l. Again the special case $\lambda=1$ gives the values found in (3.13) with $z=\pm\frac{2}{3^{\frac{3}{4}}}\sqrt{\pm l^2}$.

3.3.2. *w-dependent solution*

In general, when one chooses $b_2\neq b_4$, the constant values will be w-dependent. In this case one may solve the equations of motion and find the first three equations for s_1, s_2 and b_1 in terms of b_2, b_4 and z. Putting them into the fourth equation gives a relation between the remaining free parameters. This an equation of degree 8 for z, 6 for b_2 and 5 for b_4 — so impossible to solve!

To find a solution we restrict ourselve to a special limit of parameters. One possible solution could be found as a series of $\frac{1}{w^4}$. Also we consider the location of the horizon r_0, to be independent of w and its value is the same as w-independent solution. With these simplifications we find the following solution to the order of $\mathcal{O}\big(\frac{1}{w^4}\big)$, in the case of $\lambda=1$,

$$\begin{aligned}
b_1 &= -\frac{2}{3^{\frac{1}{4}}l}\left(1+\frac{x_1}{w^4}\right),\\
b_2 &= -\frac{2}{3^{\frac{1}{4}}l}\left(1+\frac{x_2}{w^4}\right),\\
b_4 &= -\frac{2}{3^{\frac{1}{4}}l}\left(1+\frac{x_4}{w^4}\right),\\
s_1 &= \frac{2}{3^{\frac{1}{4}}l(3+\sqrt{3})}\left(1+\frac{y_1}{w^4}\right),\\
s_2 &= \frac{l^2(3+\sqrt{3})}{12(2+\sqrt{3})}\left(1+\frac{y_2}{w^4}\right),
\end{aligned} \tag{3.15}$$

with

$$\begin{aligned}
x_1 &= -\frac{1}{13}\big(105\sqrt{3}+217\big)y_2\,, & x_2 &= -\frac{1}{13}\big(45\sqrt{3}+67\big)y_2\,,\\
x_4 &= -\big(5\sqrt{3}+7\big)y_2\,, & y_1 &= \frac{1}{13}\big(62\sqrt{3}-27\big)y_2\,,
\end{aligned} \tag{3.16}$$

where the constant y_2 although is arbitrary but can be absorbed in w by a rescaling, so we can set it to one. As we see this will produce the w-independent solution when one sends w to the infinity.

4. Conclusion

In this paper we have studied the toroidal solutions for the nonrelativistic and renormalizable theory of gravity proposed by Hořava.[1] We solved equations of motion by using an ansatz with toroidal symmetry. We show our results for general parameters in the theory and in "detailed balance."

The static case found in Ref. 16 shows the existence of black hole solutions where their location of horizon depends on the parameters of the theory, when $\Lambda_W < 0$. It shows that for $\Lambda_W > 0$ we have naked singularities.

In this paper we find the near horizon geometry of the rotating black hole solutions with small rotating parameter a with respect to $l = \sqrt{-\frac{2}{\Lambda_W}}$. So our solution is a series solution in terms of a. Also we have assume the θ to $-\theta$ symmetry. By imposing these constraints we have found a set of algebraic equations of motion. There is an interesting observation in our solutions to equations of motion. There are two types of solutions. The first one is independent of w parameter and the Cotton tensor for this solution is zero. The other solution depends on w and at $w \to \infty$ this solution returns to the first solution.

Comparing these results with those found in the two-derivative case, one observes that the location of the horizon is shifted due to the higher derivative corrections. This is in agreement with the results found for the spherical solutions in Ref. 2.

It will be interesting to find the exact rotating solution. In this case the metric will be a complicated function of θ and y. In finding our solutions we have made several assumptions, $a \ll l$, θ to $-\theta$ symmetry and in the case of w-dependent solution, the location of the horizon is considered to be independent of w. In general, there is no reason to have these constraints in the exact solution. So the near horizon of the exact solution just with the above assumptions must agree with our solutions. Note that the regularity condition and the field redefinitions for t and θ must hold in the exact solution.

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Appendix A

$$\left\{\left(\left({b_2}^2 - b_2 b_4 + {b_4}^2\right)\lambda - \frac{1}{2}{b_4}^2 - \frac{1}{2}{b_2}^2\right) l^4 + 2z(b_2 + b_4) l^2 - 6z^2\right\} s_1 s_2 + \frac{1}{4}(-1 + 3\lambda) z^3 = 0\,,$$

$$
\begin{aligned}
&l^2\kappa^4 s_1 s_2\bigg\{-\frac{19}{3}+\left(b_1-5b_2-\frac{16}{3}b_4\right)z \\
&\quad+\left(-\frac{14}{3}{b_2}^2+\left(b_1-\frac{11}{3}b_4\right)b_2+(b_1-5b_4)b_4\right)z^2\bigg\} \\
&\quad+\frac{4}{3}\bigg\{\frac{1}{8}(3\lambda-1)z^4+\bigg[-3z^3+l^2(b_1+b_4)z^2 \\
&\quad+\left(\left[-\frac{9}{2}{b_2}^2+\left(b_1+\frac{1}{2}b_4\right)b_2-\frac{1}{2}(b_1-5b_4)b_4\right]\lambda\right. \\
&\quad\left.+\frac{9}{4}{b_2}^2-\frac{1}{4}{b_4}^2-\frac{1}{2}(b_1+b_4)b_2\right)l^4 z \\
&\quad-4l^4\left(\left(b_2-\frac{1}{2}b_4\right)\lambda-\frac{1}{2}b_2\right)\bigg]s_1 s_2\bigg\}\frac{z^2w^4}{(3\lambda-1)(b_2-b_4)}=0\,, \\
&l^2\kappa^4 s_1(-4z+s_2(b_2-b_4))(1+(b_2+b_4)z) \\
&\quad+2\bigg\{\bigg[\left([b_2-(2b_2-b_4)\lambda]z-\left[\frac{1}{2}\left({b_2}^2+{b_2}^2\right)\lambda-\left({b_2}^2-b_2b_4+{b_4}^2\right)\right]s_2\right)l^4 \\
&\quad-2l^2z^2+6s_2z^2\bigg]s_1+\frac{3}{4}z^3\left(-\frac{1}{3}+\lambda\right)\bigg\}\frac{z^2w^4}{(3\lambda-1)(b_2-b_4)}=0\,, \\
&6l^2 s_1\kappa^4\bigg\{\frac{4}{3}(b_2+b_4)z^3+\left[\frac{4}{3}+\left(-5{b_2}^2+\left(b_1-\frac{11}{3}b_4\right)b_2+\left(b_1-\frac{14}{3}b_4\right)b_4\right)s_2\right]z^2 \\
&\quad+s_2\left(b_1-\frac{16}{3}b_2-5b_4\right)z-\frac{19}{3}s_2\bigg\} \\
&\quad+4\bigg\{\bigg[(-2l^2+6s_2)z^3+[\lambda(b_2+b_4)l^2-2s_2(b_1+b_2)]l^2z^2 \\
&\quad+s_2\left((9{b_4}^2+(-2b_1-b_2)b_4+b_2(-5b_2+b_1))\lambda-\frac{9}{2}{b_4}^2+(b_1+b_2)b_4+\frac{1}{2}{b_2}^2\right)l^4z \\
&\quad-4((b_2-2b_4)\lambda+b_4)s_2l^4\bigg]s_1+\frac{3}{4}(3\lambda-1)z^4\bigg\}\frac{z^2w^4}{(3\lambda-1)(b_2-b_4)}=0\,.
\end{aligned}
$$

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